

# The Numerical planetary ephemerides

# A. Fienga<sup>1</sup>,

#### <sup>1</sup>GéoAzur, Observatoire de la Côte d'Azur, France

<sup>2</sup>IMCCE, Observatoire de Paris, France



# Planetary ephemerides

Theory of planetary (and usually Moon) motions

What for ?

- celestial mechanics and reference frames
- tests of fundamental physics
- planetology: physics of asteroids, Moon
- solar physics
- preparation of space missions
- paleoclimatology and geological time scales
- other topics: preparation of stellar occultations, public outreach



### 1 Evolution and dynamical modelling

- early ages
- recent IAU resolutions
- JPL DE
- INPOP
- 2 Observational datasets
- 3 Scientific usage
- 4 more pratically...



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# Untill the 60's

- Sun Transit
- Optical Astrometry
- Transit circle, photos
- Good time resolution



ightarrow Good measurments of angles and orbital periods



### Solar system exploration

 1958, First antenna for Pionner 8 and 9 tracking



 1960, First radar tracking on planetary surfaces (Mars, Venus)





# 3 generations of planetary ephemerides

	Gaillot		D	E200	INPOP10a		
	1	913	1	.983	2	2011	
	angle	distance	angle	distance	angle	distance	
		Earth-		Earth-		Earth-	
	"	km	"	km	"	km	
Mercury	1	450	0.050	5	0.050	0.002	
Venus	0.5	100	0.050	2	0.001	0.004	
Mars	0.5	150	0.050	0.050	0.001	0.002	
Jupiter	0.5	1400	0.1	10	0.010	2	
Saturn	0.5	3000	0.1	600	0.010	0.015	
Uranus	1	12700	0.2	2540	0.100	1270	
Neptune	1	22000	0.2	4400	0.100	2200	
Pluto	1	24000	0.2	4800	0.100	2400	



### DE102, DE200, DE405, DE414

#### From 1983 to 1993: DE102, DE200

- DE102 (Newhall et al. 1983), DE200 (Standish et al.)
- Mean equator and dynamical equinox of J2000
- T<sub>eph</sub> (between TCB and TDB, Standish 1998)

#### From 1993 to 1998: DE403, DE405

- DE403: (Standish et al. 1993), DE405: (Standish et al. 1998)
- ICRF with the first VLBI Galileo data
- Asteroid modelling : 5 Bigs + 3 taxonomic densities (C,S,M)
- 1978-1982: Viking, 1986: Pioneer, Voyager, 1999: Pathfinder

#### From 2003 to 2008: DE410, DE414

- DE410: (Standish), DE414: (Konopliv et al. 2006)
- Mars rover and orbiters (MGS, Odyssey)





La XXVIème Assemblée générale de l'Union astronomique internationale,

#### Notant

- que la Recommandation 5 (1976) des Commissions 4, 8 et 31 de l'UAI a introduit, en remplacement du temps des éphémérides (TE), une famille d'échelles de temps dynamique pour les éphémérides barycentriques et une unique échelle de temps pour les éphémérides apparentes géocentriques,
- que la Résolution 5 (1979) des Commissions 4, 19 et 31 de l'UA1 a désigné ces échelles de temps par Temps dynamique burycentrique (TDB) et Temps dynamique terrestre (TDT) respectivement, cette dernière échelle de temps ayant été par la suite renommée Temps terrestre (TT) par la Résolution A4, 1991,
- que la différence entre TDB et TDT a été spécifiée comme ne comprenant que des termes périodiques, et
- 4. que les Recommandations III et V de la Résolution A4 (1991) de l'UAI (i) ont introduit l'échelle de temps-coordonnée barycentrique (TCB) pour remplacer TDB, (iii) ont reconnt que TDB était une fonction linéaire de TCB, et (iii) ont admisi que, lonsqu'une discontinuité avec les travaux antérieurs était jugée indésirable, TDB pouvait être utilisé, et la construction de la construction d

#### Reconnaissant

 que TCB est l'échelle de temps-coordonnée à utiliser dans le Système de référence céleste barycentrique,

- 2. la possibilité de réalisations multiples de TDB tel qu'il est défini actuellement,
- 3. l'utilité pratique d'une échelle de temps définie de façon non ambigué par une relation linéaire avec TCB, choisie de façon à ce que, au géocentre, la différence entre cette échelle de temps-coordonnée et le Temps terrestre (TT) reste faible pendant un long intervalle de temps.

4: Tavantage d'une cohérence avec les échelles de temps Teph utilisées pour les éphémérides du système solaire du Jet Propulsion Laboratory (JPL) et se fadisations de IDB gelies que celle de l'airhead & Bretagnon (A&A 229, 240, 1990), et

 les recommandations 2006 du Groupe de travail de l'UAI sur la "Nomenclature pour l'astronomie fondamentale" (IAU Transactions XXVIB, 2006),

#### Recommande

que, dans des situations qui demandent l'utilisation d'une échelle de temps-coordonnée qui soit reliée linéairement au Temps-coordonnée barycentrique (TCB) et reste, au géocentre, proche du Temps terrestre (TT) pendant un long intervalle de temps, TDB soit défini par la transformaiser finéalité suivante de TCB:



- JD<sub>TCS</sub> est la date Julienne TCB. Sa valeur est T<sub>0</sub>= 2443144.5003725 pour l'évènement 1977 Janvier 1 00h 00m 00s TAI, au géocentre, et il augmente de 1 par 86400 s de TCB.
- La différence entre TDB et TT, évaluée à la surface de la Terre, reste en dessous de 2 ms durant plusieurs millénaires autour de l'époque actuelle.

4: L'argument temporel utilisé pour les éphémérides DE405, qui est appelé Teph-(Standish, AéA, 336, 381, 1998), est, pour des applications pratiques, le même que TDB tel-qu'il est défini dans cette Résolution.

 Le terme constant TDB<sub>0</sub> est choisi de façon à assurer une cohérence satisfaisante avec la formule de l'aithead & Bretagnon (1990), qui est largement utilisée pour TDB – TT. n.b. La présence de TDBn signifie que TDB n'est pas synchronisé avec TT, TCG et TCB pour <u>1977 Lan-164</u>-TM<sub>2</sub> arg geocentre.

 L'usage de TCB est encouragé pour le développement des éphémérides dans le système solaire.

# DE421, DE430 : (Folkner et al. 2009, 2014)

### 2008: DE421 (Folkner et al. 2009)

- Densification of Mars orbiter data (range, VLBI), VEX, Cassini
- fit of AU,  $T_{eph} \rightarrow IAU TDB$ , fixed Sun J2
- 5 Bigs orbit integrated, 276 asteroid orbit integrated separalely,
- 5+8 fitted masses, 56 fixed, 3 taxonomic densities (C,S,M)

### 2014: DE430 (Folkner et al. 2014)

- $AU \rightarrow Fit of Sun GM (after INPOP10a)$
- TT-TDB integration (after INPOP10a)
- **343** asteroid orbit integrated and mass fitted (based on (Kunchyka 2011))
- MESSENGER (independently/simultaneoulsy from INPOP)
- 10 years of reprocessed Cassini data

DE432, ...



# The INPOP project: www.imcce.fr/inpop

2003-2007: INPOP06 (Fienga et al. 2008)

- Numerical integration with extended precision 80b
- Integration of Earth and Sun spin axis
- 300 asteroids + 3 densities + ring
- Fitted to planetary data
- INPOP(TCB) for GAIA and INPOP(TDB) from IAU 2006
- Start of the use of MEX/VEX ESA tracking data (T. Morley, F. Budnik)
- **5** GMA, 3  $\rho_{ast}$ , GM<sub>ring</sub>, sun J2



2007-2009: INPOP08 (Fienga et al. 2009): "4-D planetary ephemerides"

- Integration of TT-TDB (and TCB-TCG) as defined by IAU 2006
- INPOP and TT-TDB (and TCB-TCG) are consistent
- TCB INPOP ( with TCB IC) required for GAIA
- TT-TDB but also TCB-TCG released for users (chebychev polynomials)

$$\frac{dTCG}{dTCB} = 1 + \frac{1}{c^2}\alpha(TCB) + \frac{1}{c^4}\beta(TCB) + O\left(\frac{1}{c^5}\right) \quad (1)$$

c being the speed of light in a vacuum and where

$$\begin{aligned} \alpha(TCB) &= -\frac{1}{2}v_E^2 - \sum_{A \neq E} \frac{GM_A}{r_{EA}}, \quad (2) \\ \beta(TCB) &= -\frac{1}{8}v_E^4 + \frac{1}{2}\left[\sum_{A \neq E} \frac{GM_A}{r_{EA}}\right]^2 \\ &+ \sum_{A \neq E} \frac{GM_A}{r_{EA}} \left\{ 4\mathbf{v}_A \cdot \mathbf{v}_E - \frac{3}{2}v_E^2 - 2v_A^2 \\ &+ \frac{1}{2}\mathbf{a}_A \cdot \mathbf{r}_{EA} + \frac{1}{2}\left(\frac{\mathbf{v}_A \cdot \mathbf{r}_{EA}}{r_{EA}}\right)^2 \\ &+ \sum_{B \neq A} \frac{GM_B}{r_{AB}} \right\}. \quad (3) \end{aligned}$$



2007-2009: INPOP08 (Fienga et al. 2009)

- "4-D planetary ephemerides"
  - Resolution at each step of TT-TDB defined by IAU 2006
  - Planetary ephemerides and TT-TDB are consistent
  - TT-TDB released for users (chebychev polynomials)
- New method of fit for asteroid masses (a priori sigma)
- but with the same modeling (3 mean densities and limited number of fitted objects)
- New data sets
  - First release of Cassini normal points
  - ESOC MEX/VEX observations
- Orbit and libration of the Moon fitted to LLR
- $\blacksquare$   $\approx$  30 GMA, sun J2, 3  $\rho_{\textit{ast}}$ , AU, EMRAT, Moon



2010-2011: INPOP10a (Fienga et al. 2011, Kuchnyka et al. 2010, Somenzi et al. 2010)

- Improvement of outer planet orbits
- New asteroid modeling: no mean density, ring, 298 objects
- fit of asteroid perturbations with BVLS
- New data sets
  - Cassini VLBI points
  - ESOC MEX/VEX observations
  - Mercury flybys from Mariner and Messenger
  - Outer planet flybys
- $\blacksquare$   $GM_{\odot}$  fitted instead of AU
- Orbit and libration of the Moon fitted to LLR
- Long term solution: (Laskar et al. 2011)
- $\blacksquare$   $\approx$  145 GMA, sun J2, GM $_{\odot}$ , EMRAT, Moon



### 2013: INPOP10e (Fienga et al. 2013, Verma et al. 2013)

- GAIA last release with TCB and TDB versions
- Direct fit with constraints + a priori sigma
- Solar corona studies and corrections
- Link to ICRF by pulsar surveys
- Use of raw MGS tracking data (GINS) → First direct analysis of s/c tracking data by the INPOP team (Verma et al. 2013)
- Orbit and libration of the Moon fitted to LLR
- $\blacksquare$   $\approx$  152 GMA, GM ring, sun J2, GM $_{\odot}$ , EMRAT, Moon



2014: INPOP13c (Fienga et al. 2014, Verma et al. 2014)

- Use of raw MESSENGER tracking data (GINS) → MESSENGER tracking data analysis for constraining the Earth-Mercury distances
- $\approx$  150 GMA, GM ring, J<sup> $\odot$ </sup><sub>2</sub>, *GM*<sub> $\odot$ </sub>, EMRAT, Moon
- First global estimation of  $\beta, \gamma, J_2^{\odot}, \dot{G}/G$

Name	# Perturbers #		∉ fitted masses	Ring		$GM_{\odot}$	
	TNO	Main belt	TNO	Main belt	TNO	Main belt	
INPOP13c	0	150	0	150	0	F	F
DE430	0	343	0	343	0	0	F
EPM2011	21	301	0	21 + 3 density classes	F	F	



INPOP15b (Fienga et al. 2016), INPOP17a (Viswanathan et al. 2017)

 Numerical integration of the (Einstein-Imfeld-Hoffmann, c<sup>-4</sup> PPN approximation) equations of motion.

$$\ddot{\mathbf{x}}_{Planet} = \sum_{A \neq B} \mu_B \frac{\mathbf{r}_{AB}}{\|\mathbf{r}_{AB}\|^3} + \ddot{\mathbf{x}}_{GR}(\beta, \gamma, c^{-4}) + \ddot{\mathbf{x}}_{AST,300} + \ddot{\mathbf{x}}_{J_2^{\odot}}$$

- Adams-Cowell in extended precision
- 8 planets + Pluto + Moon + asteroids (point-mass, ring), GR, J<sup>O</sup><sub>2</sub>, Earth rotation (Euler angles, specific INPOP)
- Moon: orbit and librations
- Simultaneous numerical integration TT-TDB, TCG-TCB
- New Cassini re-analysis by JPL
- Fit to observations in ICRF over 1 cy (1914-2014)
- IERS 2003 convention



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### Space mission dependent with 65 % from s/c navigation



# INPOP17a : the earth-moon system

(Viswanathan et al. 2017)





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S/C navigation or radio-science data: range tracking

- $\blacksquare$  Range tracking: two-way light time  $\rightarrow$  distance s/c v Earth
- $\blacksquare$  Doppler  $\approx$  98% s/c navigation or radio-science data  $\rightarrow$  S/C orbit versus the planet
- S/C orbit model = Gravity field, Atmosphere, S/C specifications (shape, reflectivity, NG-A)
- $\blacksquare~$  Range  $\approx$  2 %  $\rightarrow$  s/c orbit versus Earth
- $\label{eq:scalar} {\rm \blacksquare} \ \ S/C \ \ {\rm Range} + S/C \ \ {\rm orbit} \ \rightarrow \ \ {\rm Planet} \ \ {\rm versus} \ \ {\rm Earth} \\ {\rm distances}$
- c: Solar plasma, Shapiro delay, Time scale, troposphere, ITRF to ICRF
- p: Shapiro delay, Time scale

Me	MSG <sup>c,s/c</sup>	11-13	300	10 m
Ve	VEX <sup>p</sup>	06-12	25000	10 m
Ma	MEX <sup>p</sup> MGS-MO <sup>c</sup>	00-15	50000	2 m
Sat	Cassini <sup>c</sup>	04-14	200	100 m





#### MESSENGER : (2011-2013) NASA mission to Mercury



b. Spacecraft coordinate axes

#### Results



- 1.5 yr of Doppler + range data (level 2) @ PDS
- Original orbit analysis with GINS/CNES software
- with hypothesis on Macro-model, manouvers

- accurate orbit determination / (Smith et al. 2013)
- Full fit of all planets: INPOP13a
- New constraints over  $\beta$ ,  $\gamma$ ,  $J_2^{\odot}$ ,  $\frac{\dot{G}}{G}$
- Verma et al. 2014



### S/C navigation or radio-science data: VLBI

- Very Long Base Interferometry 8GHz
- Relative positionning S/C versus ICRF QSO
- with S/C Orbit → angle Earth/planet/ICRF
- mas-level accuracy but very few points
- crucial for linking the planetary planes together

Ve	VEX	10-13	100	2 mas
Ma	MEX-MGS-MO	00-15	200	2 mas
Jup	Galileo	96-97	12	11 mas
Sat	Cassini	04-14	30	1 mas





S/C navigation or radio-science data: Flybys

- Very short arc analysis
- Combination of Doppler, Range, VLBI
- Very very few points
- crucial for outer planet orbits

	$\alpha, \delta$	ρ		
Mer	2 mas	1 m	MSG	3 (08-09)
Jup	5 mas	1.5 km	Pioneer, Cassini Voyager, Ulysses	5 (75-01)
Ura	10 mas	1000 km	Voyager	1 (86)
Nep	10 mas	2000 km	Voyager	1 (89)



#### 

## Lunar laser ranging

- Pulsed laser with light time measurements after reflection on lunar reflectors
- 7 LLR stations but 2 main stations: OCA(1.5m) and APOLLO (3.6m)
- Since 2015, 2 detection paths @ OCA: 532 nm
   (G) + 1064nm (IR)
- $\blacksquare \ \mbox{IR} = 10 \times \mbox{more points with a better space and time coverage}$







LLR



Grasse Distribution @ 532 nm

Grasse Distribution @ 1064 nm









# More than 1 century of data for planets and 40 years for the moon

- Mercury: independent analysis of MESSENGER data (Verma et al. 2014)
- Saturn: Cassini VLBI and radio tracking
- Jupiter: Galileo VLBI and 5 s/c flybys
- Venus, Mars: VEX, MEX, MGS, MRO, MO, ...



		α	δ	ρ
S/C VLBI 1990-2010	V, Ma, <b>J</b> , S	$1/10 \mathrm{mas}$	1/10 mas	
S/C Flybys 1976-2014	Me, <b>J</b> , S, U, N	0.1/5mas	0.1/5 mas	1m/2000km
S/C Range 1976-2016	Me, V, Ma, Sat			10,10,2,100 m
Direct range 1965-1997	Me,V			1 km
Optical 1914-2014	<b>J</b> , S, U, N, P	300 mas	300 mas	
LLR 1969-2017	Moon			1cm



### 1 Evolution and dynamical modelling

- early ages
- recent IAU resolutions
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- INPOP
- 2 Observational datasets
- 3 Scientific usage
  - About the new solution: INPOP17a
  - INPOP and the asteroids
  - Fundamental physics
  - other applications such as P9, solar activity...and pulsar timing

4 more pratically...



# About the new solution INPOP17a

Name	# Pe	# Perturbers #		∉ fitted masses	Ring		$GM_{\odot}$
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INPOP17a	0	157	0	157	0	F	F
DE430	0	343	0	343	0	0	F
EPM2011	21	301	0	21 + 3 density classes	F	F	

- EPM2011 older data sets without MESSENGER data and recently published Cassini VLBI
- INPOP17a data sets  $\approx$  DE430 data sets
- with 10 years of newly JPL reduced Cassini range but:
  - 2 specific data analysis for MESSENGER
  - new IR LLR @ Calern from 2015 to now
  - Improved solar conjunction adjustement
  - Weighting schema, a priori sigma

#### Differences in dynamical modeling and adjustments

Geocentric	DE4	30 - IN	POP13c	INPO	917a - I	NPOP13c	DE4	30 - IN	POP17a	-	
Differences		1980-2	020		1980-2	020		1980-2	020		
	α	δ	ρ	α	δ	ρ	α	δ	ρ		
	mas	mas	km	mas	mas	km	mas	mas	km		
Mercury	1.1	0.6	0.242	0.24	0.5	0.02	0.9	0.6	0.240		
Venus	0.5	0.17	0.025	0.23	0.5	0.03	0.4	0.4	0.024		
Mars	0.5	0.4	0.129	0.3	0.6	0.115	0.4	0.4	0.112		
Jupiter	6.8	10.2	2.190	5.3	4.5	2.784	3.8	6.6	0.737		Cassini addition in
Saturn	1.0	0.5	1.80	0.8	0.4	1.468	0.3	0.6	0.352		INPOP17a
Uranus	36.7	33.4	522.751	95.3	47.3	318.6	80.7	67.5	340.320		
Neptune	36.7	53.1	959.567	148.2	81.2	8571.2	185	55.7	9530.70		
Pluton	119.0	75.3	2412.760	236.8	42.0	2634.9	126.4	98.6	1593.011	_	

Maximum differences between INPOP17a, INPOP13c, INPOP10e and DE430 from 1960 to 2020 in in  $\alpha,\delta$  and geocentric distances.

Maximum differences between INPOP17a, INPOP13c and DE430 from 1980 to 2020 in cartesian coordinates of the earth in the BCRS.

Earth Barycentric	XYZ	VxVyVz	
Differences	km	mm.s <sup>-1</sup>	Constant accuracy for earth-SSB
INPOP13c - DE430	0.3763	0.0467	positions (~500m over 40 yrs)
INPOP17a - DE430	0.4741	0.0513	and velocities (0.05 mm/s <sup>-1</sup> over
INPOP10e - DE423	0.84	0.113	40 yrs)



# Earth-SSB INPOP17a / DE430

EMB-SSB Y INPOP17a-DE430





Time





Z [m]

Z [mm.s-1]

0.04

1980 1990





2000 Time The Numerical planetary ephemerides

2010 2020



Sun-SSB INPOP17a / DE430





→ □ ▶ → □ ▶ → 三 ▶ → 三 → ○ ○ ○



# LLR Residuals

#### Residuals comparison of INPOP versions (13C vs 17A)





# LLR residuals: why this improvement ?

- Better modeling of the fluid core rotation and interaction with the mantle
- GRAIL = 6 month lunar geodesy mission with 30km resolution for a 1200 degree gravity field
- Use of GRAIL gravity field coefficients (but only up to degree 6 in INPOP)







# LLR residuals: why this improvement ?

- Better modeling of the fluid core rotation and interaction with the mantle
- Use of GRAIL gravity field coefficients (up to 1200 degree but only up to degree 6 in INPOP

- Detection of an unexplained 6 yr signature due to the dissipation
- Detection of the solid inner core contribution ?
- (Viswanathan et al. 2017)





# INPOP and the asteroids



- How to model all these perturbations ... with unknown masses?
- Observed impact: mainly Earth-Mars distances
- Projected accelerations of asteroids over the Earth-Mars distances



# INPOP and the asteroids

From the INPOP point-of-view:

- Mars is crucial for INPOP
- big amount of perturbers with unknown masses !
- Imiting factor for PE extrapolation ...
- ... but also for other applications !

From the planetology point-of-view:

- very few asteroids with determinated masses and densities
- constraints on asteroid formation







### INPOP10a (Fienga et al. 2011) + (Kuchynka et al. 2010)

- 24635 asteroid orbits (astorb database) integrated in INPOP with very uncertain masses
- By MCS, list of the most probable 287 perturbers of inner planets + ring for the interval of observations (κ<sub>uchynka</sub> et al. 2010)



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Mass estimations with constraints BVLS (Lawson and Hanson)

- Constraints on densities between 0 to 20 g.cm<sup>-3</sup>
- Automatic selection of fitted masses
- More realistic estimations of masses and errors

 $\blacksquare pprox 145 \ {
m GMA}$ 











# Fundamental physics



# Fundamental physics

With such accuracy, the solar system is still the ideal lab for testing gravity





In Planetary and Lunar ephemerides (like INPOP), GR plays a role in

$$\begin{aligned} \Delta t_{SHAP} &= (1+\gamma) GM_{\odot}(t) ln \frac{l_0 + l_1 + t}{l_0 + l_1 - t} \\ \Delta \dot{\varpi}_{PLA} &= \frac{(2\gamma - \beta + 2) GM_{\odot}(t)}{a(1 - e^2)c^2} + \Delta \dot{\varpi}_{J_2^{\odot}}(J_2^{\odot}, a^2) + \Delta \dot{\varpi}_{AST} \\ \Delta \dot{\varpi}_{Moon} &= \frac{(2\gamma - \beta + 2) GM_{\odot}(t)}{a(1 - e^2)c^2} + \Delta \dot{\varpi}_{GEO} + \Delta \dot{\varpi}_{SEL} + \Delta \dot{\varpi}_{S,PLA} \end{aligned}$$

GR tests are then limited by

- Contributions by  $J_2^{\odot}$ , Asteroids,  $2\gamma \beta + 2$
- Lunar and Earth physics

BUT

- Decorrelation with all the planets
- Benefit of PE global fit versus single space mission



# Specific INPOP developments for testing gravity

1- Variations/Estimations of PPN  $\beta$ ,  $\gamma$ , Sun  $J_2^{\odot}$  (Fienga et al. 2011), (Fienga et al. 2015)

2- Simulation of a Pioneer anomaly type of acceleration  $\rightarrow \ddot{x}_{constant}$  (Fienga et al. 2011)

3- Supplementary advance of perihelia  $\dot{\varpi}$  and nodes  $\dot{\Omega} \rightarrow$  INPOP15a (Fienga et al. 2015, MG)

4- Equivalence Principle @ astronomical scale  

$$\rightarrow \ddot{x}_j = \frac{m_j^G}{m_j^I} F(x_i, \dot{x}_i, m_i^G, ...) = (1 + \eta) F(x_i, \dot{x}_i, m_i^G, ...)$$

With 
$$\mu_{\odot} = GM_{\odot}$$
,  $\mu_j = GM_j$  for planet  $j$ ,  
5- Estimation of  $\frac{\dot{M}_{\odot}}{M_{\odot}}$  and  $\frac{\dot{G}}{G}$  with  $\frac{\dot{\mu_{\odot}}}{\mu_{\odot}} = \frac{\dot{G}}{G} + \frac{\dot{M}_{\odot}}{M_{\odot}}$  and  $\frac{\dot{\mu}_j}{\mu_j} = \frac{\dot{G}}{G}$   
(Fienga et al. 2015)



#### 1- Estimations PPN $\beta$ , $\gamma$ , Sun $J_2^{\odot}$ including full MSG navigation data analysis

(Verma et al. 2014)





2- Simulation of a Pioneer anomaly type of acceleration ightarrow same idea for P9

$$\ddot{x}_{Planet} = \ddot{x}_{Newton} + \ddot{x}_{GR}(eta, \gamma, c^{-4}) + \ddot{x}_{AST,300} + \ddot{x}_{J_{2}^{\odot}} + \ddot{x}_{constant}$$

3- Supplementary advance of perihelia  $\dot{\varpi}$  and nodes  $\dot{\Omega}$ 

At each step of integration  $t_i$ ,

$$arpi(t_i) = arpi(t_0) + \dot{arpi}(t_i - t_0)$$
 $\Omega(t_i) = \Omega(t_0) + \dot{\Omega}(t_i - t_0)$ 
 $\ddot{x}_{Planet} = R(arpi(t_i), \Omega(t_i)) \ddot{x}_{Planet}$ 



#### 3- Supplementary advance of perihelia $\dot{\varpi}$ and nodes $\dot{\Omega}$

#### With INPOP10a

$\hat{\Omega}_{sup}$	INPOP08	INPOP10a
mas.cy <sup>-1</sup>		
Mercury		$1.4 \pm 1.8$
Venus	$200\pm100$	$0.2 \pm 1.5$
EMB	$0.0\pm10.0$	$0.0 \pm 0.9$
Mars	$0.0 \pm 2$	$\textbf{-0.05} \pm \textbf{0.13}$
Jupiter	$\textbf{-200} \pm \textbf{100}$	$-40 \pm 42$
Saturn	$\textbf{-200} \pm \textbf{100}$	$-0.1\pm0.4$

$\dot{\varpi}_{sup}$	INPOP08	INPOP10a	P09	P10
mas.cy <sup>-1</sup>				
Mercury	$-10 \pm 30$	$1.2\pm1.6$	$-3.6\pm5$	-4 ± 5
Venus	$-4 \pm 6$	$0.2\pm1.5$	$\textbf{-0.4} \pm \textbf{0.5}$	
EMB	$0.0\pm0.2$	$\textbf{-0.2}\pm0.9$	$-0.2\pm0.4$	
Mars	$0.4 \pm 0.6$	-0.04 $\pm$ 0.15	$0.1\pm0.5$	
Jupiter	$142 \pm 156$	$-41 \pm 42$		
Saturn	$-10 \pm 8$	$0.15{\pm}~0.65$	$-6 \pm 2$	$\textbf{-10}\pm15$

#### With INPOP15a

- comparisons with INPOP15a  $\rightarrow \Delta(O - C)$
- Mercury Messenger data from 2011 to 2014
- Saturn Cassini data from 2004 to 2014

$\varpi_{sup}$	$\Delta(O-C) < 5\%$	$\Delta(O-C) < 35\%$
${\sf mas.yr^{-1}}$	Internal accuracy	External accuracy
Mercury	$(0.0 \pm 1.05)$	$(0.0 \pm 3.1)$
Saturn	$(0.05 \pm 0.20)$	$(1.2 \pm 5.0)$

$$\Rightarrow \varpi_{sup}^{Mercury} \pm_1^3 \text{ mas.yr}^{-1} \text{ and } \varpi_{sup}^{Saturn} \pm_{0.2}^5 \text{ mas.yr}^{-1}$$

 $\Rightarrow$ Limits to MOND in the solar system



4- Equivalence Principle @ Earth-Moon scale  $\rightarrow \ddot{x}_j = \frac{m_j^G}{m_i^I} F(x_i, \dot{x}_i, m_i^G, ...)$ 

If  $\mathbf{r}_M = SSB$ -Moon,  $\mathbf{r}_E = SSB$ -Moon and  $\mathbf{r} = Earth$ -Moon then

$$\ddot{\mathbf{r}} = \ddot{\mathbf{r}}_{\mathbf{N},\mathbf{T}} + \mu_{S} \left[ \left( \left[ \frac{m^{G}}{m^{I}} \right]_{E} - 1 \right) \frac{\mathbf{r}_{\mathbf{E}}}{r_{E}^{3}} - \left( \left[ \frac{m^{G}}{m^{I}} \right]_{M} - 1 \right) \frac{\mathbf{r}_{\mathbf{M}}}{r_{\mathbf{M}}^{3}} \right]$$

If q is the ration between the mass of the Moon and the mass of the earth, then,

$$\ddot{\mathbf{r}} = \ddot{\mathbf{r}}_{\mathbf{N},\mathbf{T}} + \ddot{\mathbf{r}}_{\mathbf{EMB}} \times \left( \left[ \frac{m^{G}}{m'} \right]_{E} - \left[ \frac{m^{G}}{m'} \right]_{M} \right) + \frac{\ddot{\mathbf{r}}_{\mathbf{EMB}}}{(1+q)} \times \left[ \left( \left[ \frac{m^{G}}{m'} \right]_{E} - 1 \right) + q \left( \left[ \frac{m^{G}}{m'} \right]_{M} - 1 \right) \right]$$

 $\begin{bmatrix} \frac{m^{G}}{m^{I}} \end{bmatrix}_{E} - \begin{bmatrix} \frac{m^{G}}{m^{I}} \end{bmatrix}_{M} \rightarrow \text{the Universality of Free Fall (UFF)}$  $If \begin{bmatrix} \frac{m^{G}}{m^{I}} \end{bmatrix} = 1 + \eta \frac{U}{mc^{2}} \text{ with U, the self-gravity energy} \rightarrow \text{Strong Equivalence Principle (SEP)}$ 



4- Equivalence Principle @ Earth-Moon scale  $\rightarrow \ddot{x}_j = \frac{m_j^G}{m_i^G} F(x_i, \dot{x}_i, m_i^G, ...)$ 

#### Results with INPOP17a (Viswanathan et al. 2017b)





With 
$$\mu_{\odot} = GM_{\odot}$$
,  $\mu_j = GM_j$  for planet j

5- Estimation of 
$$\frac{\dot{M_{\odot}}}{M_{\odot}}$$
 and  $\frac{\dot{G}}{G}$  with  $\frac{\dot{\mu_{\odot}}}{\mu_{\odot}} = \frac{\dot{G}}{G} + \frac{\dot{M_{\odot}}}{M_{\odot}}$  and  $\frac{\dot{\mu_{j}}}{\mu_{j}} = \frac{\dot{G}}{G}$ 

$$\begin{array}{rcl} M_{\odot}(\mathbf{t}_i) & = & M_{\odot}(t_0) + (t_i - t_0) \times \dot{M}_{\odot} \\ \mathsf{G}(\mathbf{t}_i) & = & \mathsf{G}(t_0) + (t_i - t_0) \times \dot{\mathsf{G}} \end{array}$$

$$\mu_{\odot}(t_i) = G(t_i) imes M_{\odot}(t_i) \ \mu_j(t_i) = G(t_i) imes M_j$$

- by fixing  $\dot{M_{\odot}}$  or  $\dot{G} 
  ightarrow rac{\dot{\mu}}{\mu}$
- $\forall t_i, M_{\odot}(t_i) \text{ and } G(t_i) \rightarrow \ddot{x}_{Planet}, \ddot{x}_{Ast}, \ddot{x}_{Moon}$
- What values of  $\frac{\dot{\mu}}{\mu}$  (and then  $\frac{\dot{M}_{\odot}}{M_{\odot}}$  or  $\frac{\dot{G}}{G}$ ) are acceptable / data accuracy ?



# 2 approaches based on INPOP13c (Fienga et al. 2015)

- 1 Global fit including  $\frac{\mu_{\odot}}{\mu_{\odot}}$ , PPN  $\beta$ ,  $\gamma$  and  $J_2^{\odot}$ 
  - Planet CI, 290 GM<sub>ast</sub>,GM<sub>ring</sub>, GM☉, EMRAT + GR
  - Full data samples including Messenger and Cassini data
  - Correlations between parameters and correlated datasets

#### 2 - Monte Carlo + Least squares

- Exploration of other possible minima
- for one set of GRP  $(\frac{\dot{\mu_{\odot}}}{\mu_{\odot}}, \beta, \gamma J_2^{\odot}) \rightarrow$  one new fitted INPOP
- selection with 2 criteria :  $\Delta$ (O-C) < 25,50% and  $\Delta\chi^2$  < 1,2,3% (H3)
- about 36000 runs
- optimized by a genetic algorithm (2 crossovers + 1/10 mutation)
- convergence @ 30th generation

With 
$$\frac{\dot{M_{\odot}}}{M_{\odot}} = (-0.92 \pm 0.46) \times 10^{-13} \, {
m yr}^{-1} 
ightarrow rac{\dot{G}}{G}$$
 (Pinto et al. 2013)



# PPN $\beta$ , $\gamma$ , $\dot{\mu}/\mu$ , J $_2^{\odot}$ after 30 generations (Fienga et al. 2015)

Method	PPN $\beta - 1$	PPN $\gamma - 1$	Ġ/G	J <sub>0</sub>
	imes 10 <sup>5</sup>	imes 10 <sup>5</sup>	$ imes$ $10^{13}$ yr $^{-1}$	$\times$ 10 <sup>7</sup>
LS	$-4.4 \pm 5.5$	$-0.81 \pm 4.5$	$0.42\pm0.75$	$2.27\pm0.3$
MC + SGAM C1 50 %	$-0.5\pm6.3$	$-1.2 \pm 4.4$	$0.36\pm1.22$	$2.26\pm0.11$
MC + SGAM C1 25 %	$-1.6 \pm 4.5$	-0.75 ± 3.2	$0.41\pm1.00$	$2.28\pm0.08$
MC + SGAM C2 (H3)	$\textbf{-0.01} \pm \textbf{7.10}$	$-1.7\pm5.2$	$0.55\pm1.22$	$2.22\pm0.14$
MC + SGAM C2 (H2)	$0.05\pm7.12$	$-1.62 \pm 5.17$	$0.53 \pm 1.20$	$2.221 \pm 0.137$
MC + SGAM C2 (H1)	$0.11\pm7.07$	$-1.62 \pm 5.10$	$0.52\pm1.18$	$2.220 \pm 0.135$
MC + SGAM C2 (Hiter)	$0.34\pm6.91$	$-1.62 \pm 5.12$	$0.51\pm1.18$	$2.218\pm0.135$
MC + SGAM C1,C2	$-0.25 \pm 6.7$	$-1.5 \pm 4.8$	$0.49\pm1.20$	$2.24\pm0.125$

$$\begin{aligned} (\beta - 1) \times 10^5 &= 0.25 \pm \frac{7}{4} \quad (\gamma - 1) \times 10^5 &= -1.5 \pm \frac{5}{3} \\ J_2^{\odot} &= (2.24 \pm 0.15) \times 10^{-7} \\ \dot{G}/G &\pm 1 \times 10^{-13} \text{ yr}^{-1} \end{aligned}$$

# INPOP and the solar physics (Verma et al. 2013)





# About the hypothetical P9...



Des astronomes français ont réussi à préciser les directions vers lesquelles orienter les télescopes pour essayer de la dénicher la fameuse planète X.





# 1992-2014 : 22 years of KBO monitoring

### Dynamical Confinement (?)

- $\blacksquare$  Confinement for object a >150 UA and q >30 UA
- Far from Neptune zone of influence(TNO)
- (de la Fuente Marcos and de la Fuente Marcos 2014, Brown 2017) : no observational biais...yes but not clear (Shankman et al. 2017)



(Batygin and Brown 2016), (Brown and Batygin 2016)

#### Method:

- KBO mass is not sufficient for inducing this dynamical confinement
- hypothesis of a supplementary perturbing body
- N-body simulation over 4 Gyr
- integration of a disk with planetary perturbation
- Runs for different values of a,e et a',e',i'
- P9 = at least 10 x M $_{\star}$ , i = 30°, w = 138°  $\pm$ 21° , a = 700 UA, e  $\approx$  0.6,  $\Omega \approx$  113°





### (Fienga et al. 2016)

#### Method:

- addition of an acceleration induced by P9 in INPOP with
- P9 = 10 × M $_{
  m t}$ , i = 30°, w = 138°  $\pm 21^\circ$  , a = 700 UA, e pprox 0.6,  $\Omega pprox$  113°
- BUT (Batygin and Brown 2016) propose only a mean orbit for P9

#### for different positions of P9 on its mean (B&B16) orbit

- INPOP integration of planetary orbits + 300 minor bodies
- Comparaison to observations and adjustement
- most sensible data = Saturn/Cassini

	ρ	
Jupiter	1.5 km	Pioneer, Cassini,
		Voyager, Ulysses
Saturne	0.1 km	Cassini
Uranus	1000 km	Voyager
Neptune	2000 km	Voyager
Pluton	1500 km	HST, NH





### (Fienga et al. 2016)





Exclusion zones for a mean P9 orbit as proposed by (B&B16)





# to be continued ...

- P9 as a fixed body (≈ 15000 yrs OP / 10yrs Cassini data)
- map of perturbations through the whole sky  $\rightarrow$  Subaru, CFHT, VST etc...
- Improvement of INPOP with JUNO mission
- Inclusion of GAIA asteroid : GAIA frame to ICRF/INPOP
- Use of VLBA observations of millisecond pulsars (Fienga et al. 2011)



# to be continued...

P9 as a	fixed	body	$(\approx$	15000	yrs	OP
/ 10yrs	Cassi	ni data	a)			

- map of perturbations through the whole sky  $\rightarrow$  Subaru, CFHT, VST etc...
- Improvement of INPOP with JUNO mission
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	θ	η	ζ		
	mas	mas	mas		
18 MSP with radio timi	ing only				
$DE405 \rightarrow DE200$	$-0.4 \pm 0.3$	$-13\pm0.4$	$-13 \pm 0.3$		
$DE405 \rightarrow DE200$ [41]	$-1 \pm 2$	$-14 \pm 3$	$-10 \pm 3$		
$DE414 \rightarrow DE405$	$1.5 \pm 0.3$	$-1.0\pm0.4$	$-0.9 \pm 0.3$		
$DE421 \rightarrow DE405$	$1.5 \pm 0.3$	$-0.9 \pm 0.4$	$-0.8 \pm 0.3$		
$INPOP08 \rightarrow DE405$	$1.3 \pm 0.3$	$-0.3 \pm 0.4$	$-1.1 \pm 0.3$		
$INPOP10A \rightarrow DE405$	$1.6 \pm 0.3$	$-0.7 \pm 0.4$	$-0.7 \pm 0.3$		
J0737-30, J1713+07, B1	1937+21, J214	5-07 with radio	timing only		
$DE405 \rightarrow DE200$	$-0.5 \pm 0.2$	$-12 \pm 0.3$	$-13 \pm 0.18$		
$INPOP08 \rightarrow DE405$	$1.4 \pm 0.03$	$-0.03 \pm 0.05$	$-1.4 \pm 0.03$		
$INPOP10a \rightarrow DE405$	$1.7\pm0.01$	$-0.03 \pm 0.02$	$-1.0 \pm 0.01$		
J0737-30, J1713+07, B1937+21, J2145-07 with radio timing + VLBI					
· · · ·					
$DE200 \rightarrow ICRF$	$6 \pm 4$	$26 \pm 9$	$9 \pm 5$		
$DE200 \rightarrow ICRF$ [15]	$2 \pm 2$	$12 \pm 3$	$6 \pm 3$		
$DE405 \rightarrow ICRF$	$6 \pm 4$	$14 \pm 9$	$-4 \pm 5$		
$INPOP08 \rightarrow ICRF$	$4 \pm 4$	$14 \pm 9$	$-2 \pm 5$		
$INPOP10a \rightarrow ICRF$	$4 \pm 4$	$14 \pm 9$	$-2.5 \pm 5$		
$DE421 \rightarrow ICRF$	$4 \pm 4$	$14 \pm 9$	$-3.0 \pm 5$		



### 1 Evolution and dynamical modelling

- early ages
- recent IAU resolutions
- JPL DE
- INPOP
- 2 Observational datasets
- 3 Scientific usage
  - About the new solution: INPOP17a
  - INPOP and the asteroids
  - Fundamental physics
  - other applications such as P9, solar activity...and pulsar timing

### 4 more pratically...



# http://www.imcce.fr/inpop

- Chebychev polynomials
- barycentric positions, velocities of planets + sun + moon
- TT-TDB and TCG-TCB
- calceph / spice / ascii / old JPL format
  - calceph = native C library http://www.imcce.fr/fr/
    presentation/equipes/ASD/inpop/calceph/index.html
  - TCG ephemeris with the JPL format
  - 2 binary formats : little-endian and big-endian
  - 2 periods: [1900:2100] , [1000:3000]
- documentation on arXiv
- specific developpement on demand
- planetary database



### INPOP17A WILL BE ONLINE IN AUGUST 2017