Gravitational Wave Data Analysis for PTAs

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Our Likelihood is an N_{toa}D gaussian

$$\mathcal{L} \propto \exp\left(-\frac{1}{2}\vec{r}\cdot\mathbb{C}^{-1}\cdot\vec{r}\right)$$

- $\vec{r}$: residuals (data - model)
- $\mathbb{C}$: covariance matrix (noise)

likelihood is probability that our data is observed, if our model is correct

- $\max[p\,(d \mid m(\lambda))] \rightarrow$
  model most likely to have produced our data

What is the probability that this (or any) model is correct?

Bayes Theorem

Probability hypothesis $\mathcal{H}$ is true given you observed data d:

- **Posterior probability,** $\mathcal{P}$

$$p\,(\mathcal{H} \mid d) = \frac{p(\mathcal{H}) \; p\,(d \mid \mathcal{H})}{p(d)}$$

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- **Prior probability** p

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- Prior probability p
- **Likelihood,** $\mathcal{L}$

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- Prior probability p
- Likelihood, $\mathcal{L}$
- **Evidence (marginal likelihood),** $\mathcal{E}$

Need a way to compute Posterior distribution without computing Evidence

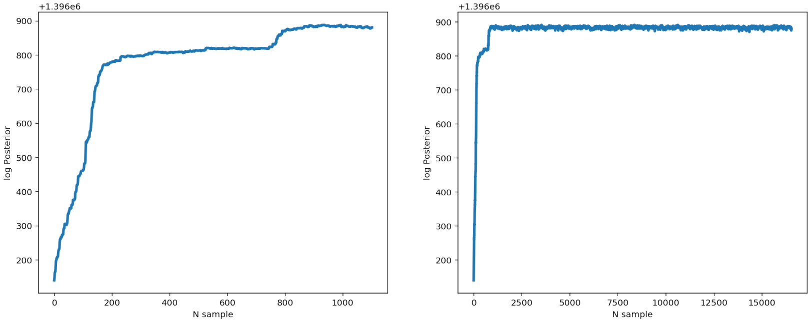
Sample from unnormalized distribution:

- Markov chain Monte Carlo
  (*e.g.* emcee, kombine, PTMCMCSampler)
- Nested Sampling (*e.g.* MultiNest)

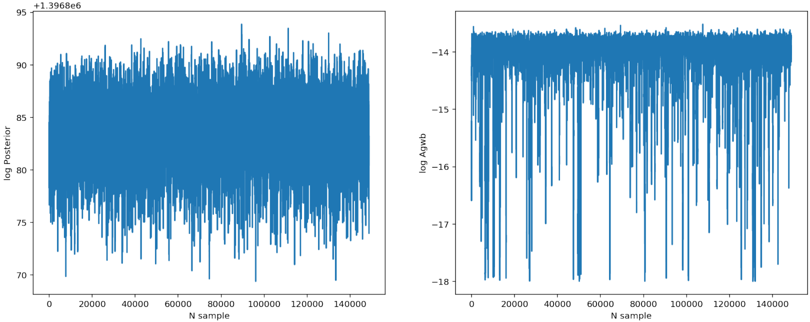
histogram of N samples $\rightarrow$ Posterior as $N \rightarrow \infty$

inference with samples

in practice first samples are biased – **burn in**



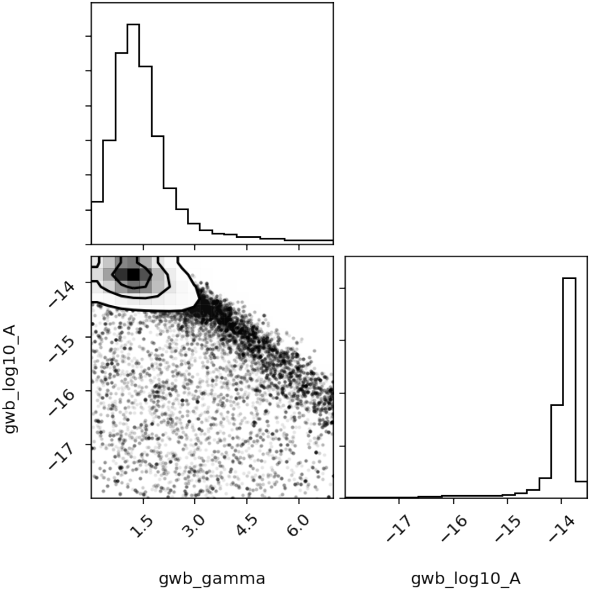
a properly converged chain of samples



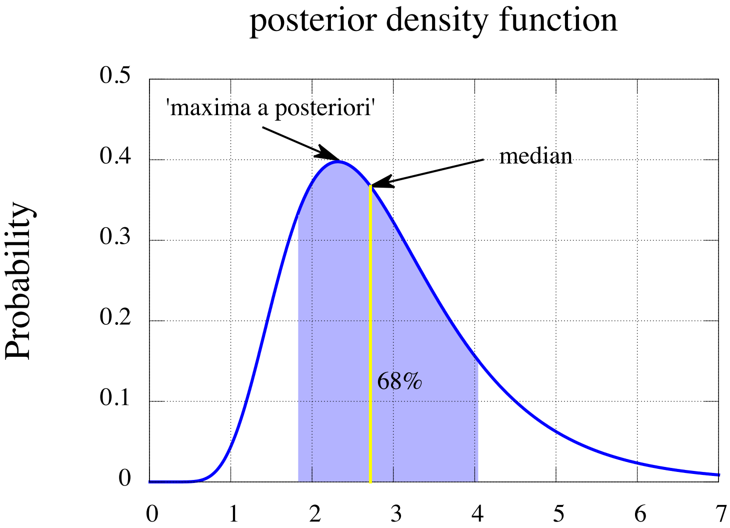
Marginalization – posterior for subset of parameters

$$\mathcal{P}(x) = \int \mathcal{P}(x, y, z)\, \mathrm{d}y\, \mathrm{d}z$$

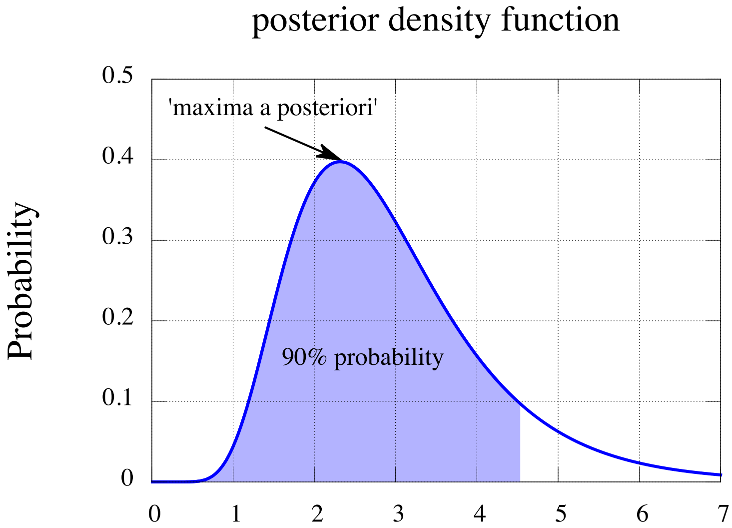
marginalization



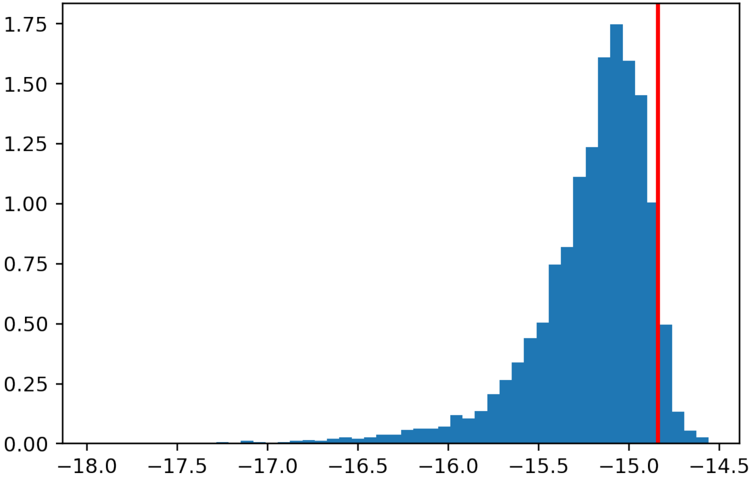
parameter estimation



Upper Limits



PTA upper limit on A_{gw} – first marginalize over all other params, then integrate to 95%



model selection

$$\mathcal{E}_1 = \int \mathcal{L}_1(x,y,z) \cdot p(x,y,z)\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$$

$$\mathcal{E}_2 = \int \mathcal{L}_2(x,y) \cdot p(x,y)\,\mathrm{d}x\,\mathrm{d}y$$

$$\mathcal{B}_{1,2} = \frac{\mathcal{E}_1}{\mathcal{E}_2}$$

Bayes factor is betting odds.

model selection – Bayes factor

$$\mathcal{B}_{1,2} = \frac{\mathcal{E}_1}{\mathcal{E}_2}$$

$\mathcal{B}$	"sigma"	
2	1σ	no evidence
20	2σ	weak evidence
2×10^6	5σ	very strong evidence

(somtimes reported as $\log \mathcal{B}$)

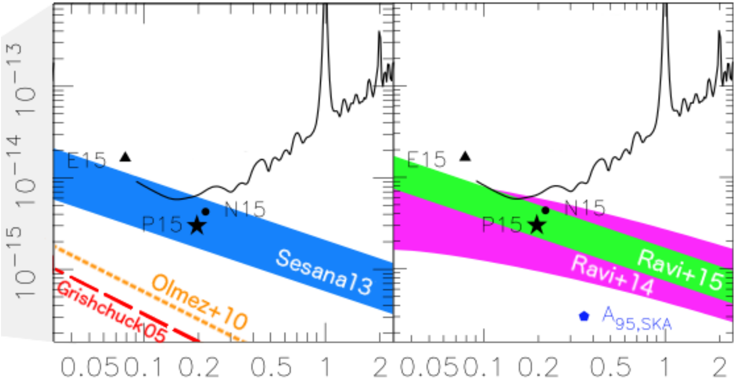
Currently no GW detection from PTAs

95 % upper limit for stochastic GW background:

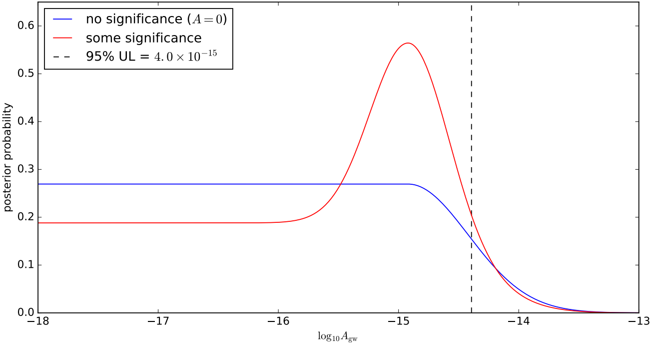
EPTA	3.0×10^{-15}	Lentati + (2015)
PPTA	1.0×10^{-15}	Shannon + (2015)
NANOGrav	1.5×10^{-15}	Arzoumanian + (2015)
IPTA	1.7×10^{-15}	Verbiest + (2016)

current ULs are ephemeris dependent

some current PTA results



Detection of GWB will be a gradual process



Detection claim must compare multiple models

How are you sure it's GWB

and not some other source of red noise?

- common red process
- correlated red process
  - quadrupole (Hellings-Downs)
  - dipole (ephemeris)
  - monopole (clocks)
- different spectral indices