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UPPER LIMITS ON THE ISOTROPIC GRAVITATIONAL RADIATION BACKGROUND FROM PULSAR TIMING ANALYSIS[1]

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ABSTRACT

A pulsar and the Earth may be thought of as end masses of a free-mass gravitational wave antenna in which the relative motion of the masses is monitored by observing the Doppler shift of the pulse arrival times. Using timing residuals from PSR 1133+16, 1237+25, 1604−00, and 2045−16, an upper limit to the spectrum of the isotropic gravitational radiation background has been derived in the frequency band 4×10^{-9} to 10^{-7} Hz. This limit is found to be $S_E = 10^{21} f^3$ ergs cm^{-3} Hz^{-1}, where S_E is the energy density spectrum and f is the frequency in Hz. This would limit the energy density at frequencies below 10^{-8} Hz to be 1.4×10^{-4} times the critical density.

Subject headings: cosmology — gravitation — pulsars

I. INTRODUCTION

The possibility that the universe could be presently filled with a measurable amount of stochastic gravitational radiation of cosmic origin has been recently considered (Zimmerman and Hellings 1980; Carr 1980; Adams *et al.* 1982). Zimmerman and Hellings find from analysis of late-time cosmological observations that this energy density could be as large as $\rho_g = 3.4\ \rho_c$, where $\rho_c \equiv 3\ H^2/(4\pi G)$ represents the critical density for a closed universe. Carr, on the other hand, notes that early-time nucleosynthesis models require a much more stringent $\rho_g \leq 10^{-4}\rho_c$. A catalog of existing direct experimental limits on the gravitational wave background (Zimmerman and Hellings 1980) finds several experiments whose sensitivities are within an order of magnitude of the 3.4 ρ_c limit without actually achieving a sensitivity which would be cosmologically significant, i.e., which could detect the cosmic background if it existed at the maximum 3.4 ρ_c level or whose null result would place new upper limits on the strength of the background.

It was first pointed out by Detweiler (1979) that a gravitational wave antenna can be formed by using the Earth and a pulsar as two free masses and monitoring their apparent motion by noting the arrival time of the pulses. An upper limit to the stochastic background is found by attributing the excess noise in the measured phase of the received pulse-train to gravitational radiation. We report here results of an analysis of new pulsar timing data. This new technique for detecting gravitational waves has succeeded in limiting the strength of the background at wave frequencies of $10^{-8.4}$ to 10^{-7} Hz to be less than the critical density by several orders of magnitude. A more complete paper, describing the details of the data analysis and discussing more fully the cosmological implications of the results, will be published in the near future (Hellings and Downs 1983).

II. DATA

The data used in this analysis are the measured arrival times of pulses from 24 pulsars, observed regularly at intervals of 1 week to 1 month over a period of about 12 years using facilities of the NASA/JPL Deep Space Network. Each data point represents a signal integration of several minutes to over 1 hour, depending on signal strength. The pulse trains were sampled typically at intervals of 50 μs to 1 ms, subsequently adding the samples modulo the pulse period. The phase of the pulse train relative to a given epoch was determined from the cross correlation of the measured pulse shape and a standard pulse-shape template. After applying several instrumental corrections, a topocentric arrival time was obtained. These arrival times were reduced to effective solar system barycentric arrival times by using the tracking station locations and Earth positions derived from JPL ephemeris DE96. Further details on the experimental techniques can be found in the paper by Downs and Reichley (1983).

The expected barycentric arrival time, t_n, of the nth pulse following epoch t_0 is given by

$$t_n = t_0 + nP_0 + \tfrac{1}{2}n^2\dot{P}_0P_0 + \tfrac{1}{6}n^3\ddot{P}_0P_0^2,$$

where P_0 is the pulsar period at time t_0 and $\dot{P}_0$ and $\ddot{P}_0$

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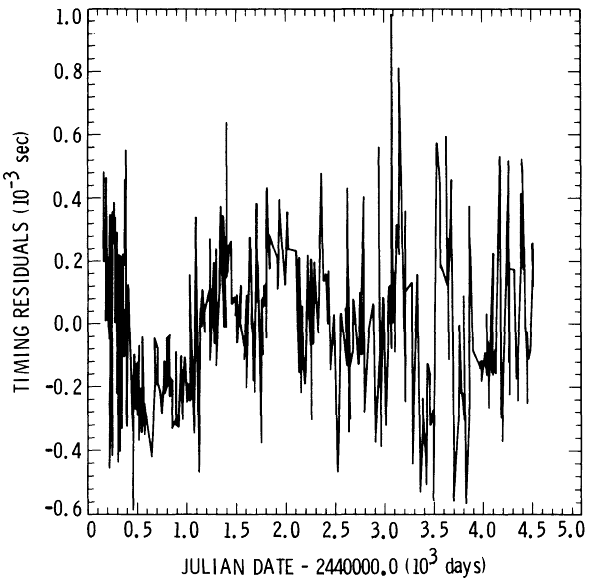

FIG. 1.—Timing residuals for PSR 1133+16

are the first and second derivatives, respectively, of P at t_0. Effects due to pulsar position, proper motion, and an arbitrary time origin are presumed removed.

The parameters t_0, P_0, $\dot{P}_0$, and $\ddot{P}_0$ are fitted to the data in a least-squares sense, resulting in a predicted t_n. Residuals, Δt_n, are formed by subtracting t_n from the measured arrival time of the nth pulse. Residuals are usually nonzero, with root mean square (rms) values of between 10 μs and 2 ms. Receiver noise fluctuations account for the residuals in *only* the weakest pulsars, while rms residuals several times that expected from receiver noise alone are found in moderate to strong pulsars. Residuals for pulsar PSR 1133+16 are shown in Figure 1.

III. ANALYSIS TECHNIQUE

a) *Gravitational Perturbations*

A weak gravitational wave traversing the Galaxy will affect the period, P, of an emitted pulse train such that the arrival time of a particular pulse will be perturbed from the arrival time expected in wave-free space. For a plane wave traveling in the positive z-direction with amplitude $h(t - z)$, we compute the fractional change in the pulse frequency, $\nu \equiv 1/P$, to be (see Estabrook and Wahlquist 1975; Hellings 1983)

$$\frac{\Delta \nu}{\nu} = \tfrac{1}{2} \cos 2\phi [1 - \cos\theta] \times [h(t) - h(t - l - l\cos\theta)], \qquad (1)$$

where l is the Earth-pulsar distance at an angle θ to the propagation direction (z-axis), and ϕ is the angle between a principle polarization vector of the wave and the projection of the pulsar position on the transverse plane (the x-y plane). The effect of a gravitational wave on the timing data is to induce fluctuations proportional to $h(t)$ in the time derivative of the phase residuals in Figure 1. Hidden at some level in the data of Figure 1 is a small stochastic contribution due to the cosmic gravitational radiation background.

In principle, gravitational waves with periods of up to the typical one-way light time between the Earth and the pulsar ($\sim 10^3$ yr) can be seen in the data. However, the total span of the data covers only a dozen years, so there is an upper limit of about 10^8 s for the periods to which the data are sensitive. It should also be noted from equation (1) that data from any pulsar contain information about $h(t)$ at the time and place of reception (i.e., at Earth) and about the value $h(t)$ had at the pulsar at the time of emission of the signal. Thus, data from any pulsar will have a gravitational wave signal in common with all other pulsars (though with an amplitude scaled by $1 - \cos\theta$) as well as a component of the signal which will be independent of the others due to the long light times between pulsars compared with the 12 yr data span. When data from several pulsars are cross-correlated, this common signal will allow one to dig into the pulsar noise to detect a possible common gravitational wave signal.

b) *Cross-Correlation*

The fractional frequency shift observed in the data on pulsar number i may be written

$$\frac{\Delta \nu_i(t)}{\nu_i} = \alpha_i h(t) + n_i(t), \qquad (2)$$

where $h(t)$ is the gravitational wave signal common to all pulsars (the $h(t)$ term on the right of eq. [1]), α_i gives the angle factor $\frac{1}{2}\cos 2\phi\ (1 - \cos\theta)$ for the ith pulsar, and $n_i(t)$ represents all noise sources unique to the pulsar, including the $h(t - l - l\cos\theta)$ term in equation (1) since each pulsar will be at a unique l_i and θ_i. It is assumed that $h(t)$ is due to an isotropic background and is continuous and stochastic. The purpose of our data analysis will be to isolate the power spectrum of $h(t)$ in the data.

When data from two pulsars are cross correlated, the result is

$$C_{ij}(\tau) = \alpha_i \alpha_j \langle h^2 \rangle + \alpha_i \langle h n_j \rangle + \alpha_j \langle n_i h \rangle + \langle n_i n_j \rangle, \qquad (3)$$

where

$$\langle h^2 \rangle \equiv \frac{1}{T} \int_{-(T-\tau)}^{T-\tau} h(t) h(t+\tau)\, dt \qquad (4)$$

and T is the length of the data span. Definitions similar to equation (4) apply for the other three terms in equation (3).

Equation (3) is valid for gravitational waves arriving from a given direction, so that α_i and α_j are well-defined numbers. To compute the cross-correlation function for two pulsars responding to an isotropic distribution of gravitational radiation, we assume that $\langle h^2 \rangle$ is independent of direction and compute the average, α_{ij}, of the angular factors $\alpha_i \alpha_j$:

$$\alpha_{ij} \equiv \frac{1}{4\pi} \int \alpha_i \alpha_j \, d\Omega = \frac{1 - \cos\gamma_{ij}}{2} \ln\left(\frac{1 - \cos\gamma_{ij}}{2}\right) - \frac{1}{6}\frac{1 - \cos\gamma_{ij}}{2} + \frac{1}{3}, \quad (5)$$

where γ_{ij} is the angle between the two pulsars.

Since the processes n_i, n_j, and $h(t)$ should all be uncorrelated over the 12 year data span, the last three terms in equation (3) should go to zero as T^{-1}. The cross correlation function may therefore be written as

$$C_{ij} = \alpha_{ij}\langle h^2 \rangle + \delta C_{ij}, \quad (6)$$

where δC_{ij}, the estimation error due to a noninfinite T, results from the last three terms of equation (3). While $\langle h^2 \rangle$ is an even function of τ, δC_{ij} will not have any particular symmetry. The noise in C_{ij} may therefore be further reduced by averaging negative and positive time lags to form

$$C'_{ij} \equiv \tfrac{1}{2}\left[C_{ij}(\tau) + C_{ij}(-\tau)\right] = \alpha_{ij}\langle h^2 \rangle + \delta C'_{ij}, \quad (7)$$

in which $\delta C'_{ij}$ is expected to be smaller than δC_{ij} by a factor of $\sqrt{2}$.

The angle factor α_{ij} scales the strength of the signal in each cross-correlation function. If several cross-correlation functions are combined in a weighted average,

$$\bar{C}(\tau) \equiv \frac{\sum \alpha_{ij} C'_{ij}}{\sum |\alpha_{ij}|} = \frac{\sum (\alpha_{ij})^2}{\sum |\alpha_{ij}|}\langle h^2 \rangle + \frac{\sum \alpha_{ij}\,\delta C'_{ij}}{\sum |\alpha_{ij}|}, \quad (8)$$

then the incoherent noise terms will be further suppressed. This is the data analysis algorithm we use below.

c) *Results*

In this *Letter*, we focus on gravitational waves of periods of a few years. We consider, therefore, those pulsars which are "quiet," i.e., are characterized by low noise in this frequency range. A future paper will analyze residuals from more pulsars and will extend the results up to higher frequencies. Data from PSR 1133+16, 1237+25, 1604−00, and 2045−16 were chosen for their low timing noise over periods of 10^7 to 10^8 s. The timing data from each pulsar were averaged into 50 day bins and differentiated to give $d(\Delta t_n)/dt = -\Delta\nu/\nu$. The resulting frequency records were then cross-correlated. The α_{ij} for each of the cross-correlations are given in Table 1. From this table it is seen that the first cross-correlation would have a relatively strong signal in it, that three others would have a somewhat weaker signal, and that two would have signals that are weaker still. The best limit came from taking the four largest α_{ij}. For these four, we have

$$\sum(\alpha_{ij})^2 / \sum|\alpha_{ij}| = 0.1620,$$

so that

$$\langle h^2 \rangle \le (0.1620)^{-1} \bar{C}(\tau). \quad (9)$$

TABLE 1

GEOMETRICAL PARAMETERS OF SELECTED PULSAR PAIRS

PULSAR PAIRS			
PSR	PSR	γ_{ij}	α_{ij}
1133+16	1237+25	17°45	0.24265
1133+16	1604−00	68°66	−0.08400
1133+16	2045−16	139°70	0.07511
1604−00	2045−16	71°04	−0.08953
1237+25	1604−00	55°87	−0.03607
1237+25	2045−16	125°32	0.01490

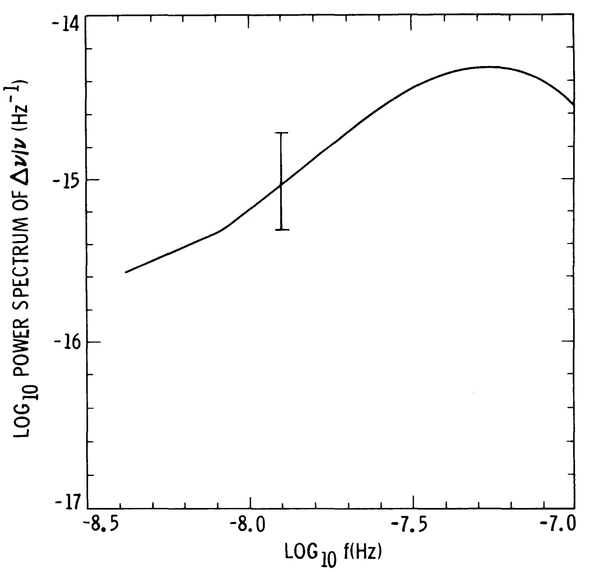

FIG. 2.—Limit on the power spectral density (S_h) of the amplitude of cosmic gravitational waves from the cross-correlations of four "quiet" pulsars. The error bar shows the rms uncertainty in the estimate of S_h.

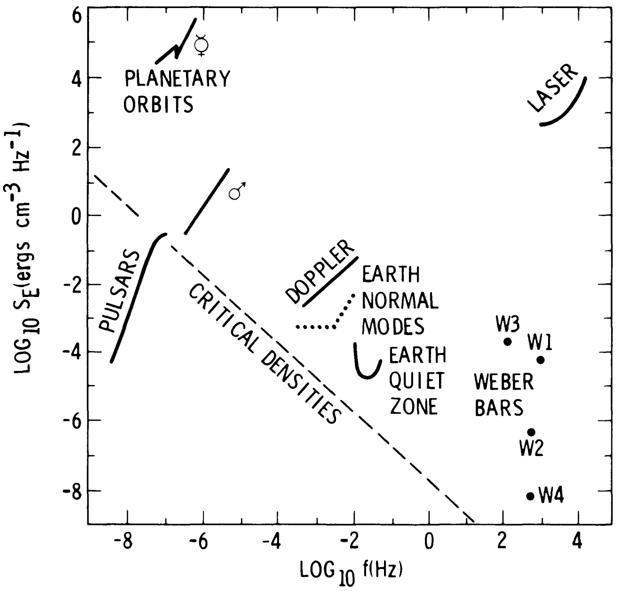

FIG. 3.—Limits on the spectrum of cosmic gravitational radiation energy density from several direct gravitational wave experiments. The line labeled "critical densities" represents the locus of peaks of a set of broad-band spectra which would each provide a critical energy density.

Computing the cosine transform of the observed $\bar{C}(\tau)$ will therefore give an upper limit to the spectrum of the gravitational wave background, since the autocovariance function, $\langle h^2 \rangle$, and the power spectrum are Fourier transform pairs. We thus have

$$S_h \leq (0.1620)^{-1} \int_0^T \bar{C}(\tau) \cos(\omega\tau)\, d\tau. \tag{10}$$

The cosine transform of the combined $\bar{C}(\tau)$, rescaled to give S_h, is shown in Figure 2. This is a smoothed estimate, with the error bar shown being approximated by the rms deviation between the smoothed and unsmoothed spectra.

IV. DISCUSSION

The impact of this result on cosmology is shown in Figure 3. Here we have formed the spectral density of gravitational wave energy density according to

$$S_E = \frac{\pi c^2}{4G} f^2 S_h \tag{11}$$

and compared it to similar results derived from several other direct gravitational wave experiments (see Zimmerman and Hellings 1980 for details). The line labeled "critical densities" represents the locus of points of the peaks of a set of broad-band gravitational wave spectra whose integrated energy density would be ρ_c. The line labeled "pulsars" is the result of the present analysis. It should be noted that it requires that the energy density in gravitational waves of frequency below 10^{-8} Hz be less than 1.4×10^{-4} times the critical density. This limit is less than 4×10^{-5} times the 3.4 ρ_c limit allowed by late-time cosmological observations. It also restricts the gravitational wave energy density in this frequency band to be less than the energy density in the microwave background radiation.

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