The Interstellar Medium

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The Interstellar Medium - Introduction

The interstellar environment is comprised of a **low-density plasma** of free electrons, known as the **interstellar medium** (ISM)

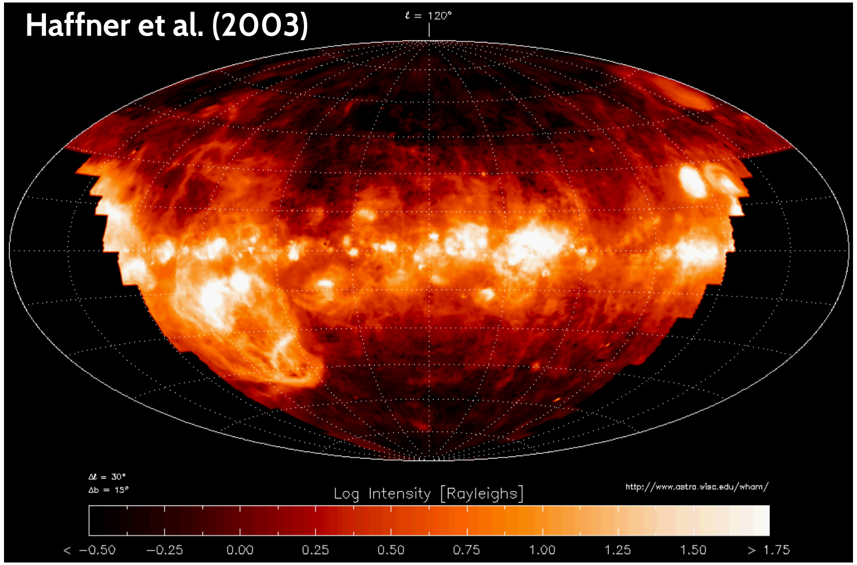
The ISM affects pulsar observations by **varying the observed pulse shape and flux** of pulsars

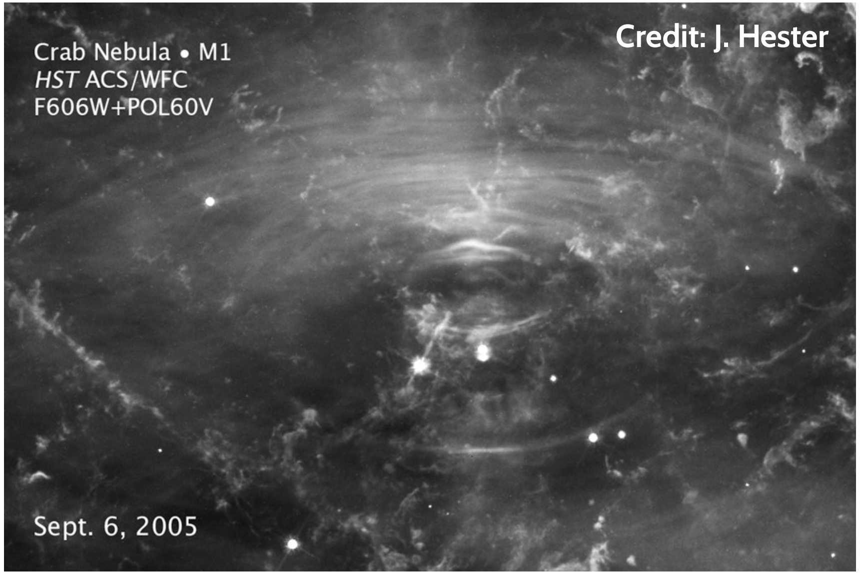
These effects allow measurements of the **total density** and **density variations** of the ISM along the line of sight to pulsars

For **precision timing** experiments, these effects must also be **mitigated**

In this talk, I'll describe the main effects of the ISM on pulse propagation:

1. Dispersion
2. Scattering
3. Scintillation





Dispersion of Radio Waves

The ISM **disperses** photons propagating through it as a function of photon frequency ν in a similar way to a prism

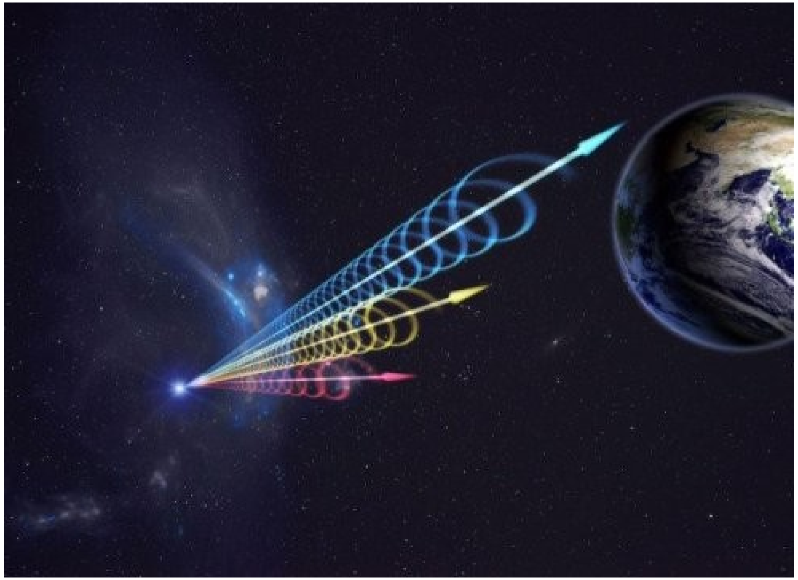
The ISM **refractive index** is given by

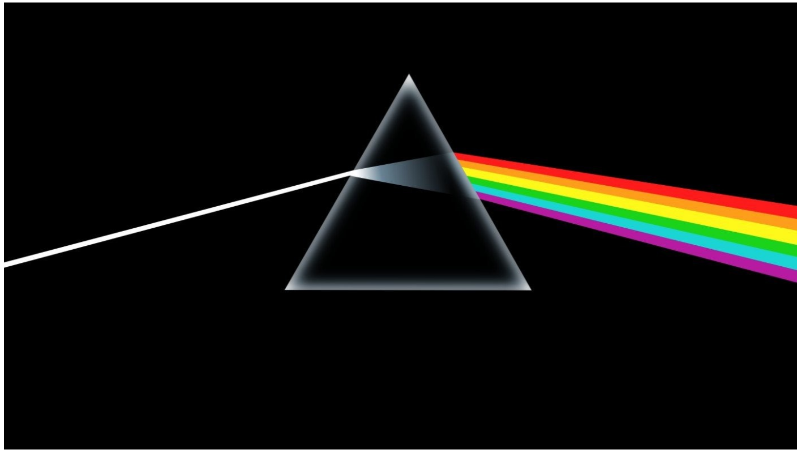
$$n = \sqrt{1 - \frac{\nu_p^2}{\nu^2}} \qquad \nu_p = \sqrt{\frac{n_e e^2}{\pi m_e}} \sim 9 \times \sqrt{n_e} \text{ kHz}$$

Using a typical electron number density value 0.05 cm^{-3}, the **plasma frequency** is ~2 kHz

i.e. **much lower** than typical observing frequencies used in radio astronomy of pulsars (~30 - 3,000 MHz).

n_e = electron number density (cm^{-3})
e = electron charge (−4.8×10^{-10} statC = 1.6×10^{-19} C)
m_e = electron rest mass (9.1×10^{-28} g)
ν_p = ISM plasma frequency
(conventional to use cgs units)





Part 1: Dispersion

Dispersion as a Function of Frequency

Photons propagating through a plasma are **dispersed** as a function of their **group velocity**

$$v_{\mathrm{g}} = cn = c\sqrt{1 - \frac{\nu_{\mathrm{p}}^2}{\nu^2}}$$

This leads to a **frequency-dependent dispersive delay** Δt along a path l, given by

$$\Delta t = \int_0^d v_{\mathrm{g}}^{-1}\,\mathrm{d}l - \frac{d}{c} = \frac{1}{c}\int_0^d \left(1 + \frac{\nu_{\mathrm{p}}^2}{2\nu^2}\right)\mathrm{d}l - \frac{d}{c}$$

Using the equation for plasma frequency, this can be simplified to

$$\Delta t = \frac{e^2}{2\pi m_{\mathrm{e}} c \nu^2}\int_0^d n_{\mathrm{e}}\,\mathrm{d}l$$

which we use to define the **dispersion measure (DM)** as

$$\boxed{\mathrm{DM} = \int_0^d n_{\mathrm{e}}\,\mathrm{d}l}$$

Dispersion as a Function of Frequency

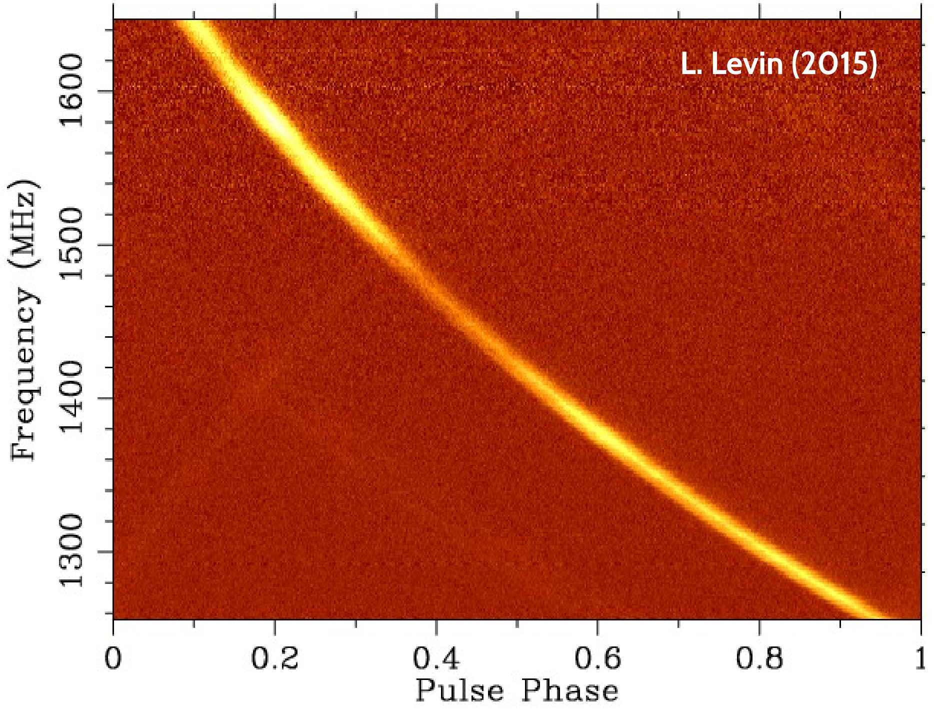
$$\boxed{\mathrm{DM} = \int_0^d n_e \, dl}$$

The DM is defined as the **integrated column density of free electrons along the line of sight**, and has units pc cm^{-3}. This allows the **frequency-dependent (in MHz) delay (in seconds)** to be written as

$$\Delta t = \frac{\mathrm{DM}}{2.41 \times 10^{-4}\, \nu^2}$$

It is useful to express this in terms of the difference between **low and high frequencies**

$$\boxed{\Delta t = \frac{\mathrm{DM}}{2.41 \times 10^{-4}} \left(\frac{1}{\nu_{\mathrm{low}}^2} - \frac{1}{\nu_{\mathrm{high}}^2} \right)}$$



For an observing bandwidth *B*, the dispersive delay causes the signal at the **low and high ends of the bandwidth to be received at different times**, by an amount (in seconds)

$$\boxed{\Delta t_{\mathrm{disp}} = 8.3 \times 10^3 \, \mathrm{DM} \, \nu^{-3} B}$$

This causes the signal to be **smeared out across the observing bandwidth** if left uncorrected

Pulsar Distance Measurements

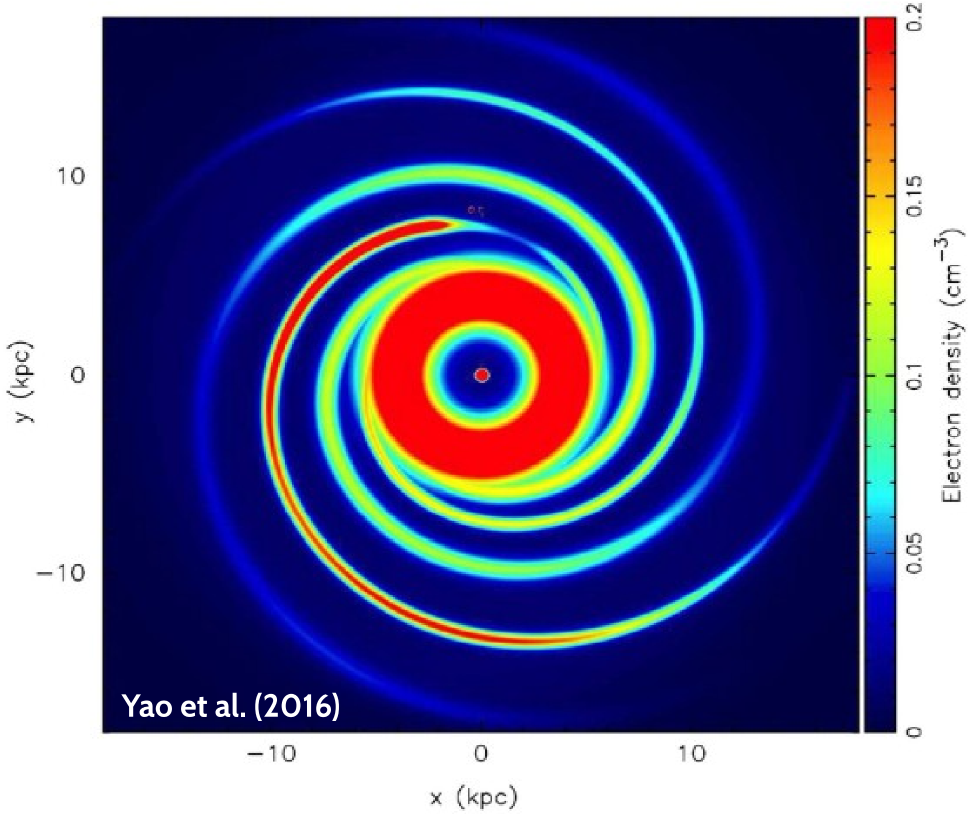
The DM is proportional to the **distance between the Earth and the pulsar**

We can therefore use the dispersive delay to **measure the average electron density of the ISM between the Earth and the pulsar**, if the distance is known (e.g. from parallax measurements

$$\boxed{n_{\mathrm{e}} = \frac{\mathrm{DM}}{d}}$$

The estimated electron density along the line of sight to the pulsar can be used to construct **models of the distribution of free electrons in the galaxy**

This can be used to **estimate distances to pulsars** which are too far away for parallax measurements to be used



Incoherent Dedispersion

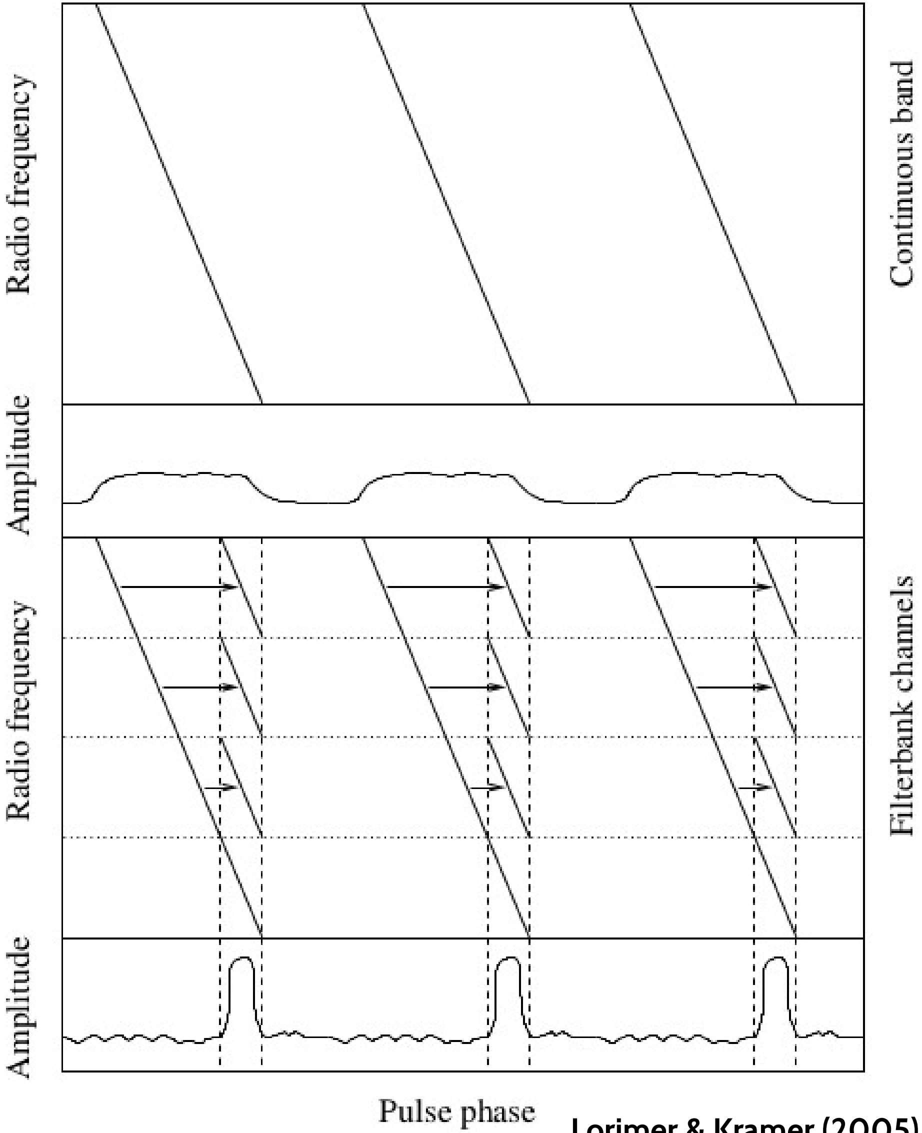
The dispersive delay can be corrected for by **dividing the observing bandwidth into sub-bands**, and applying an **appropriate delay to each channel**, so that they line up

This **removes** the inter-channel delay

This technique is known as **incoherent dedispersion**

Until approximately the mid-2000s, this was the prevalent method of removing dispersion from pulsar observations, as it is relatively **computationally inexpensive**, and can be **built into the observing backend hardware**

The **delay within each sub-band** however, **is not removed**, and will cause **profile broadening for high-DM pulsars**



Lorimer & Kramer (2005)

Coherent Dedispersion

Coherent dedispersion is a technique where the ISM effect on radio propagation is **approximated as a phase-only filter**

This allows the effect to be **modelled as a transfer function** *H*, acting on voltages $v(t)$ and $v_{int}(t)$, and **removed from data sampled at high time resolution**

In the **frequency domain**, Fourier voltages $V(f)$ and $V_{int}(f)$ are related to the transfer function $H(f)$ as

$$V(f_0 + f) = V_{\text{int}}(f_0 + f)H(f_0 + f)$$

where the **transfer function** in terms of **wavenumber** $k(f)$ and **distance** to the pulsar *d* is

$$H(f_0 + f) = \exp[-ik(f_0 + f)d]$$

Where $i = \sqrt{-1}$

The **wavenumber** of a radio signal propagating through the ISM is **frequency dependent**, and is given by

$$k(f_0 + f) = \frac{2\pi}{c}(f_0 + f)\sqrt{1 - \frac{f_p^2}{(f_0 + f)^2} \mp \frac{f_p^2 f_B}{(f_0 + f)^3}}$$

$f_p \sim 2$ kHz = plasma frequency
$f_B \sim 3$ Hz = cyclotron frequency

Coherent Dedispersion

$$k(f_0+f) = \frac{2\pi}{c}(f_0+f)\sqrt{1-\frac{f_p^2}{(f_0+f)^2} \mp \frac{f_p^2 f_B}{(f_0+f)^3}}$$

As the two fractions in the square root of the above equation have values <10^{-10} and <10^{-17} respectively for an observing frequency f = 100 MHz, these two terms can be neglected

We can then rewrite the equation as a Taylor expansion, keeping only the first terms:

$$k(f_0+f) \simeq \frac{2\pi}{c}(f_0+f)\left(1-\frac{f_p^2}{2(f_0+f)^2}\right)$$

Inserting this into the expression for the transfer function,

$$H(f_0+f) = \exp\left[\frac{2id\pi}{c}\left(f_0+f-\frac{f_p^2}{2(f_0+f)}\right)\right]$$

Coherent Dedispersion

Using the identity
$$\frac{1}{f_0+f} = \frac{1}{f_0} - \frac{f}{f_0^2} + \frac{f^2}{(f_0+f)f_0^2}$$

The transfer function can be rewritten as

$$H(f_0+f) = \exp\left\{\frac{-2id\pi}{c}\left[\left(f_0 - \frac{f_p^2}{2f_0}\right) + \left(1 + \frac{f_p^2}{f_0^2}\right) f - \frac{f_p^2 f^2}{2(f+f_0)f_0^2}\right]\right\}$$

The transfer function in this form has a **constant phase offset** as the first term, and a **time domain delay** as the second term

The third term describes a **phase rotation of the received signal**, which can be used to **recover the undispersed signal** when the inverse H^{-1} of the following transfer function is applied:

$$\boxed{H(f_0+f) = \exp\left[\frac{2id\pi}{c}\frac{f_p^2}{2(f+f_0)f_0^2}f^2\right]}$$

Or in terms of DM:

$$\boxed{H(f_0+f) = \exp\left[\mathcal{D}\frac{2\pi f^2}{(f+f_0)f_0^2}\mathrm{DM}\right]}$$

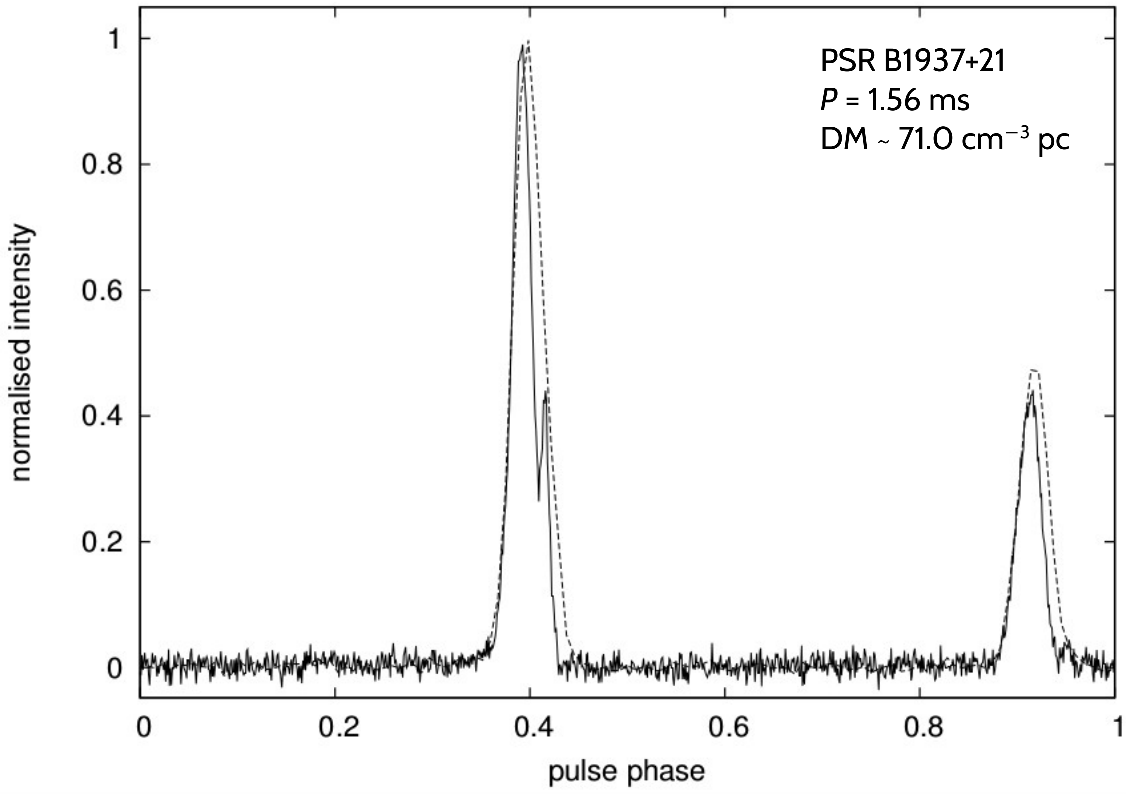
Coherent Dedispersion

Coherent dedispersion comes at the cost of being **more computationally expensive**

However, computational power is now at the point where **coherent dedispersion is the dominant technique** used in pulsar observing systems

There is **no dispersion smearing**, the dispersive effect is removed **mathematically and exactly**

The **improvement in sensitivity** compared to incoherent dedispersion is illustrated in the figure



Solid line: coherent dedispersion
Dashed line: incoherent dedispersion

Part 2: Scattering and Scintillation

Scattering of Radio Waves

Pulse scattering is caused by **inhomogeneities in electron number density** of the ISM along the line of sight

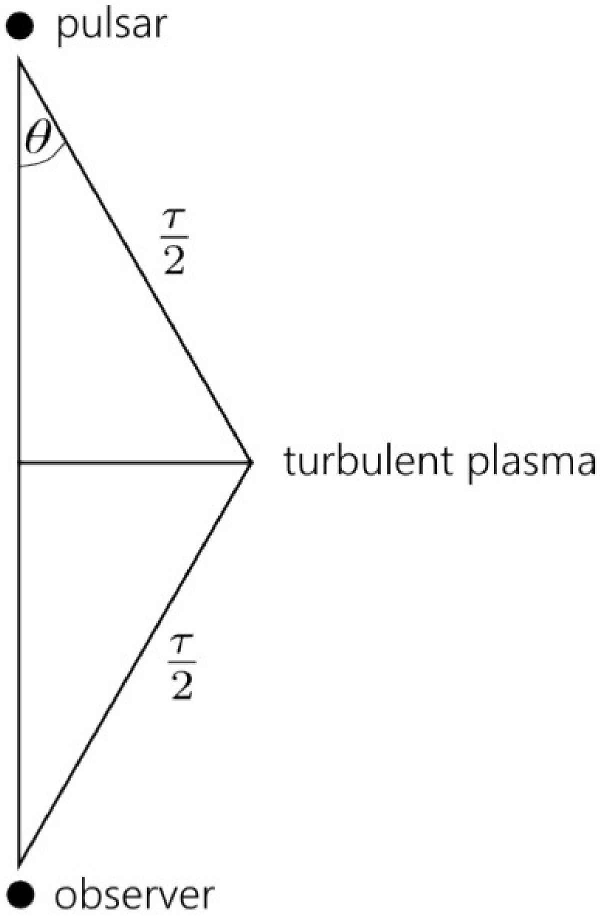
The influence of scattering on observed pulse shapes is described in a series of papers by **Williamson (1972, 1973, 1974)**, for scenarios in which the scattering is caused by different types of **scattering screen**, located **midway between the pulsar and the observer:**

1. A thin screen
2. An extended screen
3. Multiple screens

The simplest case is the **thin screen model**, which uses the approximation that all of the scattering is caused by a **thin screen located midway between the pulsar and the observer**

Although this is usually **not true in reality**, the thin screen model is often a **good approximation**

Scattering measurements allow the **inhomogeneity of the ISM** to be probed



Scattering of Radio Waves – Thin Screen Model

For a **thin scattering screen**, density irregularities along the line of sight between the observer and the pulsar scatter rays from an pulse at **random angles**

The scattered rays have a root mean square (RMS) scattering angle θ_0

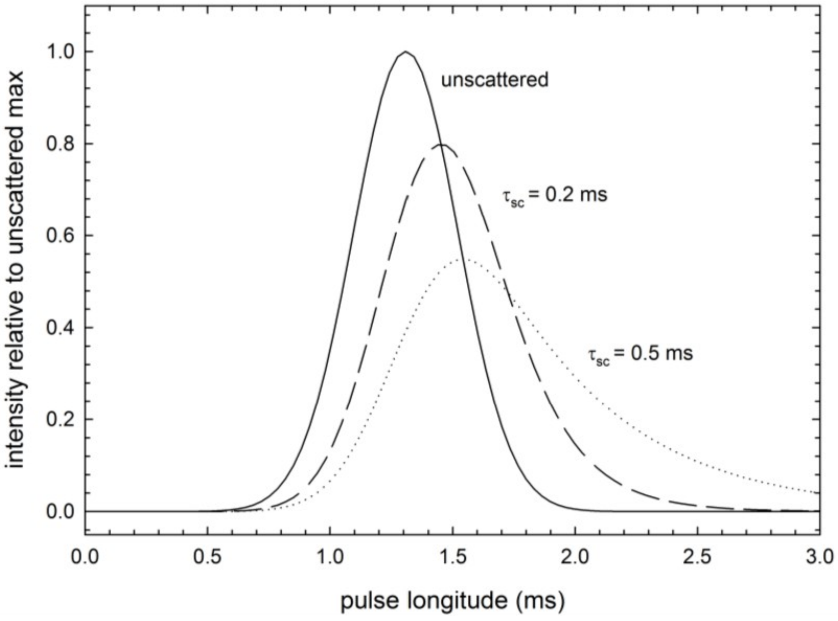
This **increases the mean distance travelled by rays** from the source to the observer and **temporally broadens the observed pulse** on the trailing edge, with an **exponential scattering tail**

The **'scattering time scale'** describes the decay constant of the scattering tail, and is given by

$$\tau_{sc} = \frac{\theta_0 D'}{2c}$$

Using the small angle approximation, the **average distance** travelled by the scattered rays is related to the distance of the scattering screen from the source by

$$D' = \frac{D(D-a)}{a}$$



D' = scattered path length

D = distance to the pulsar

c = speed of light

a = distance from the pulsar to the scattering screen (i.e. at the midpoint between pulsar and observer in the thin-screen model)

Scattering of Radio Waves – Thin Screen Model

Assuming a **Kolmogorov turbulence spectrum**, the length of the scattering tail scales with frequency as $\tau_{sc} \propto \nu^{-4.4}$

This is because scattering follows a **power law** described by

$$P(k) = C_n^2(D)k^{-\alpha}$$

where α = 4.4 for a Kolmogorov spectrum, and $k_{inner} \ll k \ll k_{outer}$

This is **proportional to the length scale over which scattering occurs** such that $k \propto \delta l^{-1}$ is the spatial wave number of the inner and outer length scales, where δl is the length scale over which variations take place

C_n^2 is the **scattering structure constant** given by

$$C_n^2 = 0.00232\ \nu^{11/3} D^{-11/6} (2\pi\tau_{sc})^{-1}$$

The **phase of the incident wave front** Φ of a scattered wave is changed according to

$$\Delta\phi \approx \sqrt{\frac{D}{D'}}\ \delta\phi = (DD')^{1/2} r_e \Delta n_e \lambda$$

$r_e = 2.8\times10^{-15}$ m = classical electron charge radius
λ = observing wavelength

Scattering of Radio Waves – Thin Screen Model

The scattering angle θ_0 is related to the **electron number density variations** by

$$\theta_0 \approx \frac{\Delta\phi\lambda}{2\pi D'} = \frac{1}{2\pi}\sqrt{\frac{D}{D'}} r_e \Delta n_e \lambda^2$$

The electron number density variation leading to an observed scattering time scale is given by

$$\Delta n_e = \left(\frac{a\tau_{sc}\nu^4 m_e^2 4\pi^2}{e^4 D}\right)^{1/2}$$

$m_e = 9.11\times10^{-31}$ kg = electron rest mass
$e = 1.6\times10^{-19}$ C = electron charge

This allows the **temporal variations in electron number density** along the line of sight to be probed, if the exponential scattering time scale can be extracted from an average pulse profile

Scattering of Radio Waves – Thin Screen Model

The **broadening of the scattered pulse** is commonly modelled mathematically by the **convolution of a function representing the intrinsic pulse with an exponential function** $g(t, \tau_{sc})$

In the **simplest case** where the intrinsic pulse is a **Gaussian** $f(t,\mu,\sigma)$, with position and width μ and σ, the scattered pulse shape is then

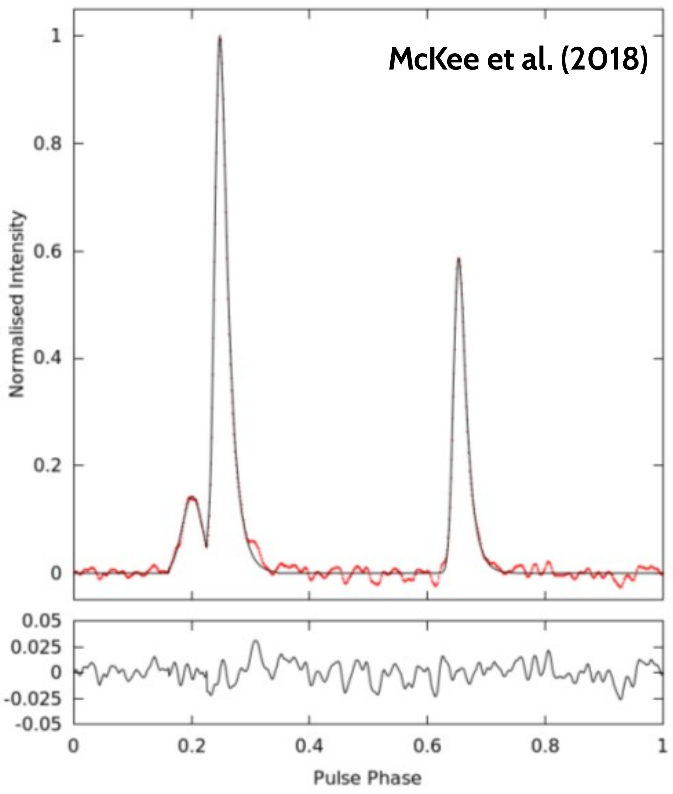
$$h(t,\mu,\sigma,\tau_{sc}) = f(t,\mu,\sigma) * g(t,\tau_{sc})$$

This convolution can be evaluated **analytically** using an **exponentially-modified Gaussian**

$$f(t,\mu,\sigma,\tau_{sc}) = h\exp\left\{\frac{-(\mu-t)^2}{2\sigma^2}\right\}\frac{\sigma}{\tau_{sc}}\sqrt{\frac{\pi}{2}}\mathrm{erfcx}\left\{\frac{1}{\sqrt{2}}\left(\frac{\mu-t}{\sigma}+\frac{\sigma}{\tau}\right)\right\}$$

with scaled complementary error function erfcx(t) and complementary error function erfc(t) terms

$$\mathrm{erfcx}(t) = \exp(t^2)\mathrm{erfc}(t) \qquad \mathrm{erfc}(t) = 1-\mathrm{erf}(t) = \frac{2}{\sqrt{\pi}}\int_t^\infty \exp(-t^2)\,\mathrm{d}x$$



This analytical form of the convolved function has been shown to **closely approximate** the shape of a scattered pulse that is intrinsically Gaussian, and fitting the function to an observed pulse allows the **parameters of the intrinsic Gaussian to be recovered**

Scintillation

Electron number density variations due to **turbulence in the ISM** cause scattered rays to interfere with each other, producing an **interference pattern** which leads to apparent **intensity variations**

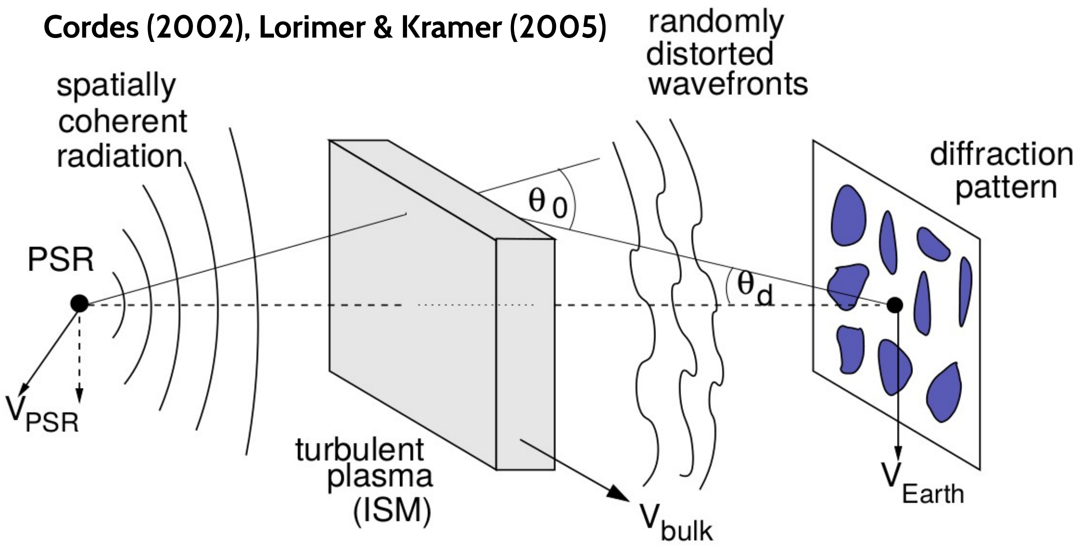
The **scintillation bandwidth** Δf_s is given by

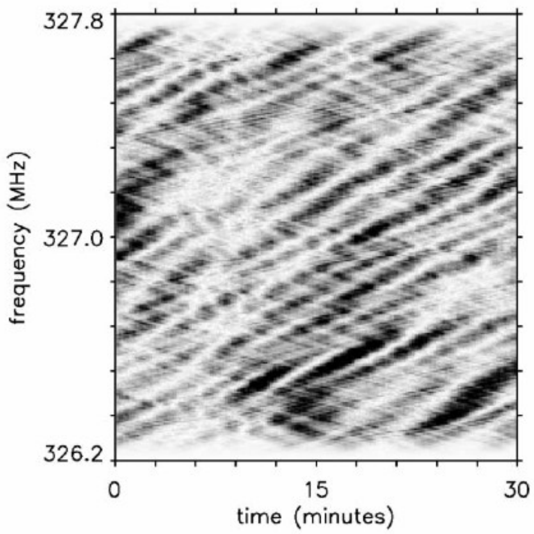
$$\Delta f_{\rm s} = \frac{8\pi^2 D' c}{D^2 (\Delta n_{\rm e})^2 \lambda^4}$$

D = distance to the pulsar
D' = scattered ray path length
λ = observing wavelength
c = speed of light

As scintillation is symptomatic of **variations in electron number density**, it allows for the **variability of the local ISM** to be studied on **short time scales** through the scattering strength structure constant $C_n^{\,2}$

Cordes (2002), Lorimer & Kramer (2005)





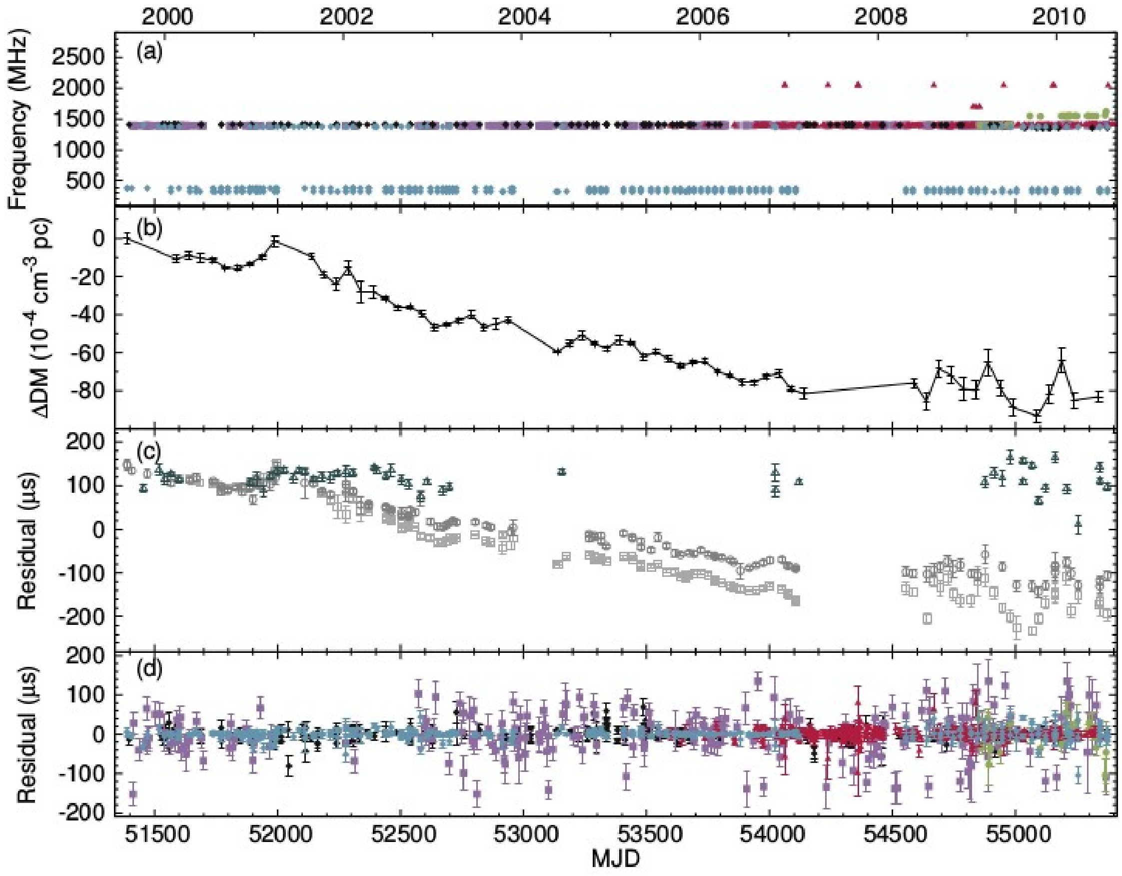
Hill et al. (2005)

Part 3: Examples

Example: Correcting for DM Variations

Measuring the **delay between TOAs at widely-separated observing frequencies** allows **variations** in the DM to be measured

If **high-precision DM measurements** can be made, it may be possible to **correct for the dispersive effect** on a TOA-by-TOA basis



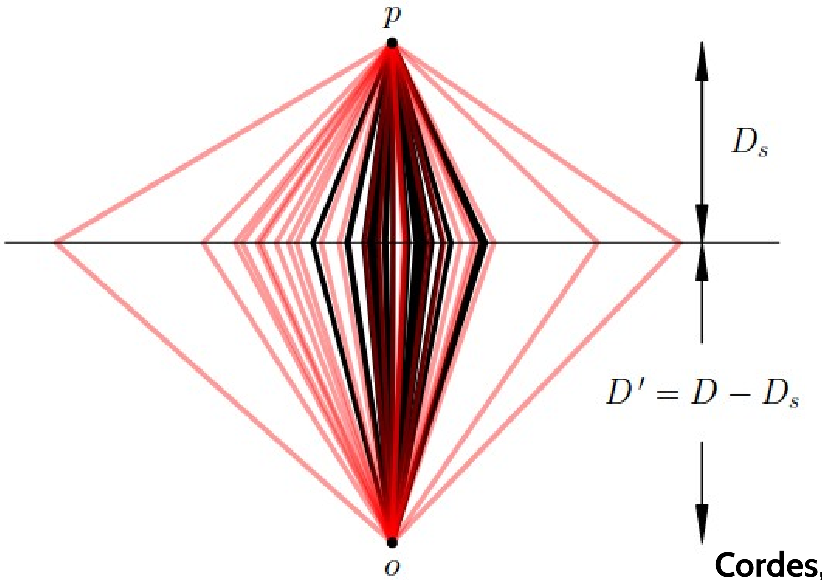
Janssen, McKee, et al. (in prep)

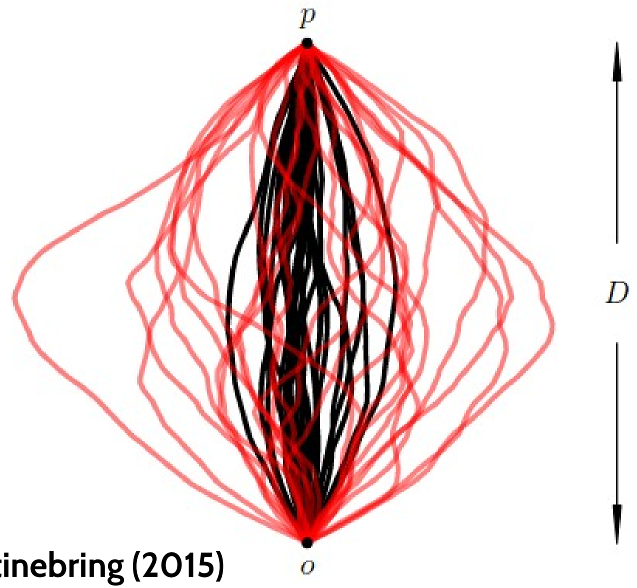
Example: Correcting for DM Variations

However, scattering causes scattered rays to sample **different paths** through the ISM

This may mean that DMs measured at **low frequencies** are **incompatible** with those at **high frequencies**

This could **prevent** high-precision DM measurements from low frequency observations being used to corrects for DM at higher frequencies





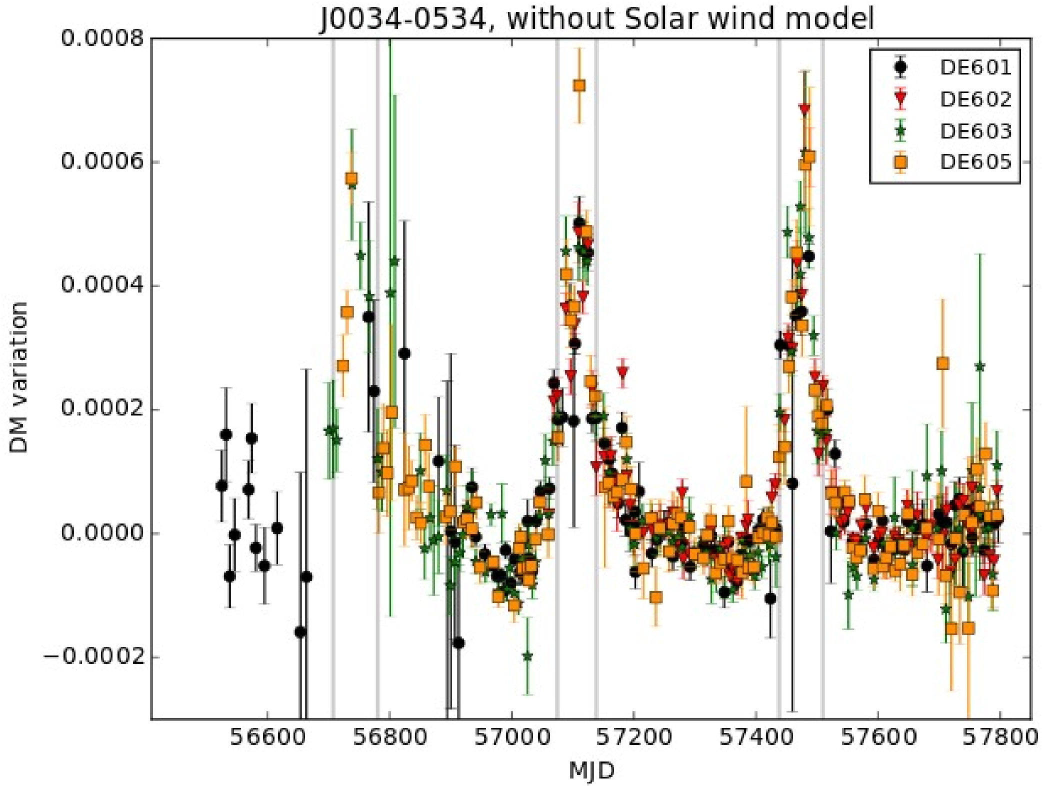
Cordes, Shannon, & Stinebring (2015)

Example: Mapping the Solar Wind

At very low frequencies, high-precision DM measurements can be made by **comparing the delay across the observing bandwidth**

In the case of **GLOW** (the German LOFAR stations used independently from the rest of the array), **high cadence observations** are made of a set of millisecond pulsars

Some of the pulsar pass **close to the sun** – this allows the DM contribution from the **Solar wind** to be mapped out



Tiburzi & Verbiest (2017)

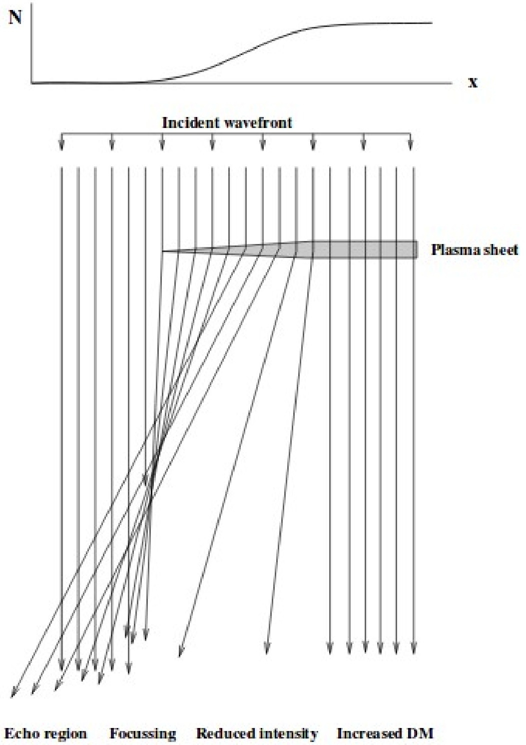
Example: Extreme Scattering Events

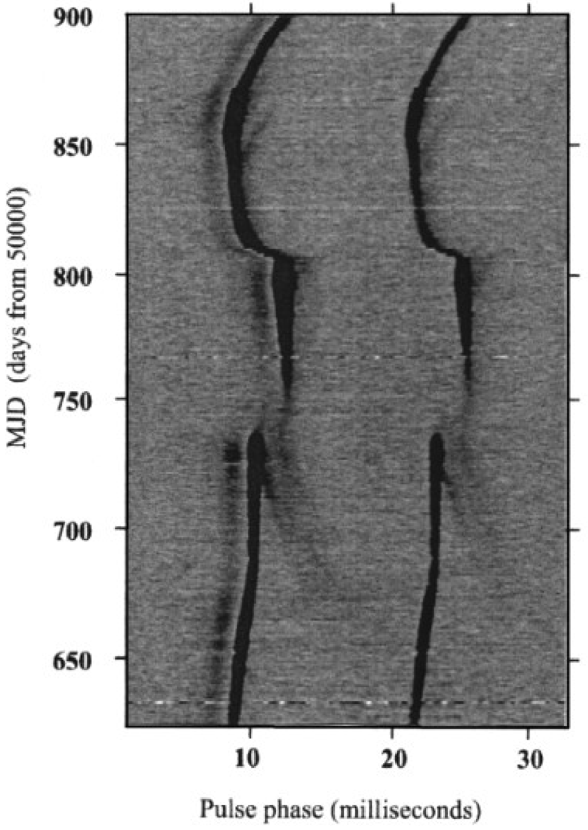
Scattering **isn't always caused by random variations** in electron number density

Occasionally, **dense structures in the ISM** (clouds, filaments, etc.) **cross the line of sight to the pulsar**

This causes a **sudden** and **large** amount of scattering, which is **difficult to model**

In this example, a **plasma 'wedge'** in the Crab Nebula crosses the line of sight to the pulsar, causing **extreme frequency-dependent effects**





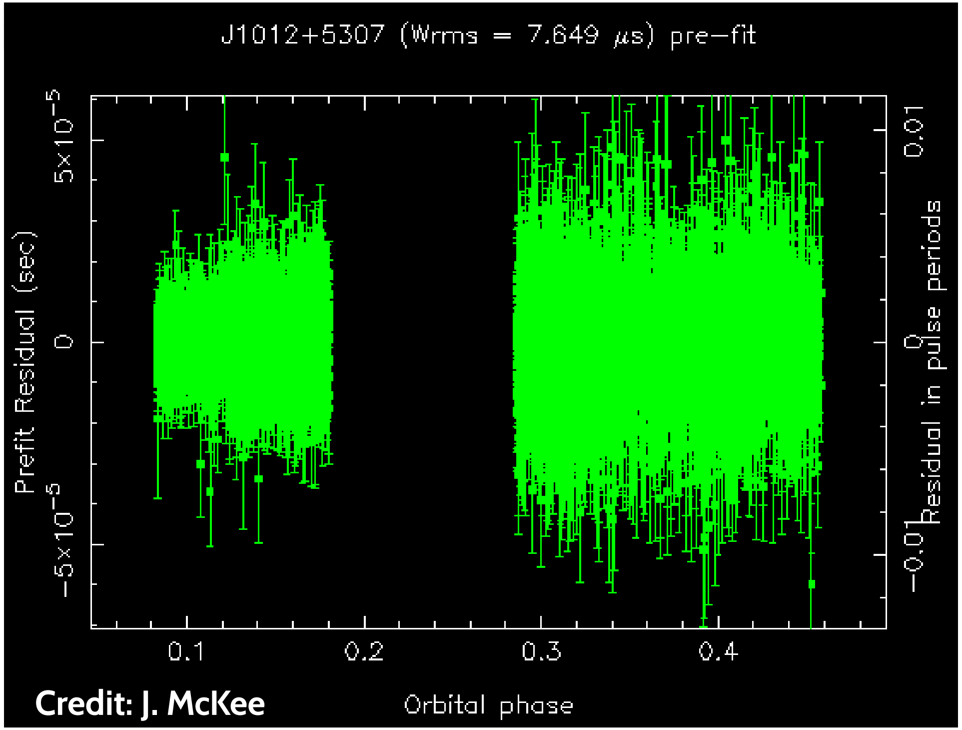
Graham-Smith, Lyne, & Jordan (2011)

Example: Scintillation Boosting S/N

Scintillation can cause pulsars to **appear extremely bright**, and allow very high precision TOAs to be generated at short integrations

The plot shows the timing residuals from PSR J1012+5307, during a period of very pronounced scintillation

TOAs are made using 10-second integrations!



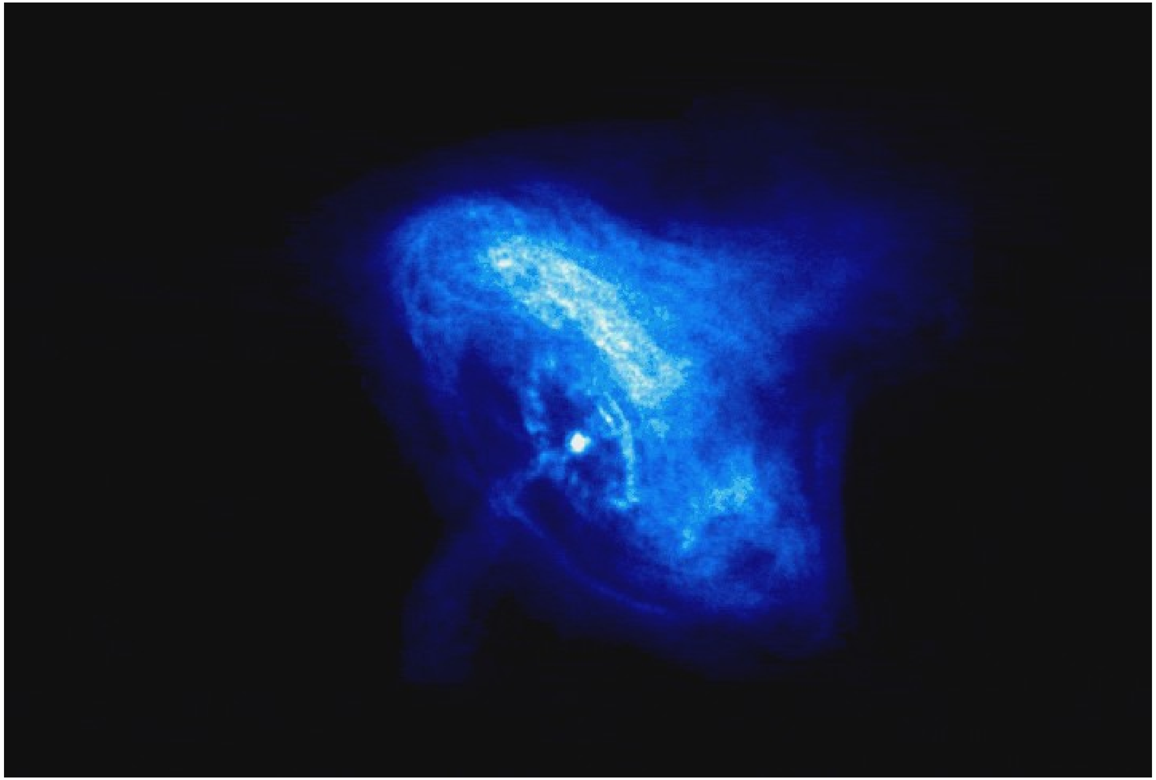
Summary

ISM effects are highly frequency-dependent

Pulsar observations allow highly-sensitive measurements of variations in the interstellar environment

The effects can contaminate pulsar timing experiments, but an improved understanding of the interstellar medium may allow for these effects to be removed

Thank you!



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