

# The Galactic central black hole and the no-hair theorem

Éric Gourgoulhon

Laboratoire Univers et Théories (LUTH)  
CNRS / Observatoire de Paris / Université Paris Diderot  
Paris Sciences et Lettres Research University  
92190 Meudon, France

<http://luth.obspm.fr/~luthier/gourgoulhon/>

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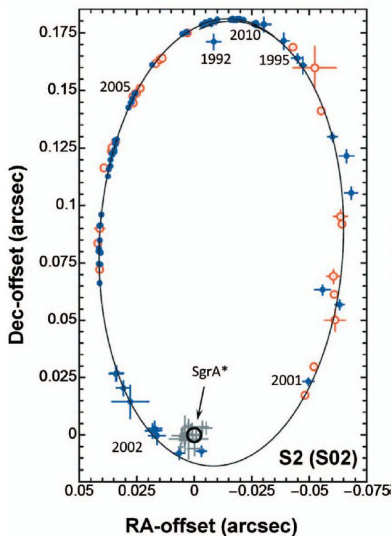
# Outline

- 1 The black hole at the Galactic center
- 2 Definition and main properties of black holes
- 3 The Kerr black hole
- 4 The no-hair theorem

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## The black hole at the centre of our galaxy : Sgr A\*



[ESO (2009)]

Mass of Sgr A\* black hole deduced from stellar dynamics :

$$M_{\text{BH}} = 4.3 \times 10^6 M_{\odot}$$

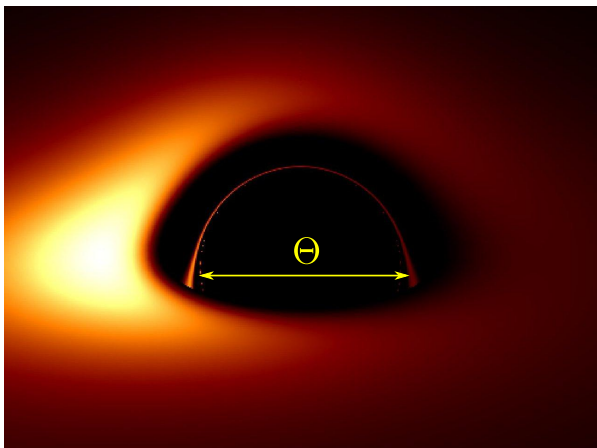
← Orbit of the star S2 around Sgr A\*

$$P = 16 \text{ yr}, \quad r_{\text{per}} = 120 \text{ UA} = 1400 R_{\text{S}}, \\ V_{\text{per}} = 0.02 c$$

[Genzel, Eisenhauer &amp; Gillessen, RMP 82, 3121 (2010)]

Next periastron passage : mid 2018

## Can we see it from the Earth ?



Angular diameter of the event horizon of a Schwarzschild BH of mass  $M$  seen from a distance  $d$  :

$$\Theta = 6\sqrt{3} \frac{GM}{c^2 d} \simeq 2.60 \frac{2R_S}{d}$$

Image of a thin accretion disk around a Schwarzschild BH  
 [Vincent, Paumard, Gourgoulhon & Perrin, CQG 28, 225011 (2011)]

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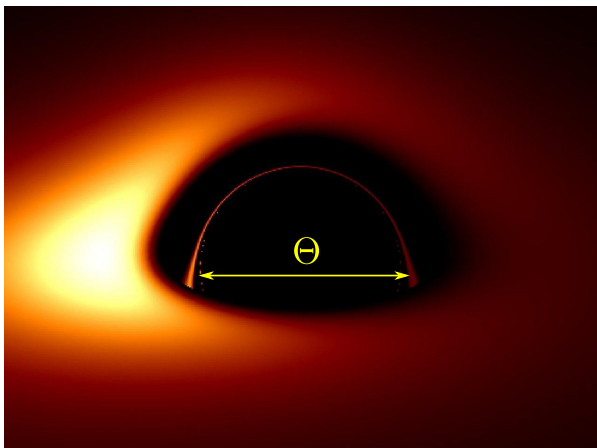


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Largest black holes in the Earth's sky :

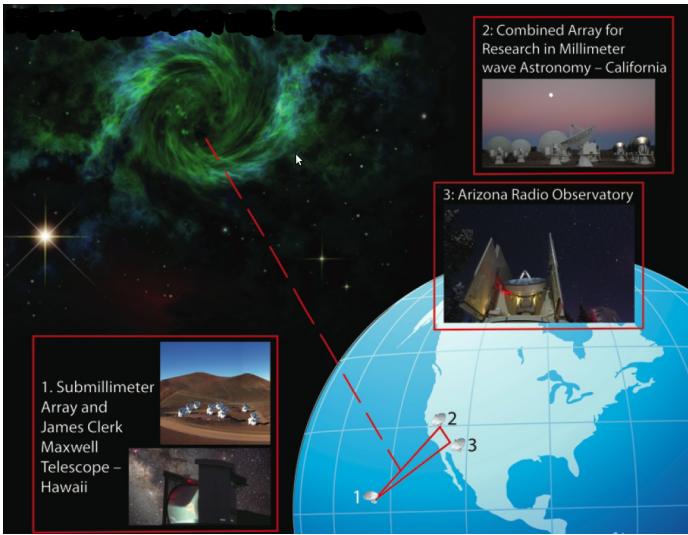
**Sgr A\*** :  $\Theta = 53 \mu\text{as}$

**M87** :  $\Theta = 21 \mu\text{as}$

**M31** :  $\Theta = 20 \mu\text{as}$

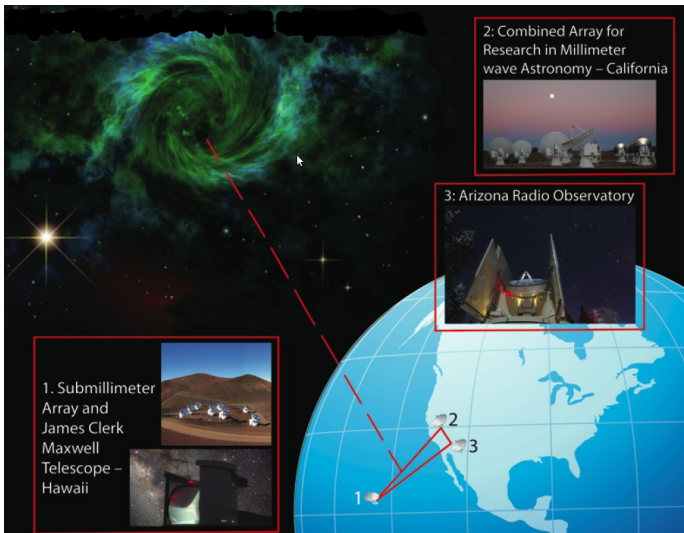
*Remark* : black holes in X-ray binaries are  $\sim 10^5$  times smaller, for  $\Theta \propto M/d$

# Reaching the $\mu\text{as}$ resolution with VLBI



Very Large Baseline Interferometry (VLBI) in (sub)millimeter waves

Existing American VLBI network [Doeleman et al. 2011]

Reaching the  $\mu\text{as}$  resolution with VLBI

Very Large Baseline Interferometry (VLBI) in (sub)millimeter waves

The best result so far : VLBI observations at 1.3 mm have shown that the size of the emitting region in Sgr A\* is only  $37 \mu\text{as}$

[Doeleman et al., *Nature* 455, 78 (2008)]

Existing American VLBI network [Doeleman et al. 2011]



# The near future : the Event Horizon Telescope

To go further :

- shorten the wavelength : **1.3 mm**  $\rightarrow$  **0.8 mm**
- increase the number of stations; in particular add ALMA

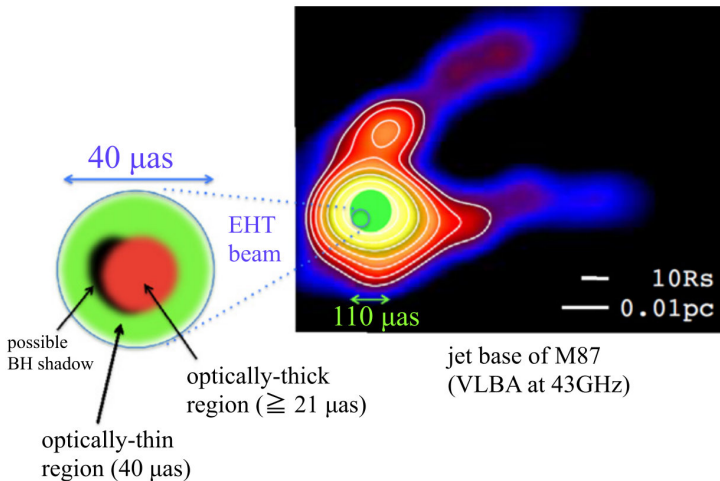


Atacama Large Millimeter Array (ALMA)

part of the **Event Horizon Telescope (EHT)** to be completed by 2020

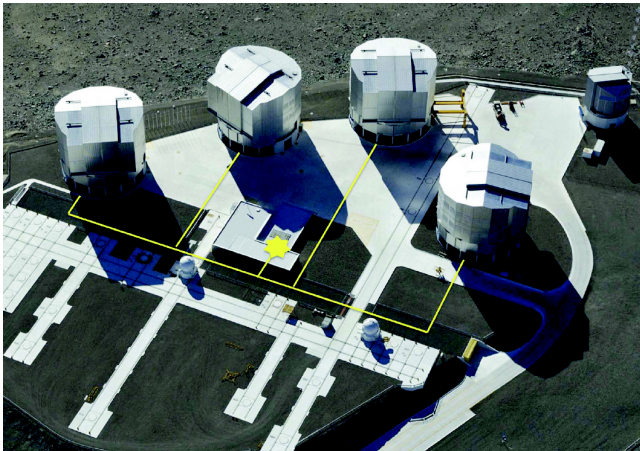
**August 2015 : VLBI observations involving ALMA and VLBA**

## VLBA and EHT observations of M87



[Kino et al., ApJ 803, 30 (2015)]

# Near-infrared optical interferometry : GRAVITY



[Gillessen et al. 2010]

GRAVITY instrument at VLT (2016)

Beam combiner (the four 8 m telescopes + four auxiliary telescopes)

astrometric precision on orbits :  $10 \mu\text{as}$

cf. Guy Perrin's talk next Wednesday

# Near-infrared optical interferometry : GRAVITY



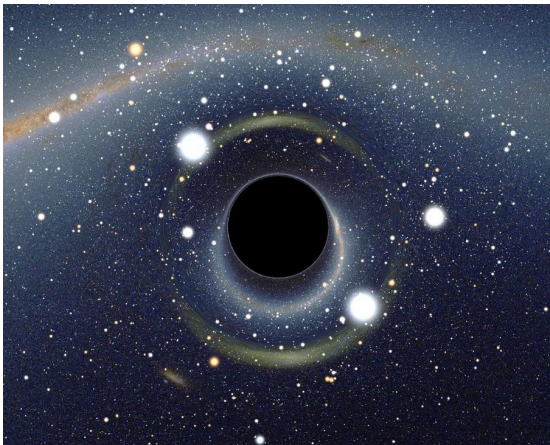
[MPE/GRAVITY team]

July 2015 : GRAVITY  
shipped to Chile and  
successfully assembled  
at the Paranal  
Observatory  
Fall 2016 : observations  
have started !

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# What is a black hole ?



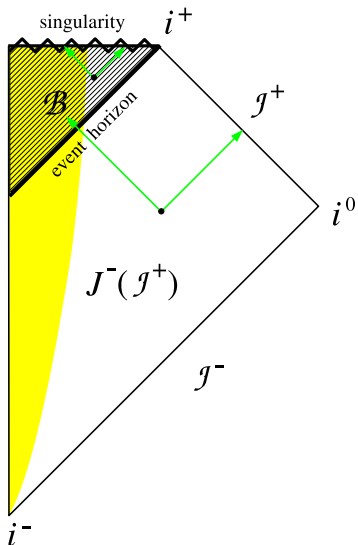
[Alain Riazuelo, 2007]

... for the layman :

A **black hole** is a region of spacetime from which nothing, not even light, can escape.

The (immaterial) boundary between the black hole interior and the rest of the Universe is called the **event horizon**.

# What is a black hole ?



Textbook definition [Hawking & Ellis (1973)]

**black hole :**  $\mathcal{B} := \mathcal{M} - J^-(\mathcal{I}^+)$

where

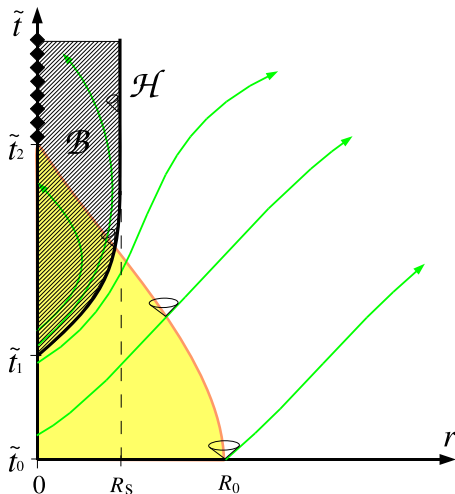
- $(\mathcal{M}, g)$  = asymptotically flat manifold
- $\mathcal{I}^+$  = (complete) future null infinity
- $J^-(\mathcal{I}^+) =$  causal past of  $\mathcal{I}^+$

i.e. black hole = region of spacetime from which light rays cannot escape to infinity

**event horizon :**  $\mathcal{H} := \partial J^-(\mathcal{I}^+)$   
(boundary of  $J^-(\mathcal{I}^+)$ )

$\mathcal{H}$  smooth  $\implies \mathcal{H}$  null hypersurface

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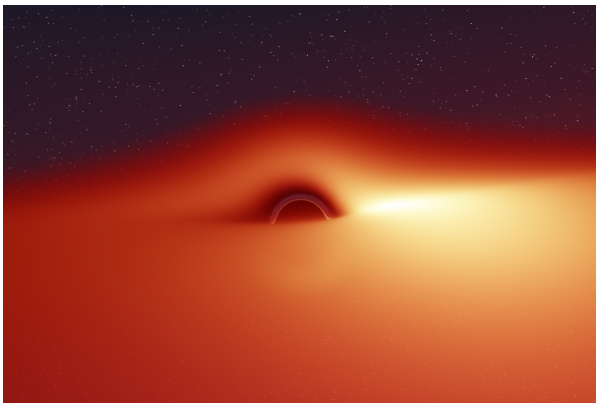


# What is a black hole ?

... for the astrophysicist : a very deep gravitational potential well

Release of potential gravitational energy by **accretion** on a black hole : up to 42% of the mass-energy  $mc^2$  of accreted matter !

NB : thermonuclear reactions release less than 1%  $mc^2$



Matter falling in a black hole forms an **accretion disk**  
[Lynden-Bell (1969),  
Shakura & Sunayev (1973)]

[J.-A. Marck (1996)]

# Main properties of black holes (1/3)

- In general relativity, a black hole contains a region where the spacetime curvature diverges : **the singularity** (*NB : this is not the primary definition of a black hole*). The singularity is inaccessible to observations, being hidden by the event horizon.

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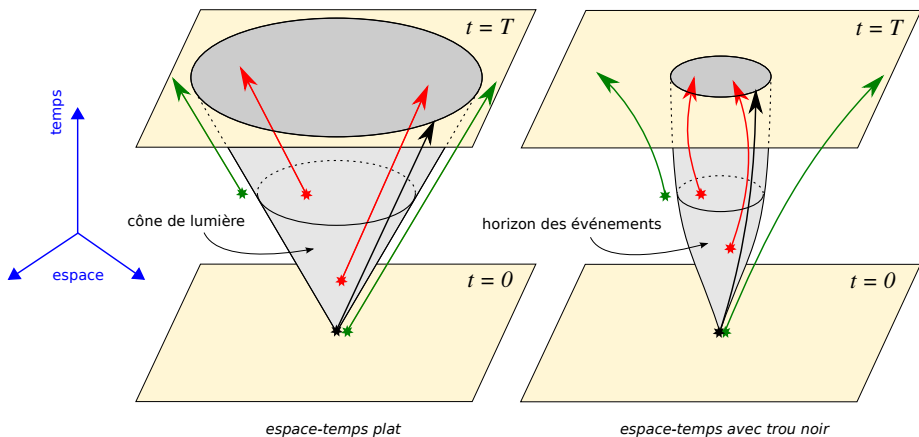
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- The singularity marks the **limit of validity of general relativity** : to describe it, a quantum theory of gravitation would be required.
- The event horizon  $\mathcal{H}$  is a **global structure** of spacetime : no physical experiment whatsoever can detect the crossing of  $\mathcal{H}$ .

## Main properties of black holes (2/3)

The event horizon as a null cone



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$\implies$  a black hole is a **compact object** :  $\frac{M}{R}$  large, not  $\frac{M}{R^3} !$
- Due to the non-linearity of general relativity, **black holes can form in spacetimes without any matter**, by collapse of gravitational wave packets.

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## The Kerr solution

Roy Kerr (1963)

$$g_{\alpha\beta} dx^\alpha dx^\beta = - \left( 1 - \frac{2GMr}{c^2 \rho^2} \right) c^2 dt^2 - \frac{4GMa r \sin^2 \theta}{c^2 \rho^2} c dt d\varphi + \frac{\rho^2}{\Delta} dr^2 \\ + \rho^2 d\theta^2 + \left( r^2 + a^2 + \frac{2GMa^2 r \sin^2 \theta}{c^2 \rho^2} \right) \sin^2 \theta d\varphi^2$$

where

$$\rho^2 := r^2 + a^2 \cos^2 \theta, \quad \Delta := r^2 - \frac{2GM}{c^2} r + a^2 \quad \text{and} \quad r \in (-\infty, \infty)$$

→ spacetime manifold :  $\mathcal{M} = \mathbb{R}^2 \times \mathbb{S}^2 \setminus \{r = 0 \ \& \ \theta = \pi/2\}$

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→ Schwarzschild solution as the subcase  $a = 0$  :

$$g_{\alpha\beta} dx^\alpha dx^\beta = - \left( 1 - \frac{2GM}{c^2 r} \right) c^2 dt^2 + \left( 1 - \frac{2GM}{c^2 r} \right)^{-1} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2)$$

# Basic properties of Kerr metric

- Asymptotically flat ( $r \rightarrow \pm\infty$ )
- Stationary : metric components independent from  $t$
- Axisymmetric : metric components independent from  $\varphi$
- Not static when  $a \neq 0$
- Contains a black hole  $\iff 0 \leq a \leq m$ , where  $m := GM/c^2$   
 event horizon :  $r = r_+ := m + \sqrt{m^2 - a^2}$
- Contains a curvature singularity at  $\rho = 0 \iff r = 0$  and  $\theta = \pi/2$

# Physical meaning of the parameters $M$ and $J$

- **mass  $M$**  : *not* a measure of the “amount of matter” inside the black hole, but rather a *characteristic of the external gravitational field*  
→ measurable from the orbital period of a test particle in far circular orbit around the black hole (*Kepler's third law*)

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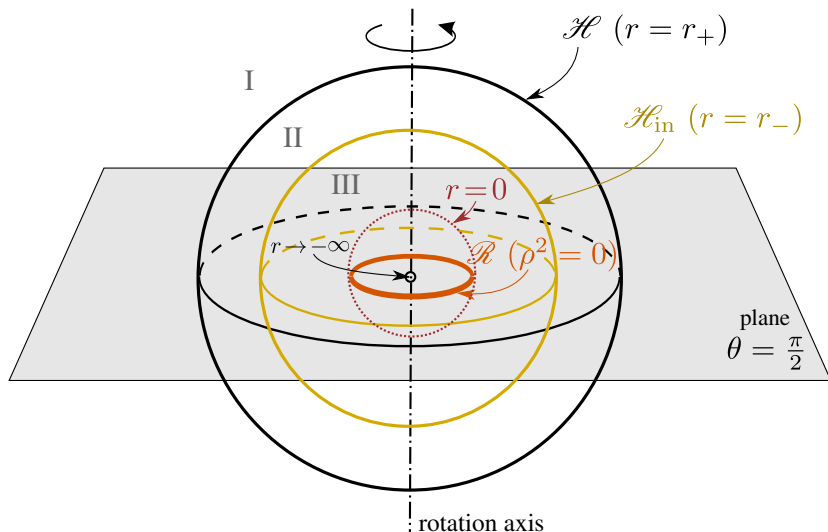
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*Remark* : the **radius** of a black hole is not a well defined concept : it *does not* correspond to some distance between the black hole “centre” and the event horizon. A well defined quantity is the **area** of the event horizon,  $A$ .

The radius can be then defined from it : for a Schwarzschild black hole :

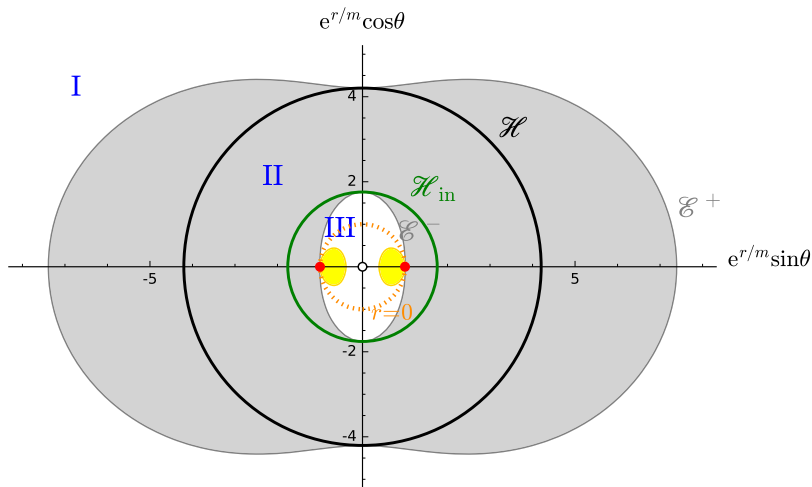
$$R := \sqrt{\frac{A}{4\pi}} = \frac{2GM}{c^2} \simeq 3 \left( \frac{M}{M_{\odot}} \right) \text{ km}$$

## Kerr spacetime

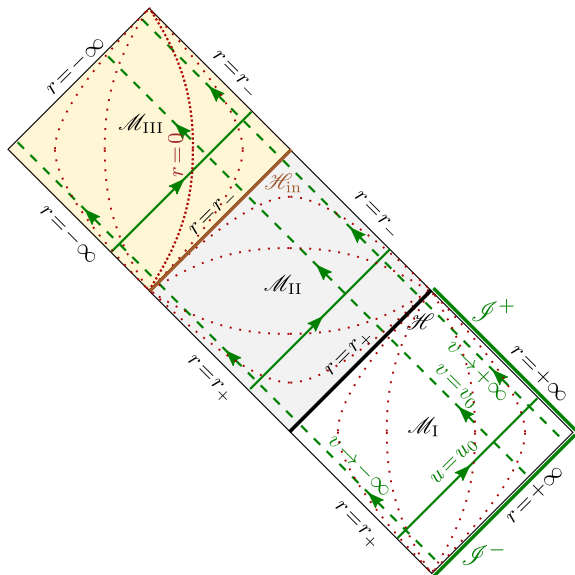


Slice  $t = \text{const}$  of the Kerr spacetime viewed in O'Neill coordinates  $(R, \theta, \varphi)$ , with  $R := e^r$ ,  $r \in (-\infty, +\infty)$ .

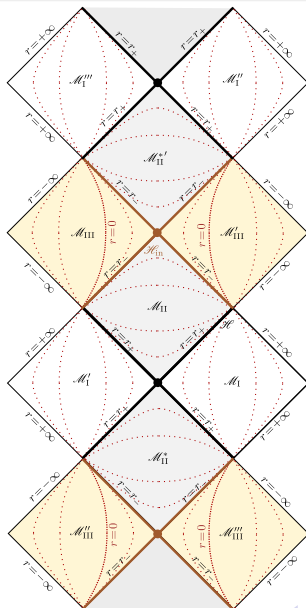
## Kerr spacetime : ergoregion and Carter time machine

Meridional view of a section  $t = \text{const}$  of Kerr spacetime with  $a/m = 0.90$

## Conformal diagram of Kerr spacetime

with  $\mathcal{M} = \mathbb{R}^2 \times \mathbb{S}^2 \setminus \mathcal{R}$ 

## Carter-Penrose diagram of the maximal analytic extension



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# The no-hair theorem

Dorochkevitch, Novikov & Zeldovitch (1965), Israel (1967), Carter (1971), Hawking (1972)

Within 4-dimensional general relativity, a stationary black hole in an otherwise empty universe is necessarily a **Kerr-Newmann black hole**, which is an **electro-vacuum solution** of Einstein equation described by only 3 parameters :

- the total mass  $M$
- the total specific angular momentum  $a = J/(Mc)$
- the total electric charge  $Q$

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Astrophysical black holes have to be electrically neutral :

- $Q = 0$  : **Kerr solution (1963)**

Other special cases :

- $a = 0$  : **Reissner-Nordström solution (1916, 1918)**
- $a = 0$  and  $Q = 0$  : **Schwarzschild solution (1916)**
- $a = 0$ ,  $Q = 0$  and  $M = 0$  : **Minkowski metric (1907)**

# The no-hair theorem : the precise mathematical statement

Any spacetime  $(\mathcal{M}, g)$  that

- is 4-dimensional
- is asymptotically flat
- is stationary
- is a solution of the vacuum Einstein equation
- contains a black hole with a connected regular horizon
- does not contain any closed timelike curve in the domain of outer communications
- is analytic

has a domain of outer communications that is isometric to the domain of outer communications of the Kerr spacetime.

*domain of outer communications* : black hole exterior

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**Possible improvements** : remove the hypotheses of analyticity and non-existence of closed timelike curves (analyticity released recently but only for slowly rotating black holes

[Alexakis, Ionescu & Klainerman, *Duke Math. J.* **163**, 2603 (2014)])

# The Kerr metric is specific to black holes

## Spherically symmetric (non-rotating) case :

### Birkhoff theorem

*Within 4-dimensional general relativity, the spacetime outside any spherically symmetric body is described by Schwarzschild metric*

⇒ No possibility to distinguish a non-rotating black hole from a non-rotating dark star by monitoring orbital motion or fitting accretion disk spectra

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## Rotating axisymmetric case :

*No Birkhoff theorem*

Moreover, no “reasonable” matter source has ever been found for the Kerr metric (the only known source consists of two counter-rotating thin disks of collisionless particles [Bicak & Ledvinka, PRL 71, 1669 (1993)])

⇒ The Kerr metric is specific to rotating black holes (in 4-dimensional general relativity)

# Lowest order no-hair theorem : quadrupole moment

Asymptotic expansion (large  $r$ ) of the metric in terms of multipole moments

$(\mathcal{M}_k, \mathcal{J}_k)_{k \in \mathbb{N}}$  [Geroch (1970), Hansen (1974)] :

- $\mathcal{M}_k$  : mass  $2^k$ -pole moment
- $\mathcal{J}_k$  : angular momentum  $2^k$ -pole moment

$\implies$  For the Kerr metric, all the multipole moments are determined by  $(M, a)$  :

- $\mathcal{M}_0 = M$
- $\mathcal{J}_1 = aM = J/c$
- $\mathcal{M}_2 = -a^2 M = -\frac{J^2}{c^2 M}$  (\*)  $\leftarrow$  mass quadrupole moment
- $\mathcal{J}_3 = -a^3 M$
- $\mathcal{M}_4 = a^4 M$
- $\dots$

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- ...

Measuring the three quantities  $M$ ,  $J$ ,  $\mathcal{M}_2$  provides a compatibility test w.r.t. the Kerr metric, by checking (\*)



# Theoretical alternatives to the Kerr black hole

## Within general relativity

The compact object is not a black hole but

- boson stars
- gravastar
- dark stars
- ...

## Beyond general relativity

The compact object is a black hole but in a theory that differs from GR :

- Einstein-Gauss-Bonnet with dilaton
- Chern-Simons gravity
- Hořava-Lifshitz gravity
- Einstein-Yang-Mills
- ...

# Is general relativity unique?

Yes if we assume

- a 4-dimensional spacetime
- gravitation only described by a metric tensor  $g$
- field equation involving only derivatives of  $g$  up to second order
- diffeomorphism invariance
- $\nabla \cdot T = 0$  ( $\implies$  weak equivalence principle)

The above is a consequence of **Lovelock theorem (1972)**.

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- field equation involving only derivatives of  $g$  up to second order
- diffeomorphism invariance
- $\nabla \cdot T = 0$  ( $\implies$  weak equivalence principle)

The above is a consequence of **Lovelock theorem (1972)**.

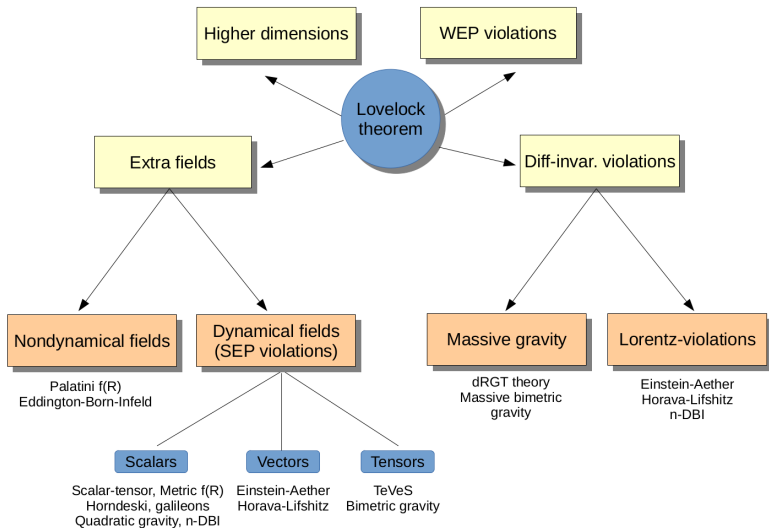
However, GR is certainly not the ultimate theory of gravitation :

- it is not a quantum theory
- cosmological constant / dark energy problem

GR is generally considered as a low-energy limit of a more fundamental theory :

- string theory
- loop quantum gravity
- ...

## Extensions of general relativity



[Berti et al., CGQ 32, 243001 (2015)]

# Alternative theories of gravity

Class of **metric theories** of gravity, described by the action

$$S = S_{\text{grav}} + S_{\text{mat}}(\mathbf{g}, \Psi_1, \Psi_2, \dots)$$

$\mathbf{g}$  : spacetime metric,  $\Psi_1, \Psi_2, \dots$  : matter fields

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General relativity :

$$S_{\text{grav}} = \frac{1}{16\pi G} \int R \sqrt{-g} d^4x \quad (\text{Einstein-Hilbert action})$$

$R$  : scalar curvature of metric  $\mathbf{g}$  :  $R := g^{\mu\nu} R^\sigma_{\mu\sigma\nu}$

$R^\alpha_{\beta\mu\nu}$  : Riemann curvature tensor of  $\mathbf{g}$

# Scalar-tensor theories

Gravity action depends on a scalar field  $\phi$  in addition to the spacetime metric  $g$  :

$$S_{\text{grav}} = S_{\text{grav}}(g, \phi) = \frac{1}{16\pi G} \int \left[ \phi R - \frac{\omega(\phi)}{\phi} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \phi^2 V \right] \sqrt{-g} d^4x$$

Special case : Jordan-Fierz-Brans-Dicke theory :  $\omega(\phi) = \text{const}$

**No-hair theorem** : for a *real* scalar field  $\phi$ , the only black hole solution is Kerr

However, for *complex* scalar fields, **hairy black hole** solutions exist [Herdeiro & Radu, arXiv:1403.2757 (2014)]

## Einstein-Gauss-Bonnet with dilaton

Gravity action is quadratic in the curvature :

$$S_{\text{grav}} = \frac{1}{16\pi G} \int \left[ R + e^{\gamma\phi} (R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}) - \frac{\beta}{2} (g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + 2V(\phi)) \right] \sqrt{-g} d^4x$$

Low energy expansion of **string theory**



## Chern-Simons gravity

Gravity action is quadratic in the curvature :

$$S_{\text{grav}} = \frac{1}{16\pi G} \int \left[ R + \frac{\alpha}{4} \phi R^*_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - \frac{\beta}{2} (g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + 2V(\phi)) \right] \sqrt{-g} d^4x$$

Low energy expansion of **string theory** or **loop quantum gravity**

# Test : are astrophysical black holes Kerr black holes ?

- GR  $\implies$  Kerr BH (no-hair theorem)
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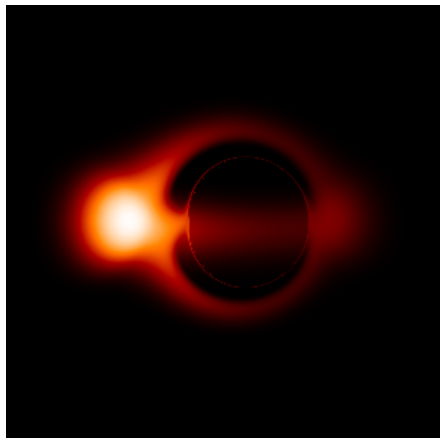
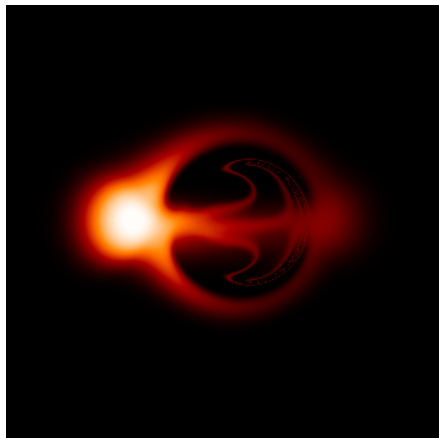
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- pulsar orbiting Sgr A\* : the Holly Grail !

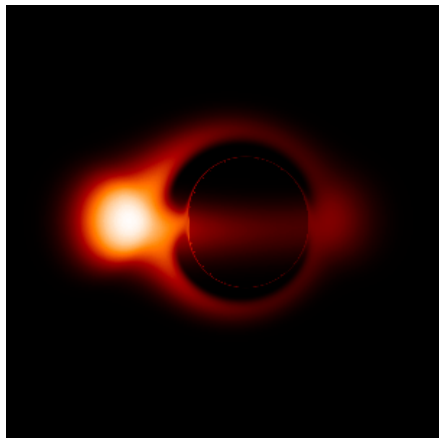
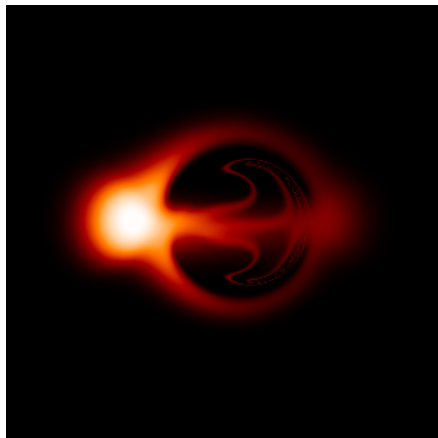
## An example : comparison with a boson star

Kerr BH  $a/M = 0.9$ Boson star  $k = 1, \omega = 0.70 m/\hbar$ 

[Vincent, Meliani, Grandclément, Gourgoulhon &amp; Straub, CQG 33, 105015 (2016)]



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*More examples next Wednesday...*