Astrophysical sources of gravitational waves: supermassive black hole binaries

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EPTA

INTERNATIONAL PULSAR TIMING ARRAY
IPTA

Consider a small metric perturbation

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \qquad h_{\mu\nu} \ll 1$$

The linearization of the EEs results in a wave equation

$$\Box \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu}$$

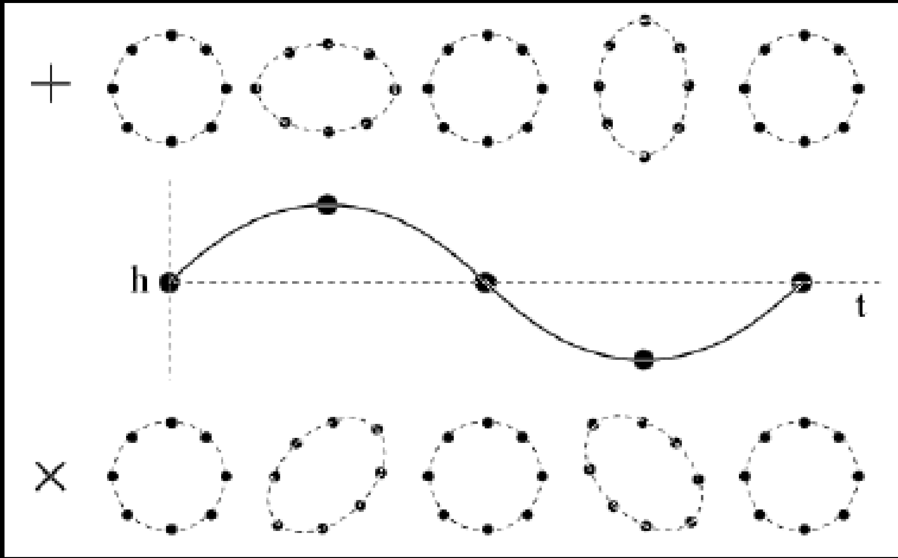
The solution is a wave travelling
At the speed of light:
GRAVITATIONAL WAVES

$$g_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 + h_+^{TT} & h_\times^{TT} \\ 0 & 0 & h_\times^{TT} & 1 - h_+^{TT} \end{pmatrix}$$

$$\bar{h}^{ij}(t, r) = \frac{2G}{c^4} \left[\frac{d^2}{dt^2} q^{ij} \left(t - \frac{r}{c} \right) \right]$$

They are proportional to the
Second derivative of the mass
quadrupol moment and they carry
an energy given by

$$L_{gw} = \frac{G}{5c^5} \left\langle \sum_{ij} \frac{d^3}{dt^3} Q_{ij} \left(t - \frac{x}{c} \right) \frac{d^3}{dt^3} Q^{ij} \left(t - \frac{x}{c} \right) \right\rangle$$

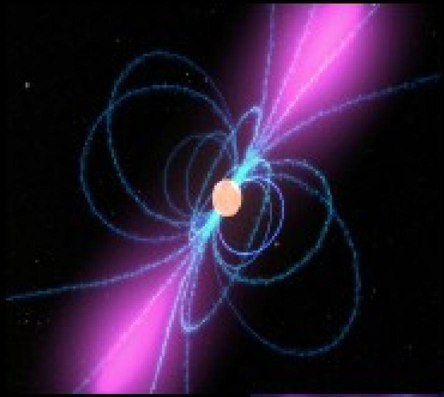


GWs are transversal and have two independent polarizations

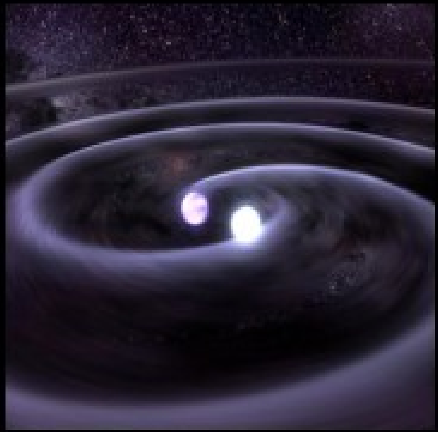
Gravitational wave sources

Massive compact systems with a time varying mass quadrupole momentum:

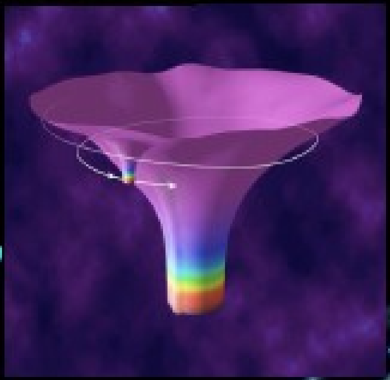
1-collapses and explosions (supernovae, GRBs)



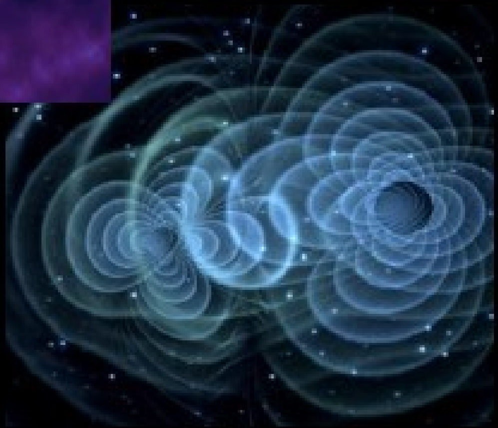
2-rotating asymmetric objects (pulsars, MSPs)



3-binary systems:

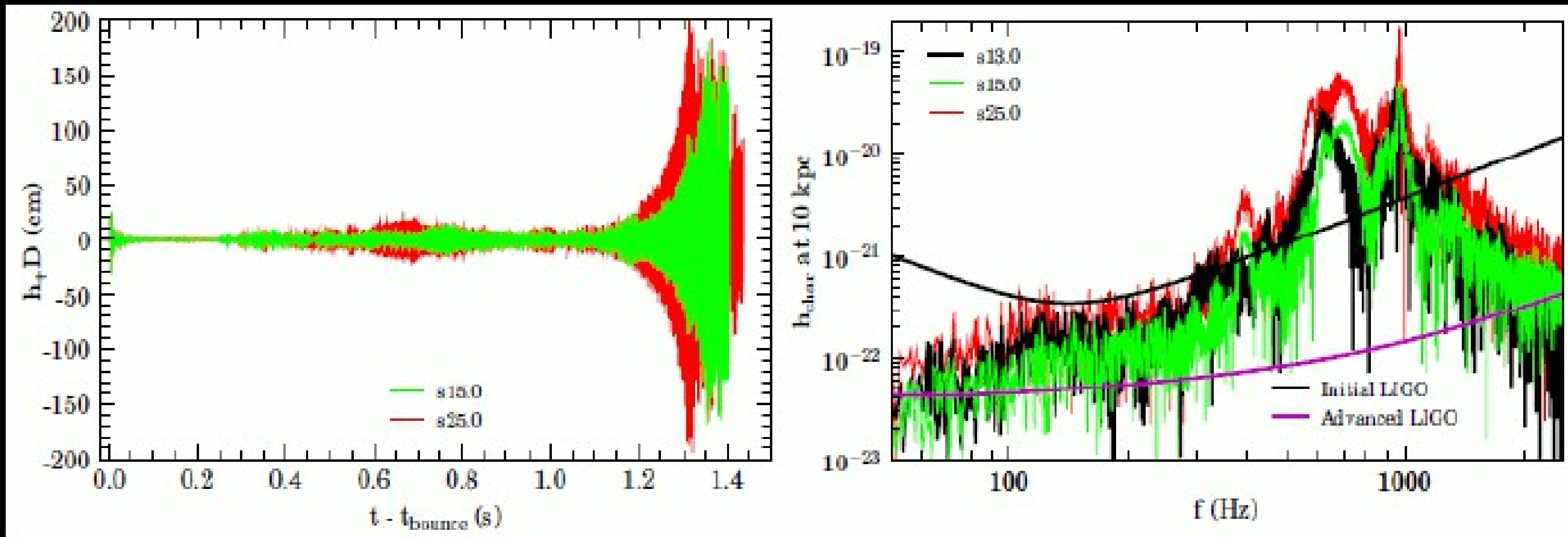


- **a-stellar compact remnants (WD-WD, NS-NS, NS-BH, BH-BH)**
- **b-extreme mass ratio inspirals (EMRIs), CO falling into a massive black hole**
- **c-massive black hole binaries (MBHBs) forming following galaxy mergers**

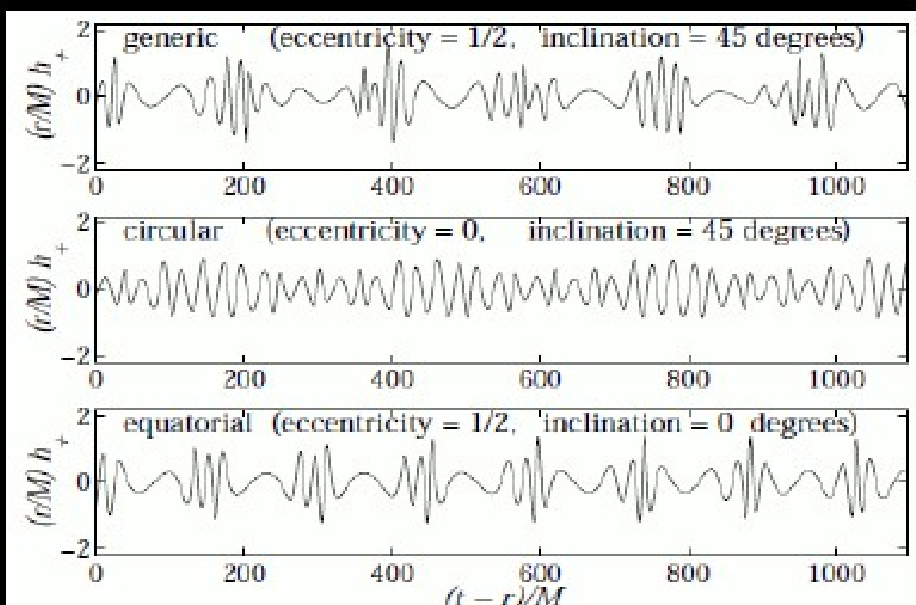


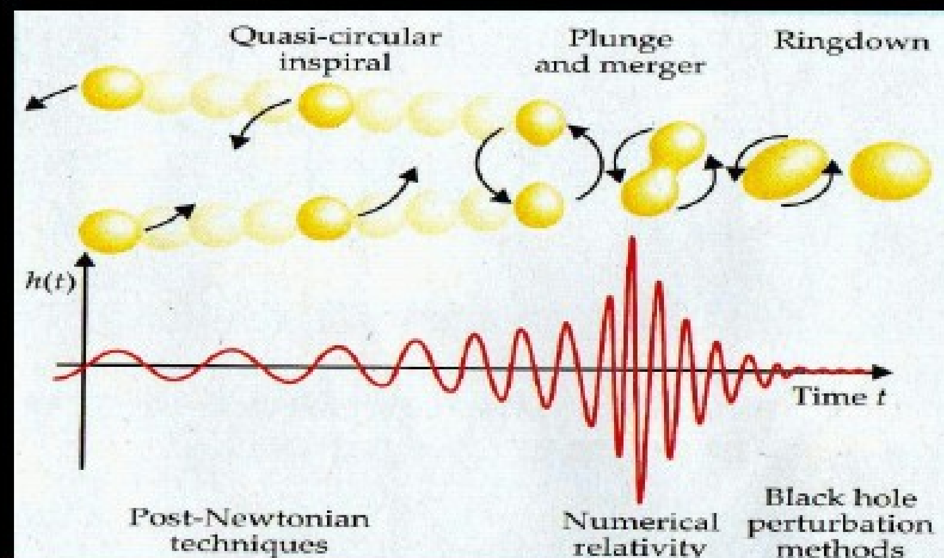
Example of gravitational waveforms

Supernova explosion (credits C. Ott)



EMRIs (credits Drasco & Hughes)





Black hole binaries

Heuristic scalings

We want compact accelerating systems
Consider a BH binary of mass M, and semimajor axis a

$$h \sim \frac{R_S}{a}\frac{R_S}{r} \sim \frac{(GM)^{5/3}(\pi f)^{2/3}}{c^4 r}$$

In astrophysical scales

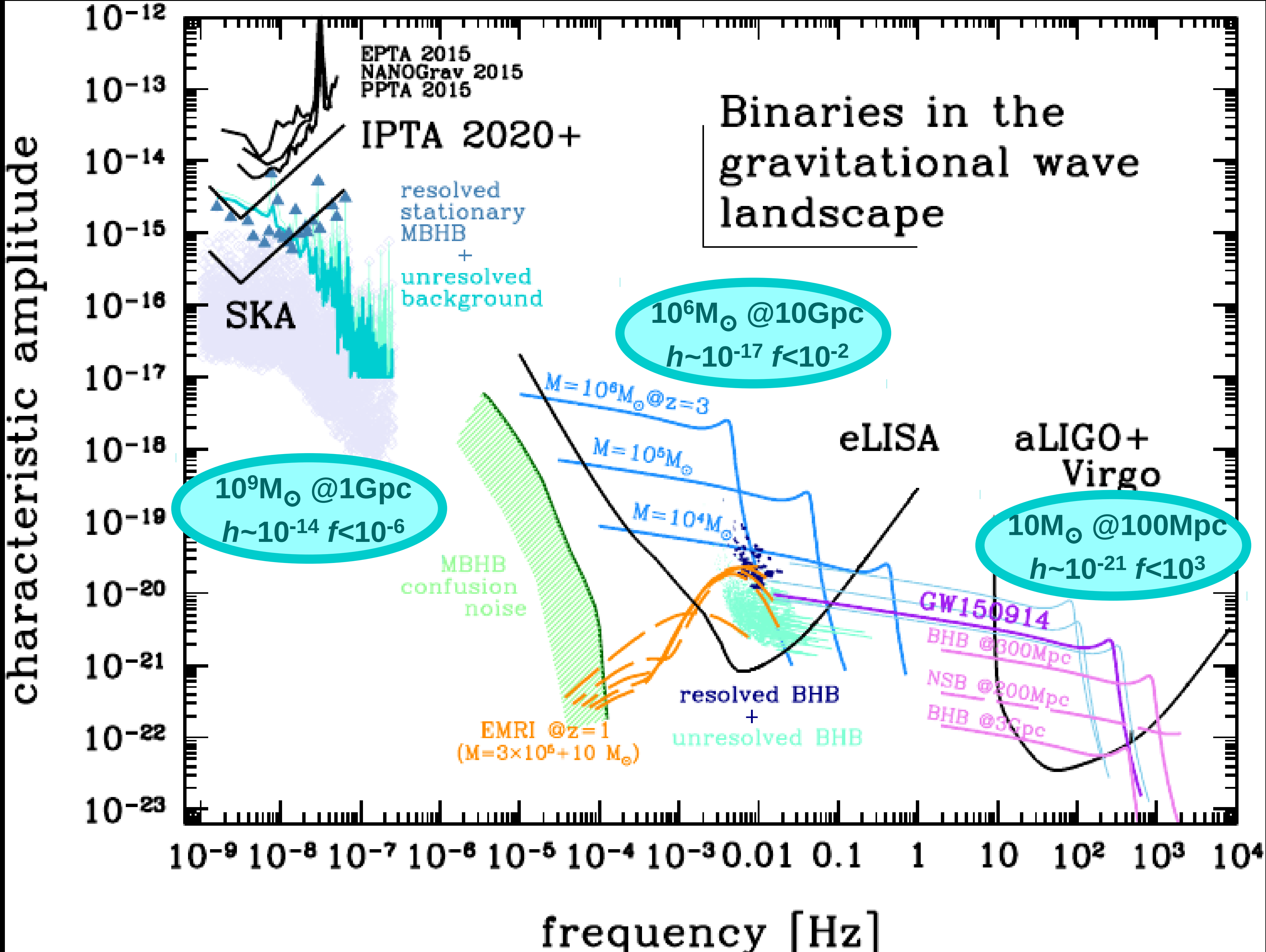
$$h \sim 10^{-20}\frac{M}{M_\odot}\frac{\mathrm{Mpc}}{D}$$

$$f \sim \frac{c}{2\pi R_s} \sim 10^4\mathrm{Hz}\frac{M_\odot}{M}$$

10 $M_\odot$ binary at 100 Mpc: $h \sim 10^{-21}$, $f < 10^{3}$

10^6 $M_\odot$ binary at 10 Gpc: $h \sim 10^{-18}$, $f < 10^{-2}$

10^9 $M_\odot$ binary at 1Gpc: $h \sim 10^{-14}$, $f < 10^{-5}$



Binaries in the gravitational wave landscape
EPTA 2015
NANOGrav 2015
PPTA 2015
IPTA 2020+
SKA
resolved stationary MBHB + unresolved background
10⁶M⊙ @10Gpc
h~10⁻¹⁷ f<10⁻²
10⁹M⊙ @1Gpc
h~10⁻¹⁴ f<10⁻⁶
10M⊙ @100Mpc
h~10⁻²¹ f<10³
M=10⁶M⊙@z=3
M=10⁵M⊙
M=10⁴M⊙
eLISA
aLIGO+ Virgo
MBHB confusion noise
GW150914
BHB @300Mpc
NSB @200Mpc
BHB @3Gpc
resolved BHB + unresolved BHB
EMRI @z=1
(M=3×10⁵+10 M⊙)
characteristic amplitude
frequency [Hz]

Observational facts

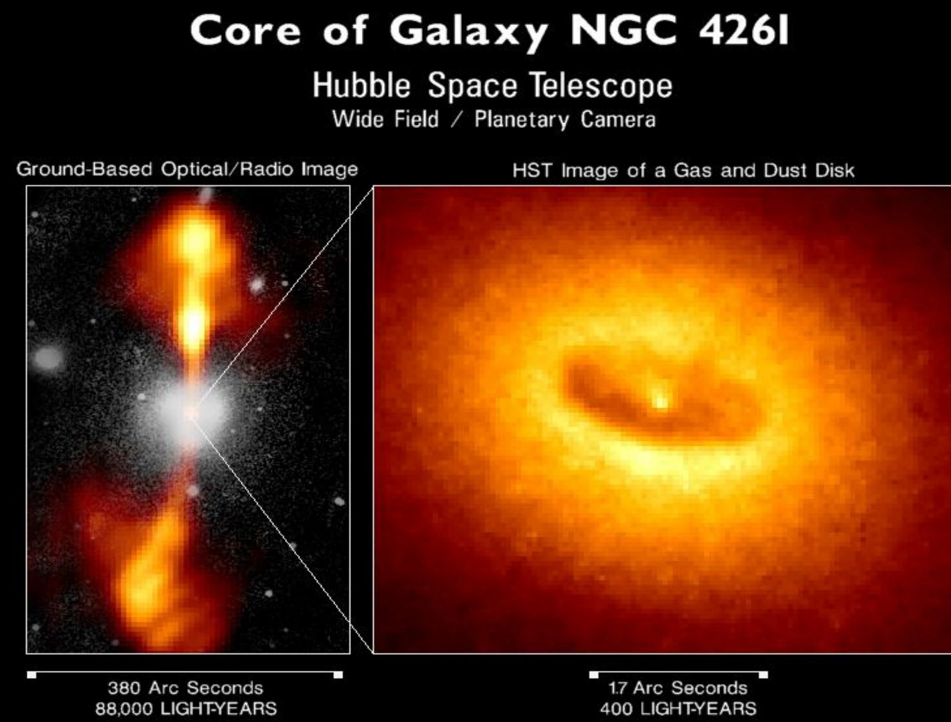
1- In all the cases where the inner core of a galaxy has been resolved (i.e. In nearby galaxies), a massive compact object (which I'll call Massive Black Hole, MBH for convenience) has been found in the center.

2- MBHs must be the central engines of Quasars: the only viable model to explain this cosmological objects is by means of gas accretion onto a MBH.

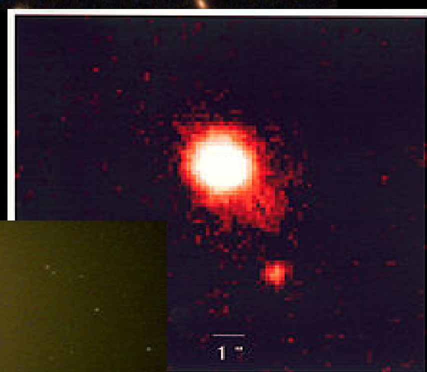
3- Quasars have been discovered at z~7, their inferred masses are ~10^9 solar masses!

THERE WERE 10^9 SOLAR MASS BHs WHEN THE UNIVERSE WAS <1Gyr OLD!!!

MBH formation and evolution have profound consequences for GW astronomy



Core of Galaxy NGC 4261
Hubble Space Telescope
Wide Field / Planetary Camera
Ground-Based Optical/Radio Image
HST Image of a Gas and Dust Disk
380 Arc Seconds
88,000 LIGHT-YEARS
1.7 Arc Seconds
400 LIGHT-YEARS



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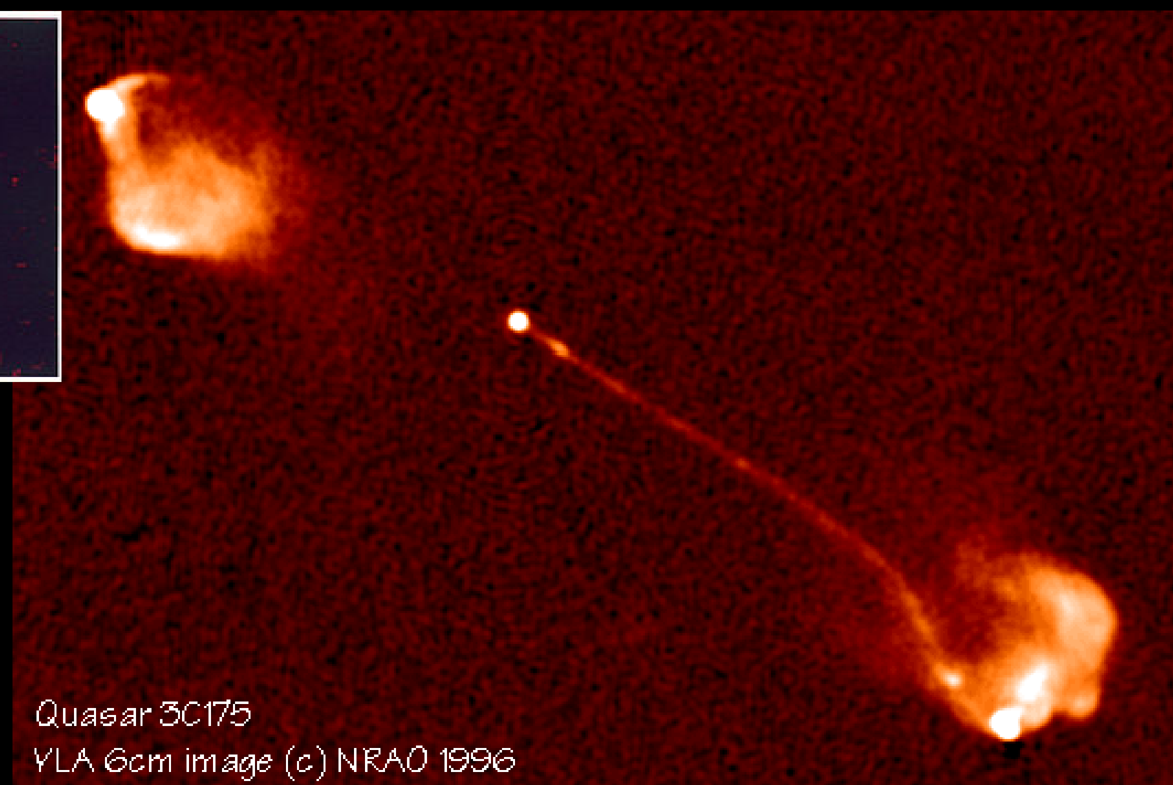
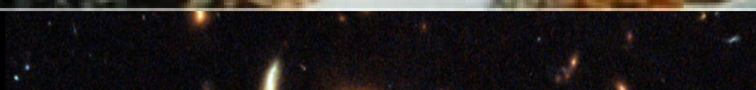
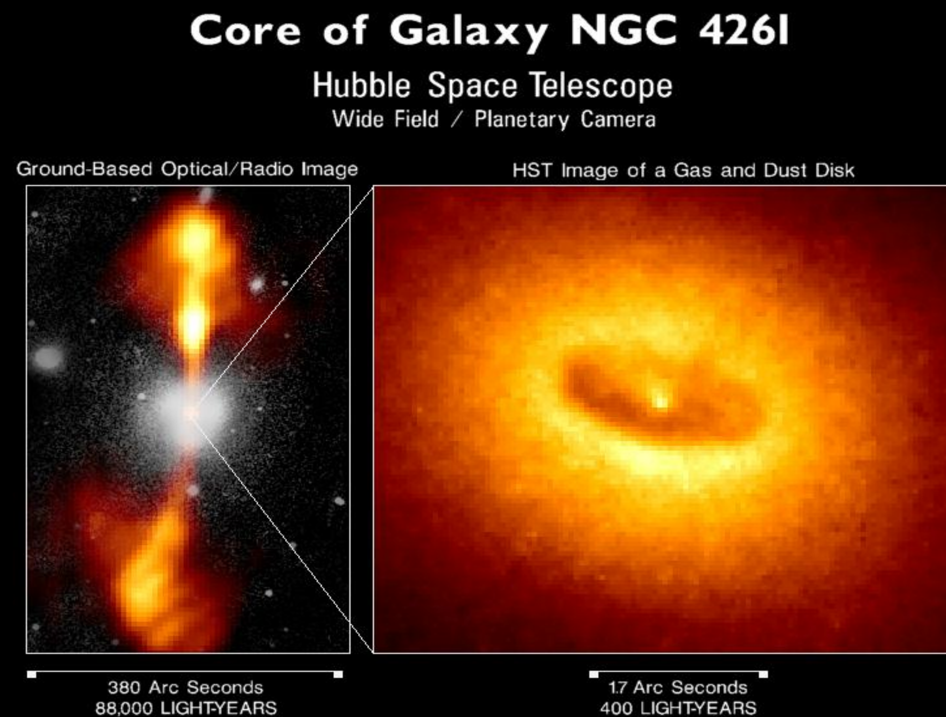


Quasar 3C175
VLA 6cm image (c) NRAO 1996

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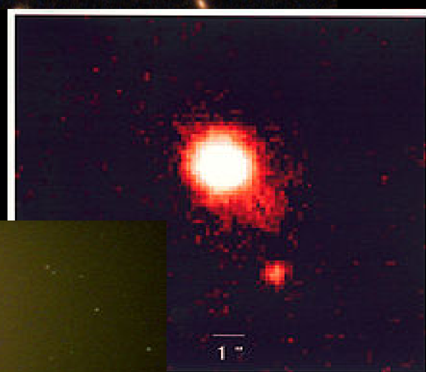
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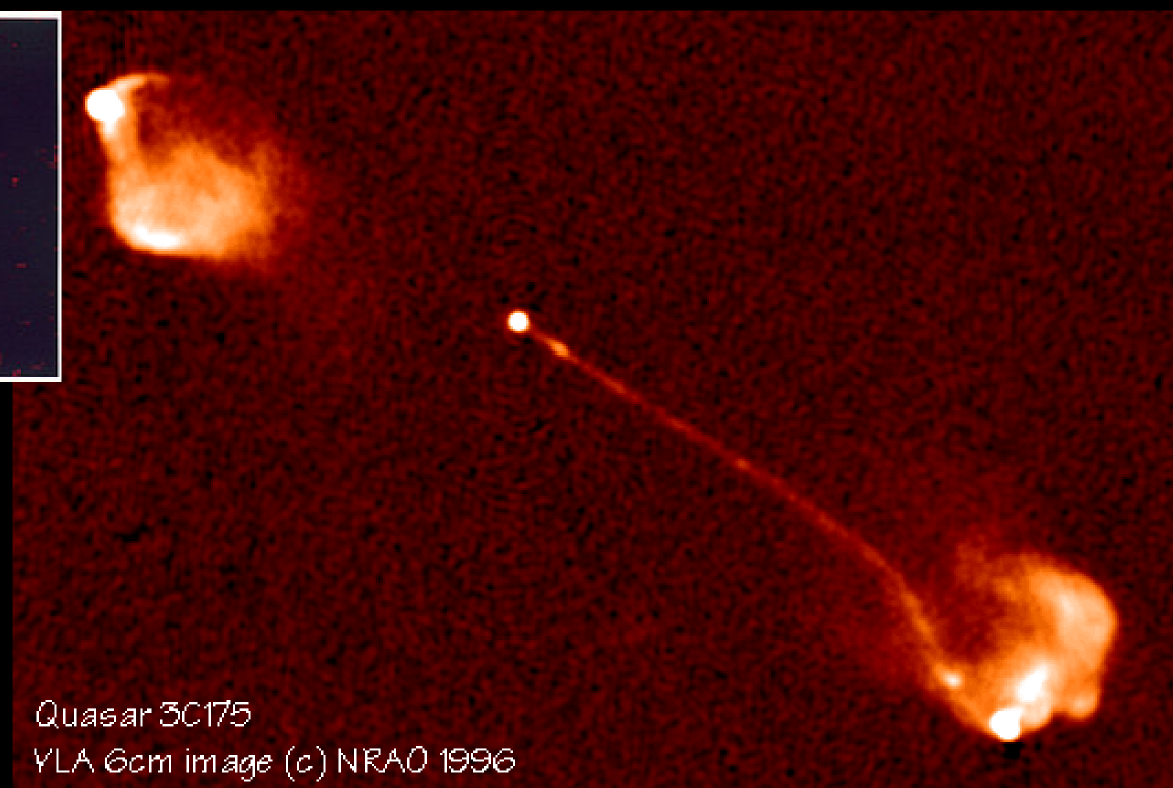
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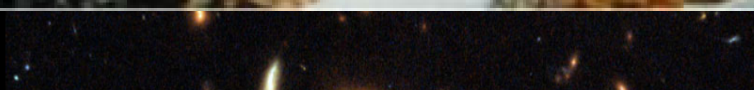
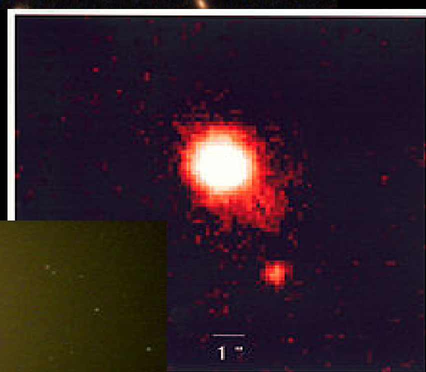
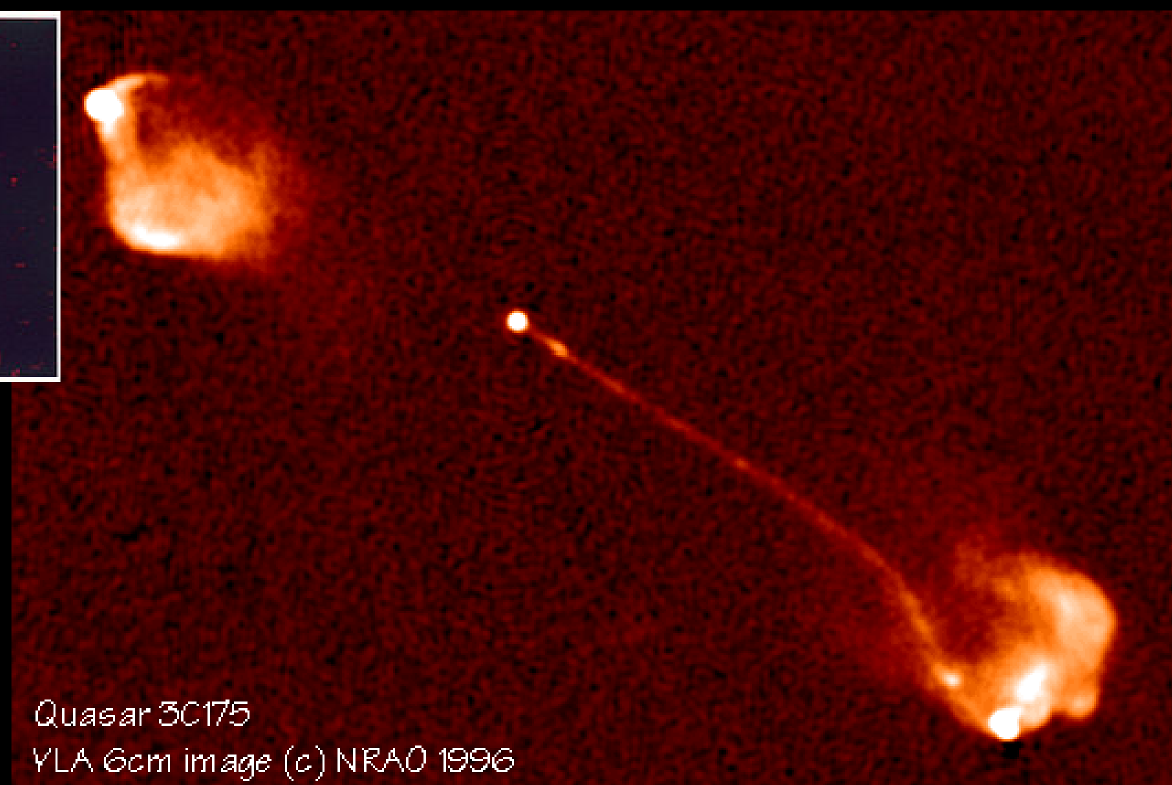
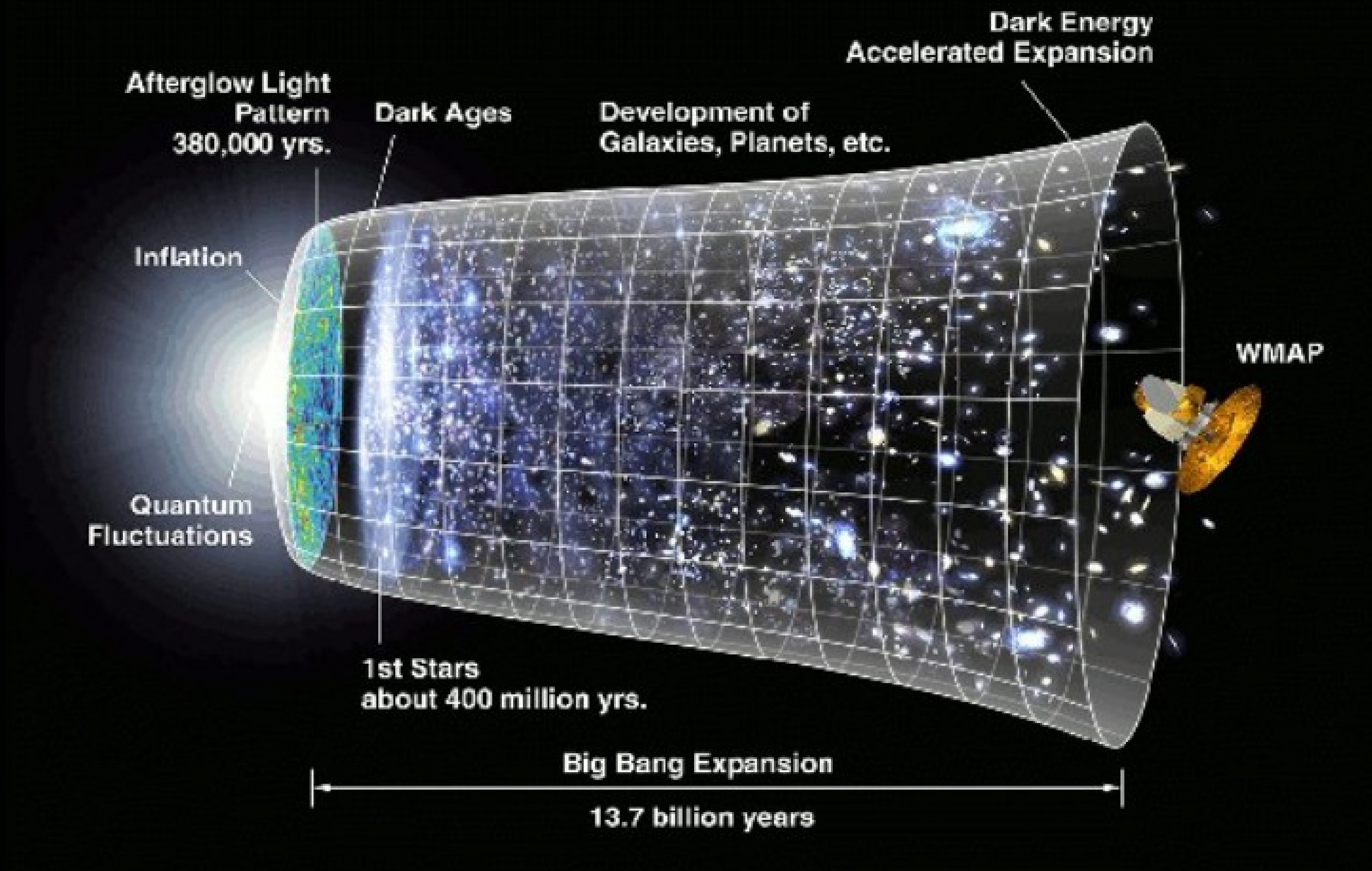




Quasar 3C175
VLA 6cm image (c) NRAO 1996

Cosmology in two slides

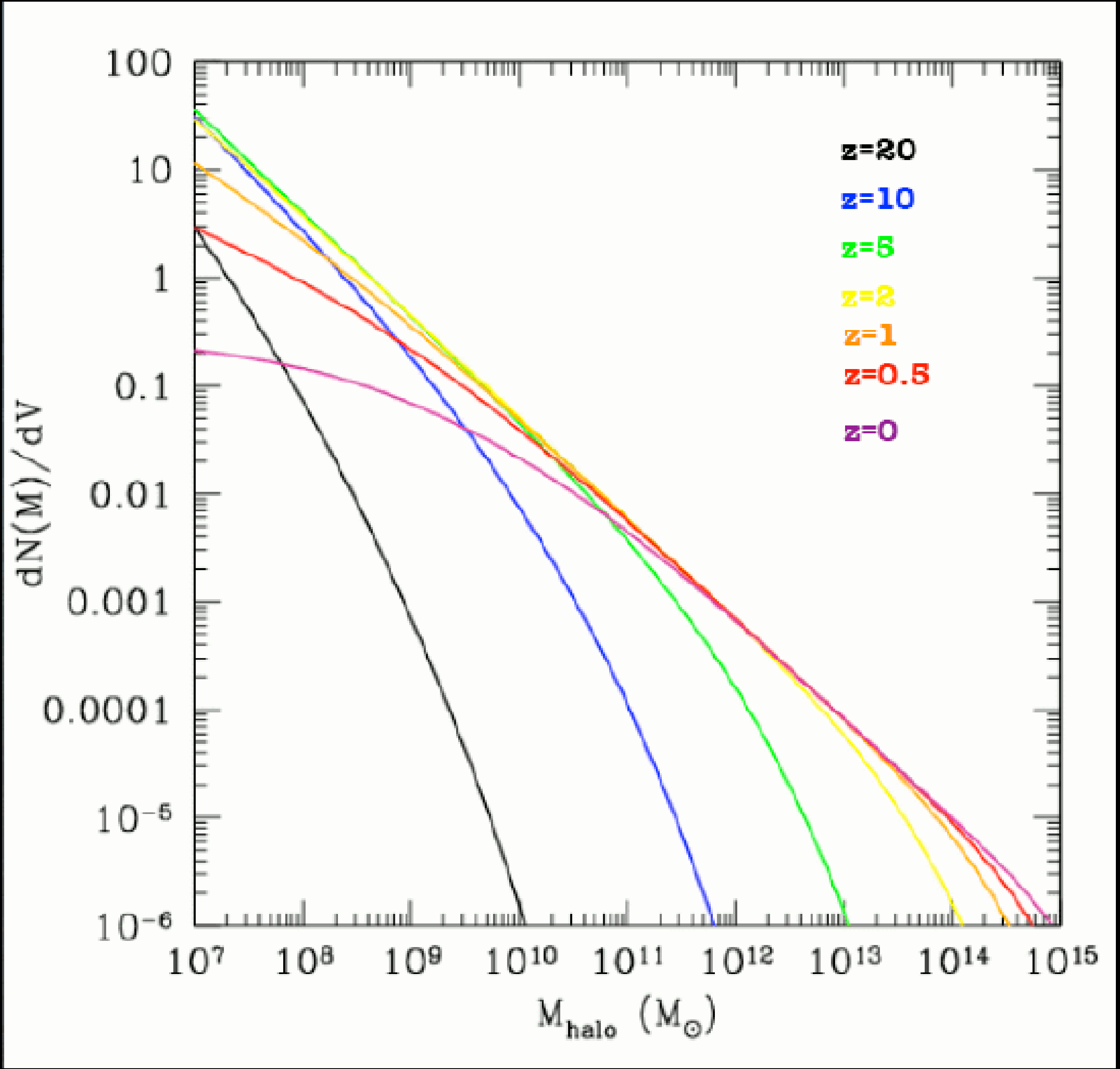
According to our best cosmological models, we live in a *ΛCDM Universe*. The energy content of the Universe is *27%* in the form of *ordinary matter* (~3% baryons, ~24% dark matter) and *73%* in the form of a *cosmological constant* (or Dark energy, or whatever), which would be responsible of the accelerated expansion.



The typical halo mass is an increasing function of time: bottom-up or

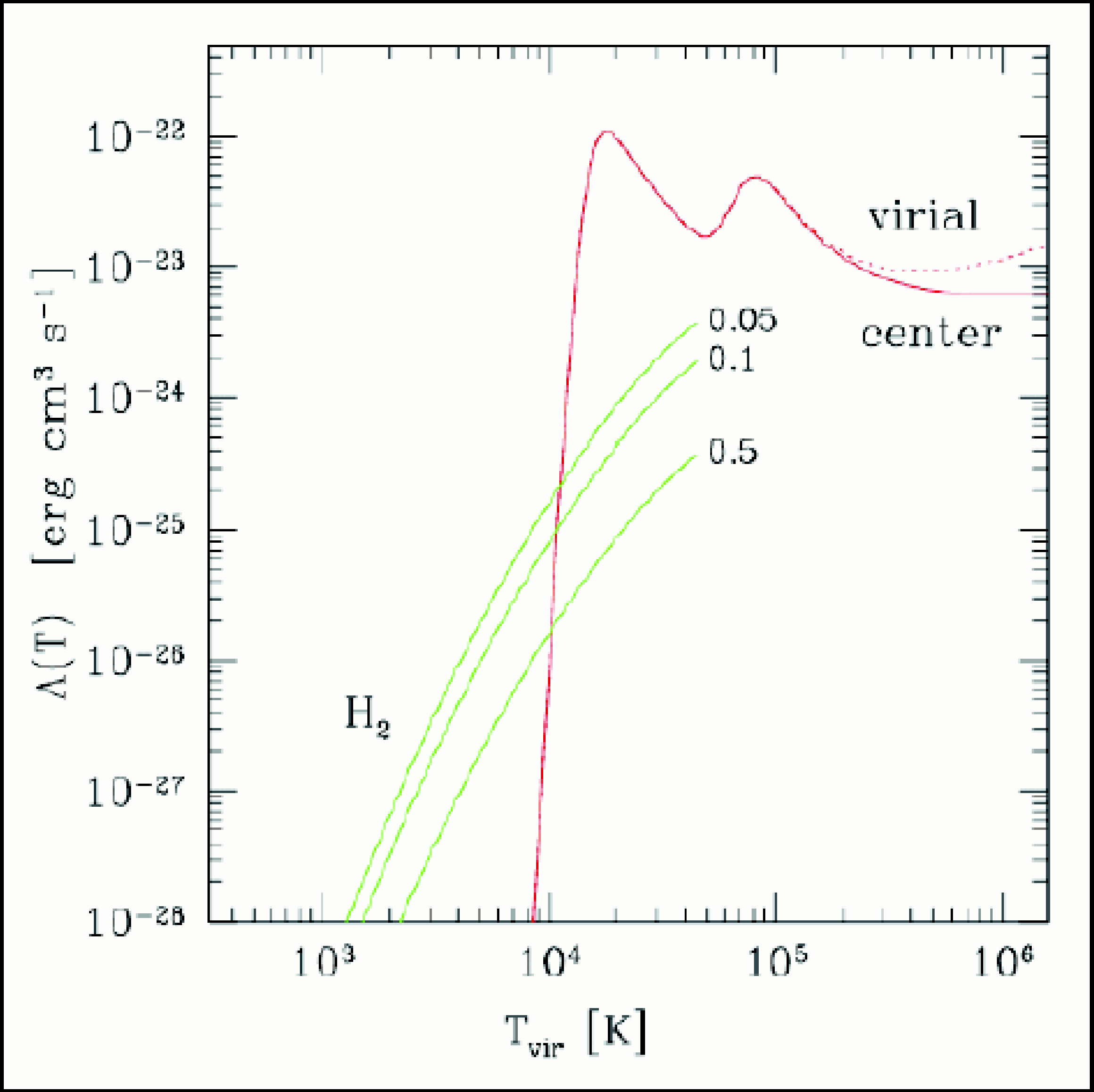
HIERARCHICAL structure formation!

The halo mass function evolves in time (redshift) with larger halos forming at lower redshifts (later times).



$$T_{\rm vir} = 1.98 \times 10^4 \left(\frac{\mu}{0.6}\right)\left(\frac{M}{10^8\ h^{-1}\ M_\odot}\right)^{2/3}\left[\frac{\Omega}{\Omega(z)}\ \frac{\Delta_c}{18\pi^2}\right]^{1/3}\left(\frac{1+z}{10}\right)\ \mathrm{K}$$

What happens to the baryons? In the early Universe most of the baryonic matter is in form of hot *atomic (H) or molecular (H_2) Hydrogen*.



Baryons need to cool down (i.e. loose energy) in order to condense in dense structures and form stars.

The only way to cool down is through transition between different atomic or molecular levels.

We need to excite high energy levels to radiate this energy away.

The only way is collisional excitation: *we need high temperatures!!!*

Atomic Hydrogen can cool only at temperatures>10^4K, while H_2 can cool already at 10^3K.

Seed BH formation from POPIII stars

H_2 cooling → $T=10^3$ K → $z=30$ $M_{DM}=10^6\ M_\odot$

↓

Subsonic collapse

↓

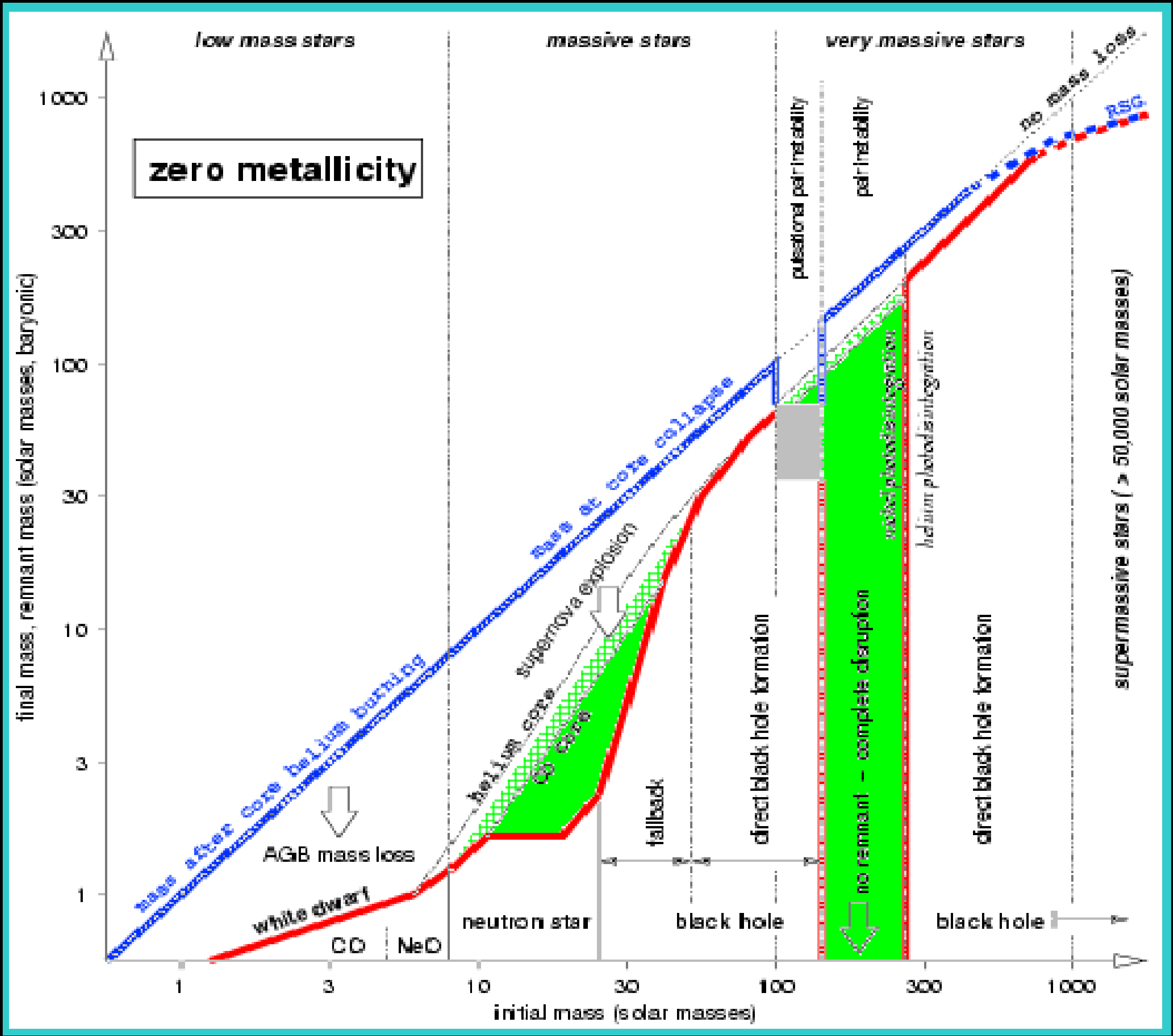
No fragmentation

↓

Formation of VMSs

↓

Intermediate mass seed BHs



Heger & Woosley 2002

Seed BH formation from direct collapse

A seed BH can directly form following the collapse of a giant gas cloud.
Two problems:

1- *you need to dissipate the angular momentum of the cloud*

$$\frac{GM^2}{R} \simeq Mc^2 \rightarrow R \simeq \frac{GM}{c^2} \qquad R_{\rm Sch} = 2\frac{GM}{c^2}$$

Angular momentum can halt the collapse when the rotational support equals the gravitational binding energy

$$\frac{J^2}{MR^2} \approx \frac{GM^2}{R} \rightarrow R \approx \frac{J^2}{GM^3} \approx \frac{GM}{v^2} \rightarrow \boxed{R_J \approx \left(\frac{c}{v}\right)^2 R_{\rm Sch}}$$

You need *J*~0, or an efficient way to dissipate *J*.

2- *you need to avoid star formation*

a-if you form stars you have less gas to feed the BH
b-stars are collisionless: you don't dissipate *J* efficiently anymore
c-supernovae blow away gas.

It turns out that both scenarios are viable, and form BH seeds in relatively massive halos (10^7-10^9 solar masses) at high redshift.

POPIII SCENARIO:

-seed BH mass~10^2 solar masses

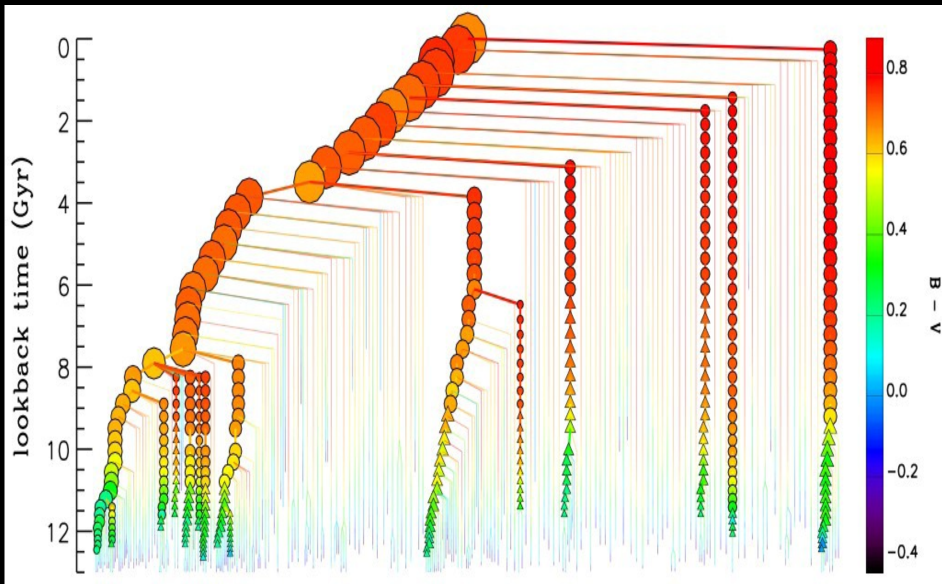
-at redshift 15-20

DIRECT COLLAPSE SCENARIO:

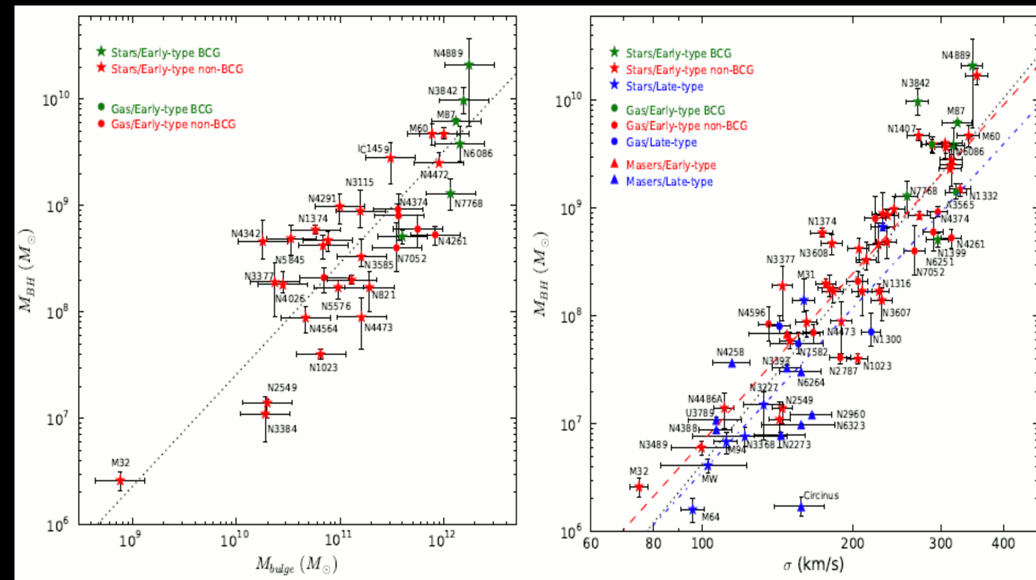
-seed BH mass~10^4-10^5 solar masses

-at redshift 15-10

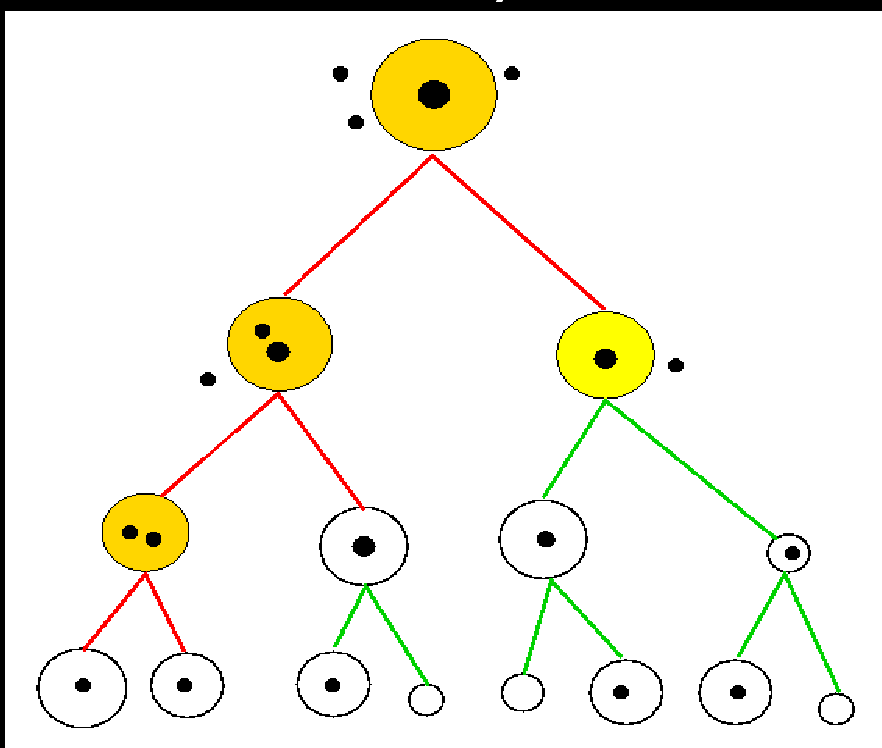
Structure formation in a nutshell



(From de Lucia et al. 2006)

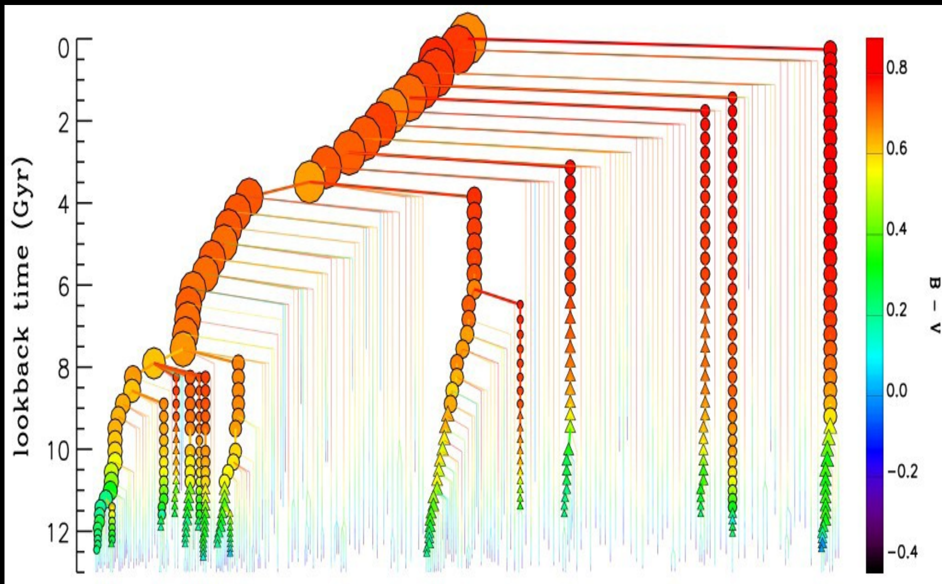


(Ferrarese & Merritt 2000, Gebhardt et al. 2000)



(Menou et al 2001, Volonteri et al. 2003)

Structure formation in a nutshell



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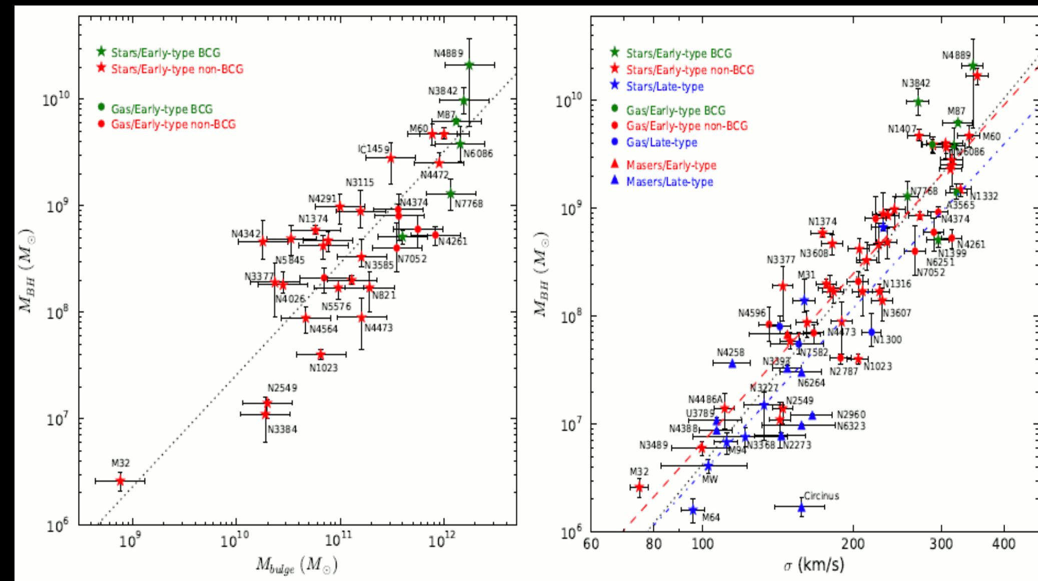

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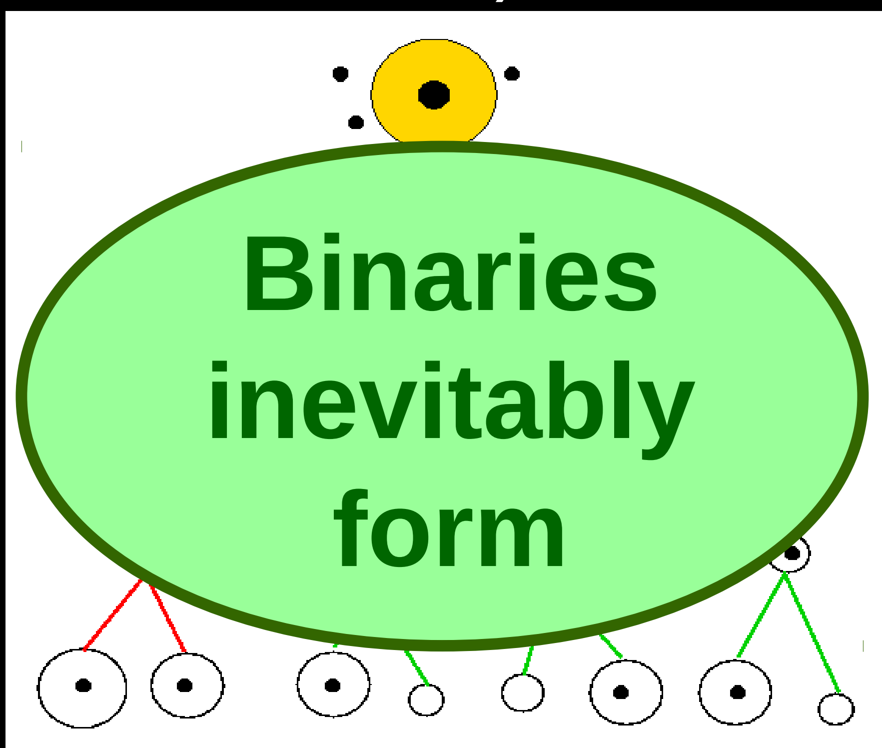


Binaries inevitably form

(Menou et al 2001, Volonteri et al. 2003)

*Where and when do the first MBH seeds form?
*How do they grow along the cosmic history?
*What is their role in galaxy evolution?
*What is their merger rate?
*How do they pair together and dynamically evolve?

Accretion

During mergers, gravitational instabilities drive cold gas toward the galactic nucleus, this gas can form a thin disk around the MBH, starting the accretion process.

Now consider a flux of proton with density ρ being accreted onto a BH of mass *M*. The accreting material emits radiation with a luminosity *L*. Equating the gravitational force (acting on the accreting material) to the force due to the radiation pressure (exerted by the outward radiation emitted by the accretion disk itself)

$$F_g = \frac{GMm}{r^2},$$

$$F_l = \frac{L\sigma_T}{4\pi r^2 c},$$

one found an equilibrium condition (in the spherical limit), which is commonly known as *Eddington accretion limit*, described by the *Eddington luminosity*:

$$L_{\rm Edd} = \frac{4\pi G M m_{\rm p} c}{\sigma_{\rm T}}$$

L_{EDD}=1.38x10^{38} erg/s for a solar mass BH and scales as the BH mass. A 10^9 solar mass MBH shines with a luminosity of about 10^{47} erg/s (10^{14} Suns or 1000 MWs)!!!!!

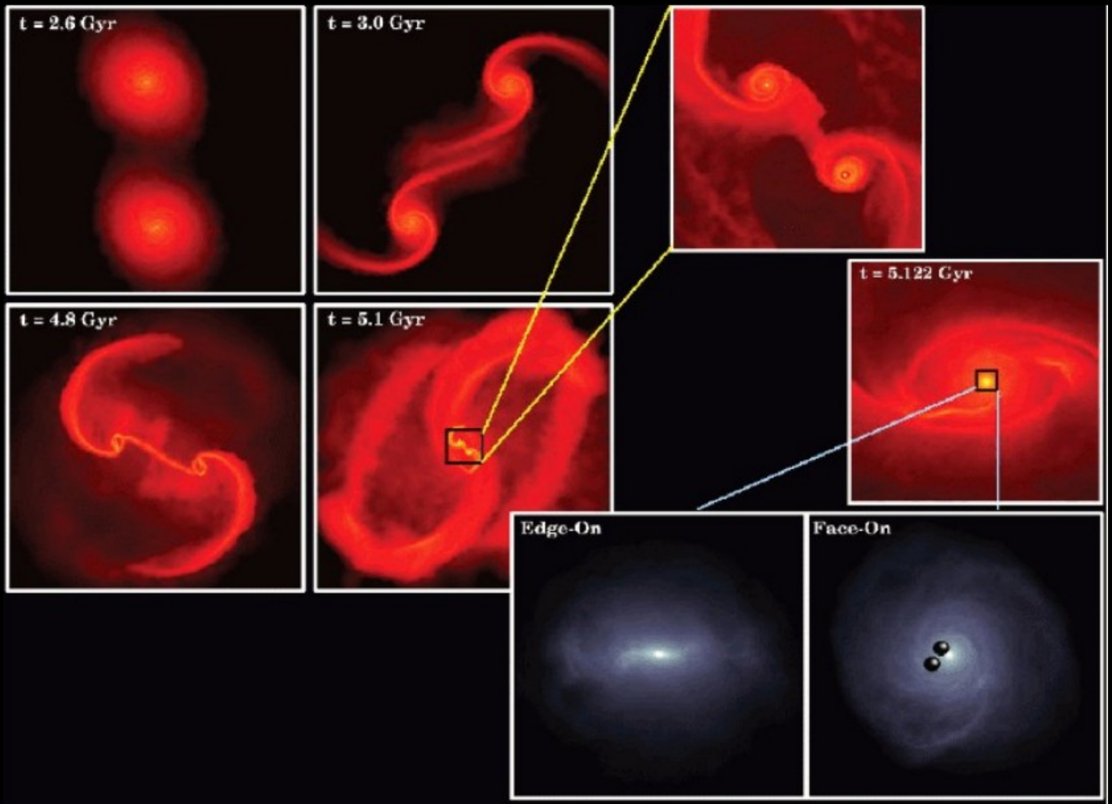
This imply an accretion in mass given by:

$$\frac{dM}{dt} = 2.5 \times 10^{-8} \left(\frac{M}{M_\odot}\right) M_\odot \mathrm{yr}^{-1}$$

MBHs CAN EFFICIENTLY INCREASE THEIR MASS!!!!!!

Mergers

Observations



Simulation

1. **dynamical friction** (Lacey & Cole 1993, Colpi et al. 2000)

- from the interaction between the DM halos to the formation of the BH binary
- determined by the global distribution of matter, driven by stars and/or gas
- efficient only for ***major mergers*** against mass stripping

2. **hardening of the binary** (Quinlan 1996, Miloslavljevic & Merritt 2001, Sesana et al. 2007, Escala et al. 2004, Dotti et al. 2007)

- ***3 bodies interactions*** between the binary and the surrounding stars
- the binding energy of the BHs is larger than the thermal energy of the stars
- the SMBHs create a ***stellar density core ejecting the background stars***
- ***Dynamical drag*** caused by a thick ***circumbinary disk***

3. **emission of gravitational waves** (Peters 1964)

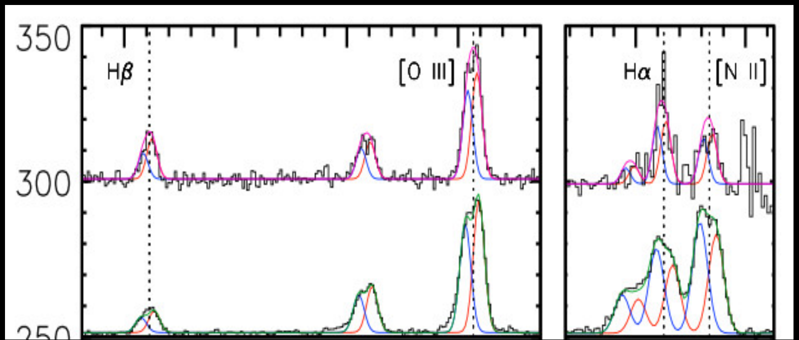
- takes over at subparsec scales
- leads the binary to coalescence

The two MBH separation has to decay from 10 kpc to 10^{-6}pc

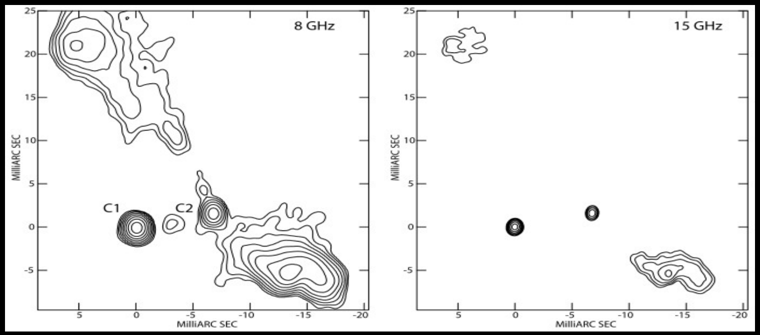
DYNAMICAL RANGE OF TEN ORDER OF MAGNITUDE!!!!!

But do we see them?

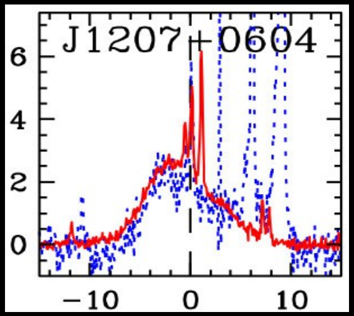
10 kpc: double quasars
(Komossa 2003)

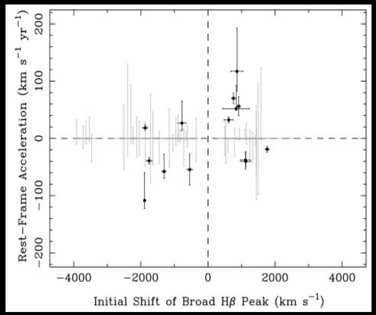


1 kpc: double peaked NL
(Comerford 2013)

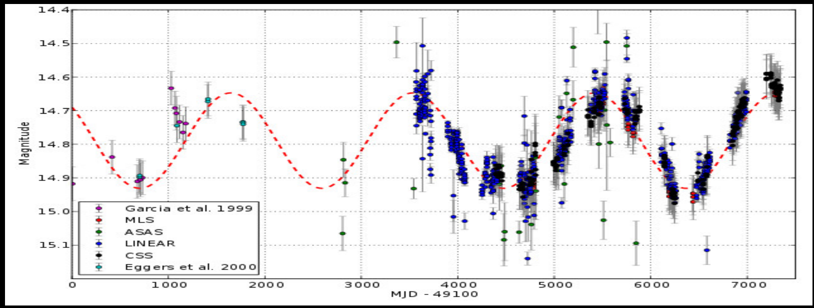


10 pc: double radio cores
(Rodriguez 2006)

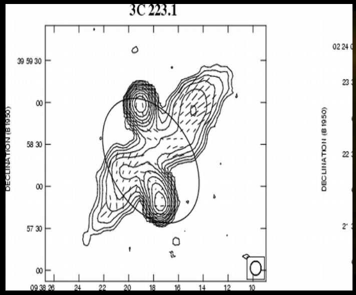




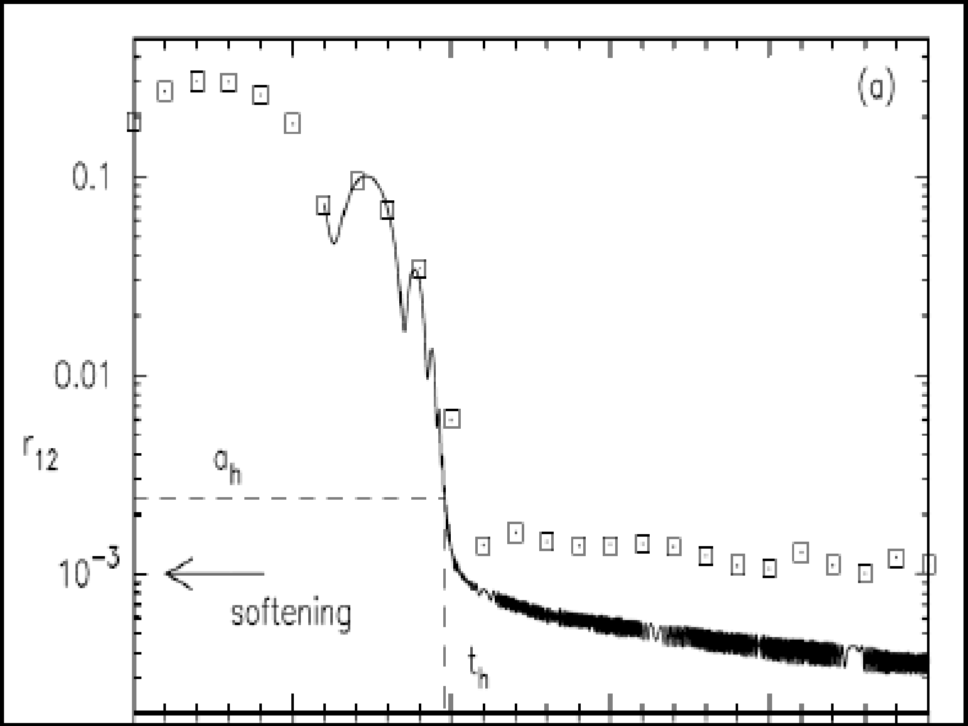
1 pc: -shifted BL (Tsalmatzsa 2011)
-accelerating BL (Eracleous 2012)



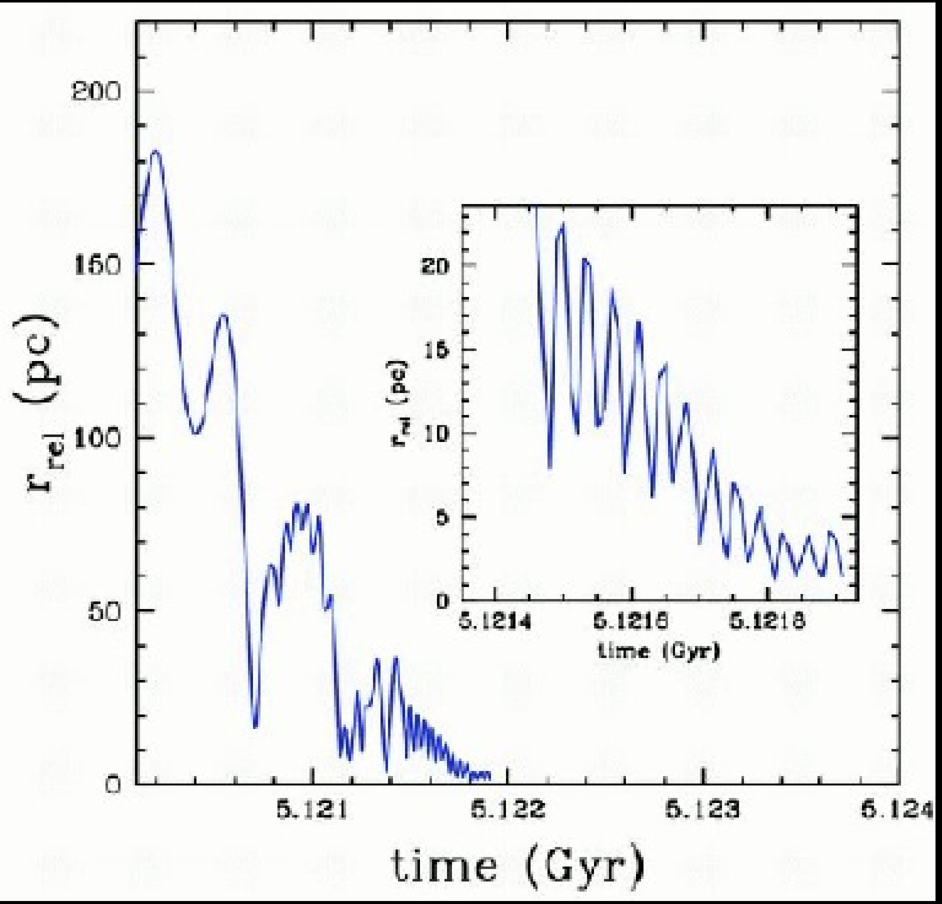
0.01 pc: periodicity (Graham 2015)



0.0pc:-X-shaped sources (Capetti 2001)
-displaced AGNs (Civano 2009)



From Milosavljevic & Merritt 2001



From Colpi & Dotti 2009

Dynamical friction is initially very efficient in shrinking the binary, but on parsec scales the mechanism is no longer efficient:

BINARY STALLING?

I-Dynamical friction: 10kpc-1pc

Consider a BH with mass M_{BH} moving with velocity V in a surrounding distribution of field star with a density ρ_* and a Maxwellian velocity distribution with dispersion σ. The drag exerted by the stars on the BH is given by:

$$\mathbf{F}_{\rm DF} = -4\pi \ln \Lambda G^2 M_{\rm BH}^2 \rho_* \left[\mathrm{erf}\left(\frac{V}{\sqrt{2}\sigma}\right) - \left(\sqrt{\frac{2}{\pi}}\frac{V}{\sigma}\right) \exp\left(-\frac{V^2}{2\sigma^2}\right)\right] \frac{\mathbf{V}}{V^3}$$

- in the limit V->0 this force is proportional to V
- in the limito of $V \gg \sigma$ this force is proportional to $1/V^2$
- the drag is maximum for $V=\sigma$

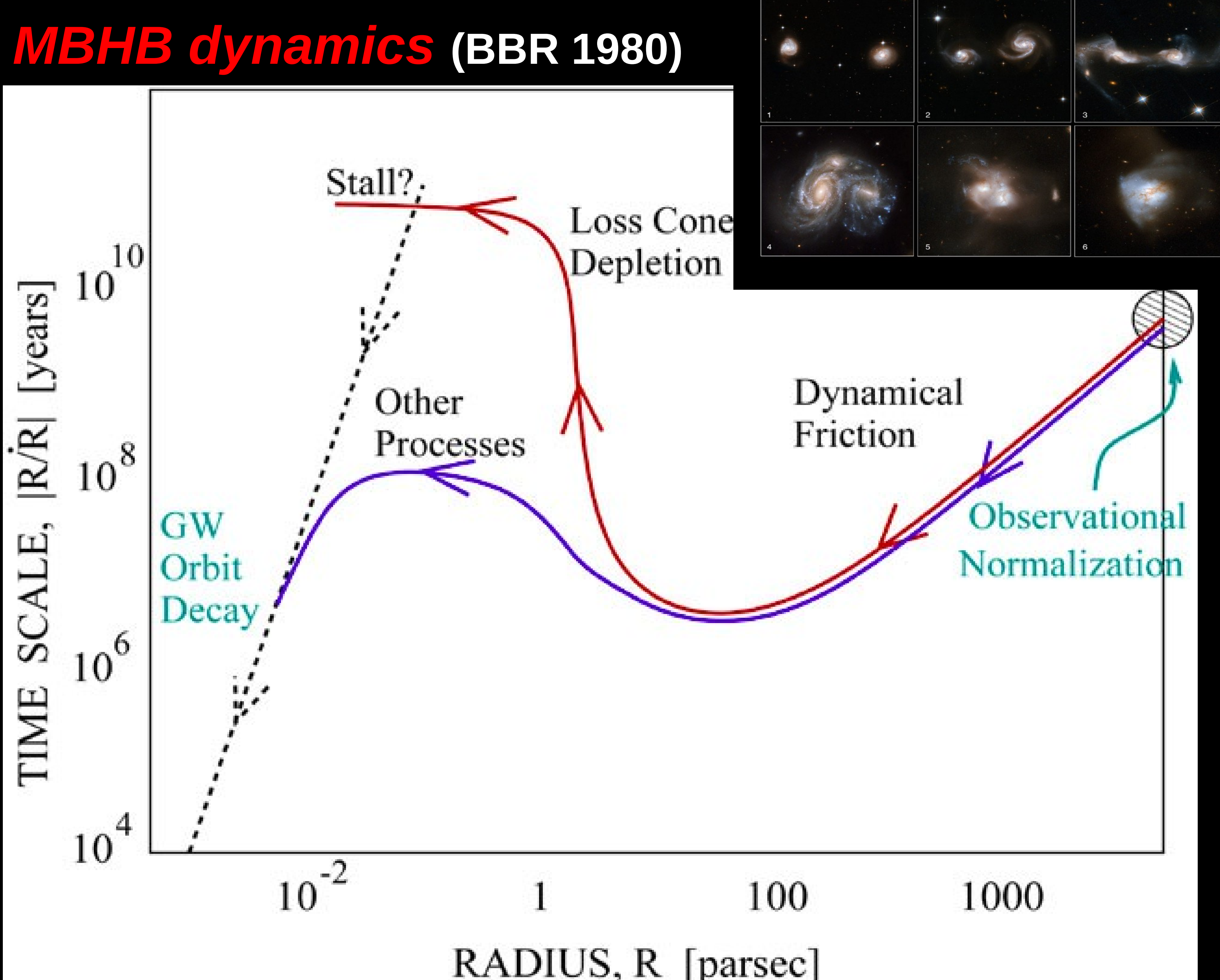
In a gaseous medium the formula is similar:

$$\mathbf{F}_{\rm DF}^{\rm gas} = -4\pi \ln\left[\frac{b_{\rm max}}{b_{\rm min}} \frac{(\mathcal{M}^2-1)^{1/2}}{\mathcal{M}}\right] G^2\, M_{\rm BH}^2 \rho_{\rm gas} \frac{\mathbf{V}}{V^3}, \quad \text{for} \quad \mathcal{M} > 1$$

$$\mathbf{F}_{\rm DF}^{\rm gas} = -(4/3)\pi G^2\, M_{\rm BH}^2 \rho_{\rm gas} \mathcal{M}^3 \mathbf{V}/V^3 \propto M_{\rm BH}^2 \rho_{\rm gas}\, \mathbf{V}/c_{\rm s}^3 \text{ for } \mathcal{M} \ll 1$$

but now $\mathcal{M} = V/c_{\rm s}$ is the gas speed of sound.
Again the drag is maximum when $V=c_s$, and is comparable to the stellar case.

MBHB dynamics (BBR 1980)



II-The hardening phase: "final parsec problem". 1pc-0.01pc

Dynamical friction is efficient in driving the two BHs to a separation of the order

$$a_h \simeq 0.31\,\mathrm{pc}\;M_{2,6}^{1/2}\sqrt{\frac{q}{1+q}}$$

GW emission takes over at separation of the order

$$a_{GW} \approx 0.0014\,\mathrm{pc}\;\left(\frac{M M_1 M_2}{10^{18.3}\;\mathrm{M_\odot}^3}\right)^{1/4} F(e)^{1/4}\, t_9^{1/4}$$

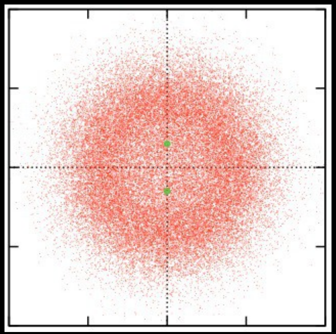
The ratio can be written as

$$\frac{a_h}{a_{GW}} \approx 2.5\times 10^2\left(\frac{q}{1+q}\right)^{3/4} F(e)^{-1/4}\, M_6^{-1/4}\, t_9^{-1/4}$$

STELLAR DRIVEN BINARIES

assuming stars are supplied to the binary loss cone at a constant rate:

$$\frac{da}{dt} = \frac{a^2 G\rho}{\sigma} H$$



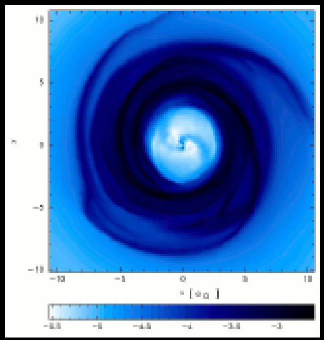
$$dt/d\ln f \propto f^{2/3} M_1^{2/3}$$

$$h_c \propto M_1^2 q f.$$

GAS DRIVEN BINARIES

self-consistent solution for the binary-disk interaction with no leakage in the cavity:

$$\frac{da}{dt} = \frac{2\dot{M}}{\mu}(a a_0)^{1/2}.$$



$$dt/d\ln f \propto f^{-1/3} M_1^{1/6}$$

$$h_c \propto M_1^{7/4} q^{3/2} f^{1/2}$$

Transition frequency

$$f_{\mathrm{star/GW}} \approx 5 \times 10^{-9} M_8^{-7/10} q^{-3/10}$$

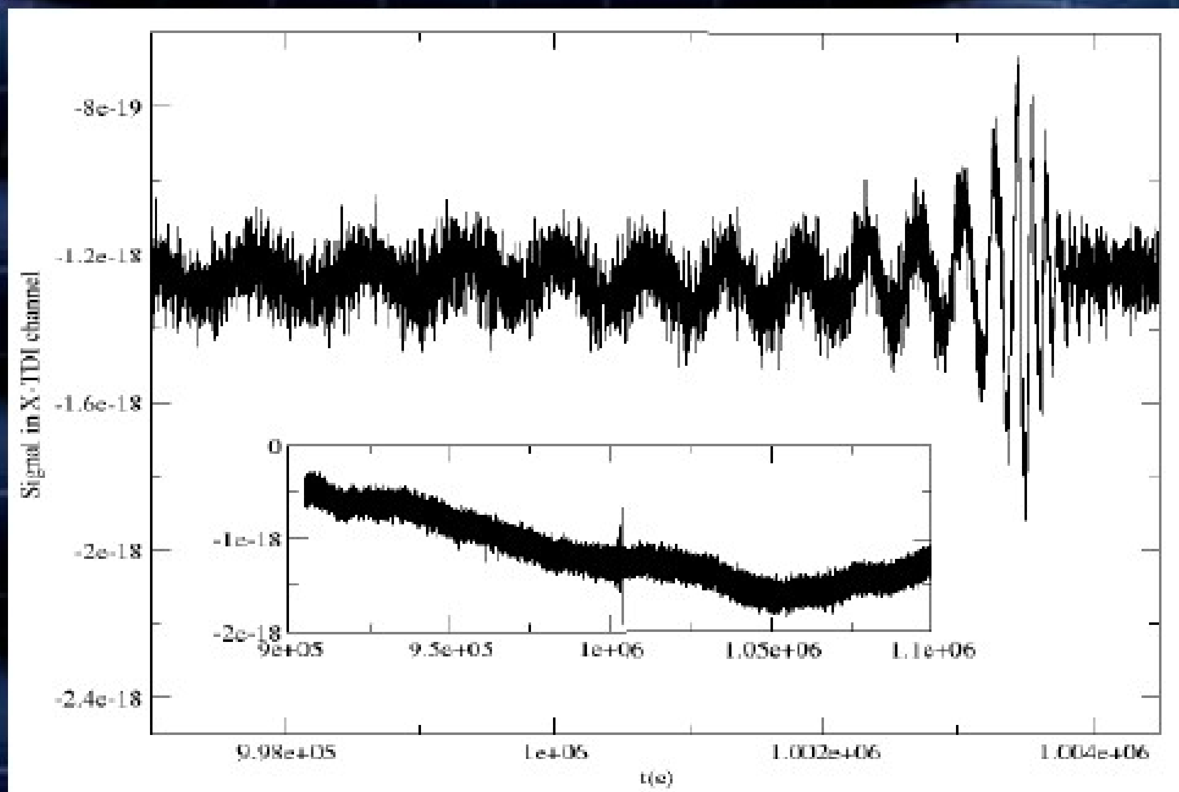
$$f_{\mathrm{gas/GW}} \approx 5 \times 10^{-9} M_8^{-37/49} q^{-69/98}$$

III-Gravitational wave emission 0.01pc-coalescence

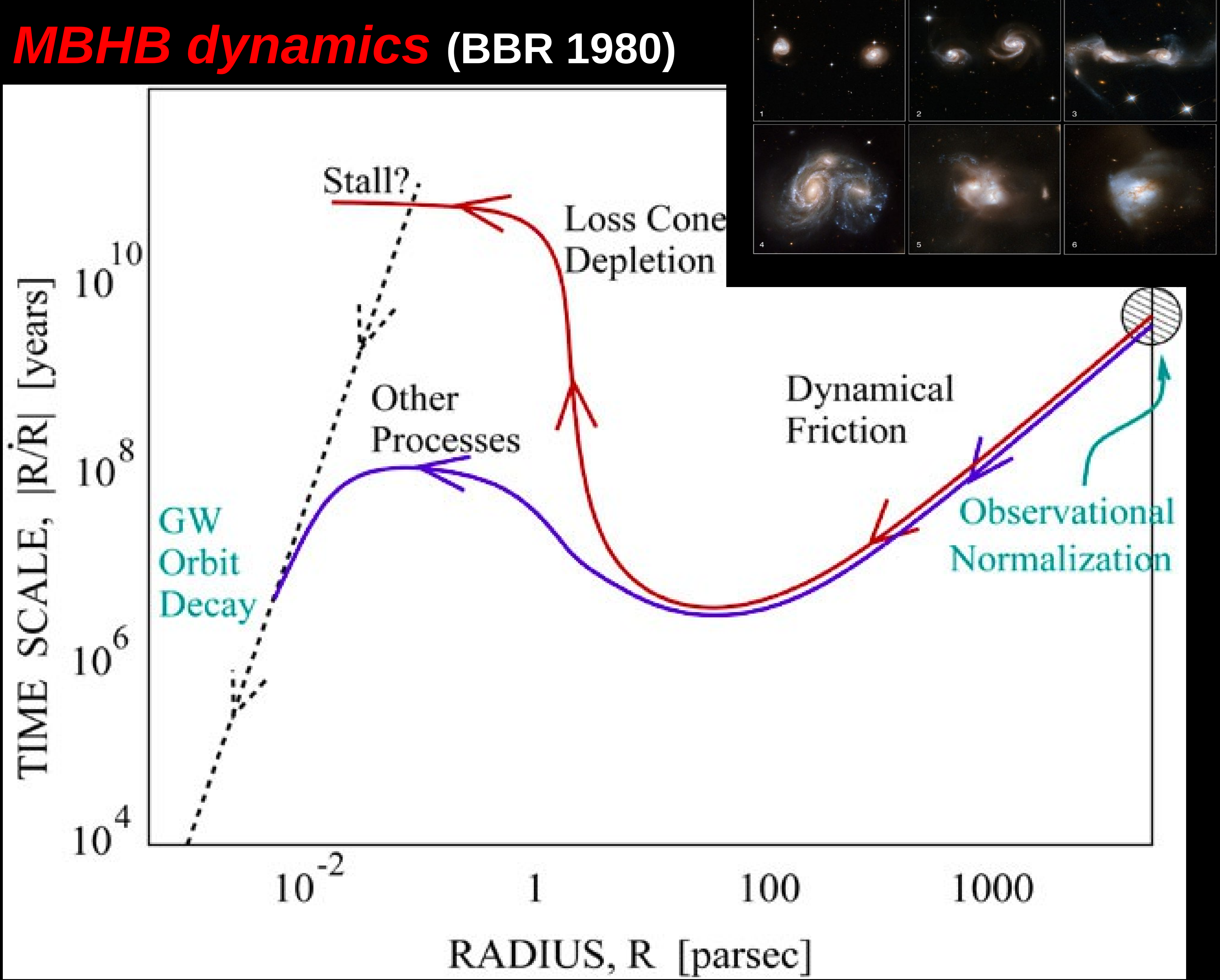
If the binary overcome the final parsec problem then it coalesces on a timescale given by:

$$t_{\rm GW} = \frac{5c^5}{256G^3}\frac{a^4}{M_1M_2MF(e)} \approx 0.25{\rm Gyr}\left(\frac{MM_1M_2}{10^{18.3}\ {\rm M_\odot}^3}\right)^{-1} F(e)^{-1}\left(\frac{a}{0.001{\rm pc}}\right)^4$$

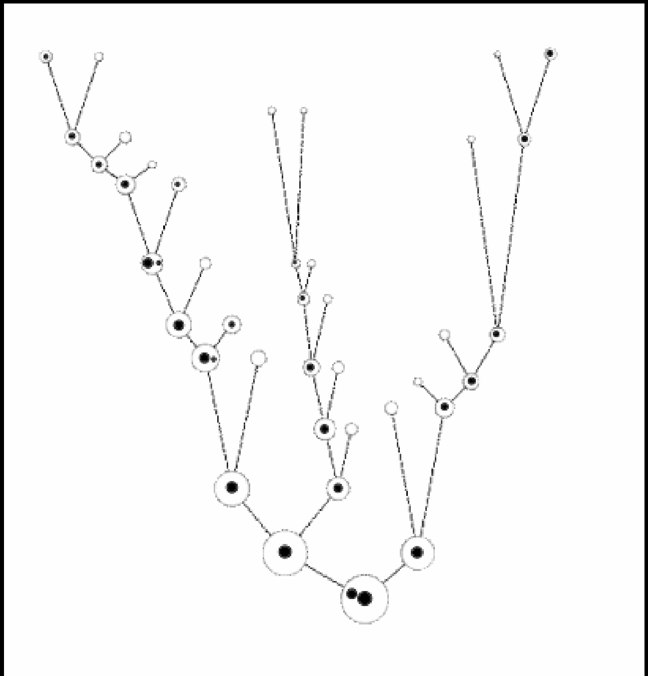
producing the *loudest gravitational wave signals in the Universe!*



MBHB dynamics **(BBR 1980)**



The expected GW signal in the PTA band



The GW characteristic amplitude coming from a population of circular MBH binaries

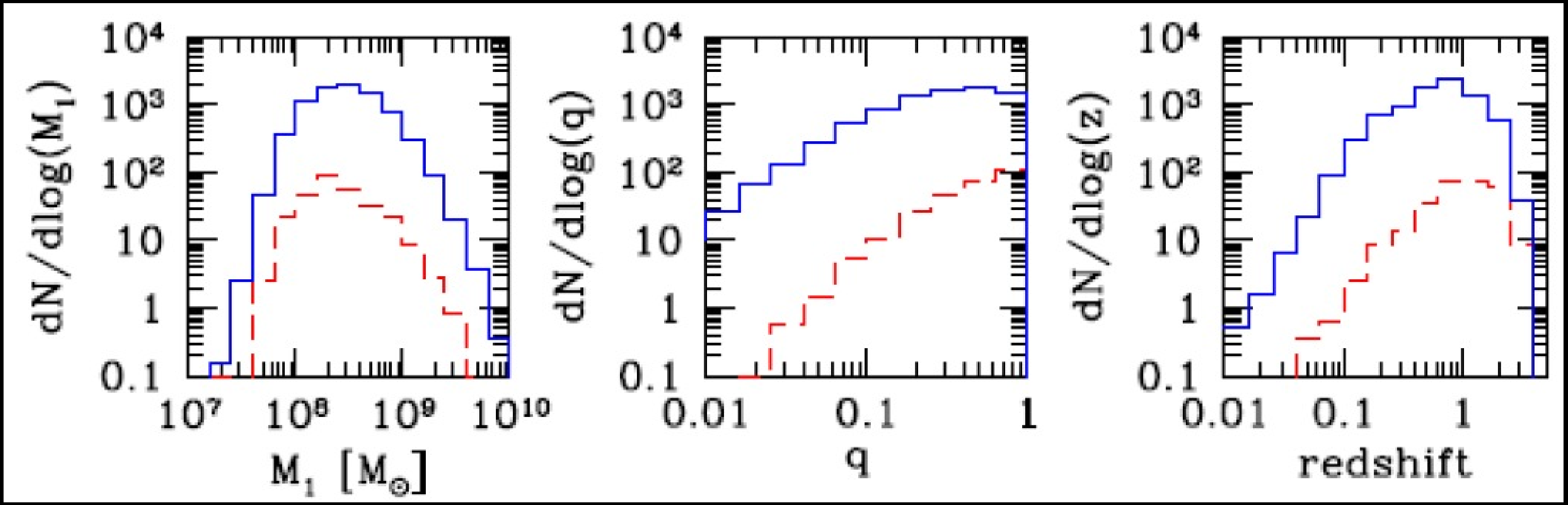
$$h_c^2(f) = \int_0^\infty dz \int_0^\infty d\mathcal{M} \, \frac{d^3N}{dz d\mathcal{M} d\ln f_r} h^2(f_r)$$

$$\delta t_{\rm bkg}(f) \approx h_c(f)/(2\pi f)$$

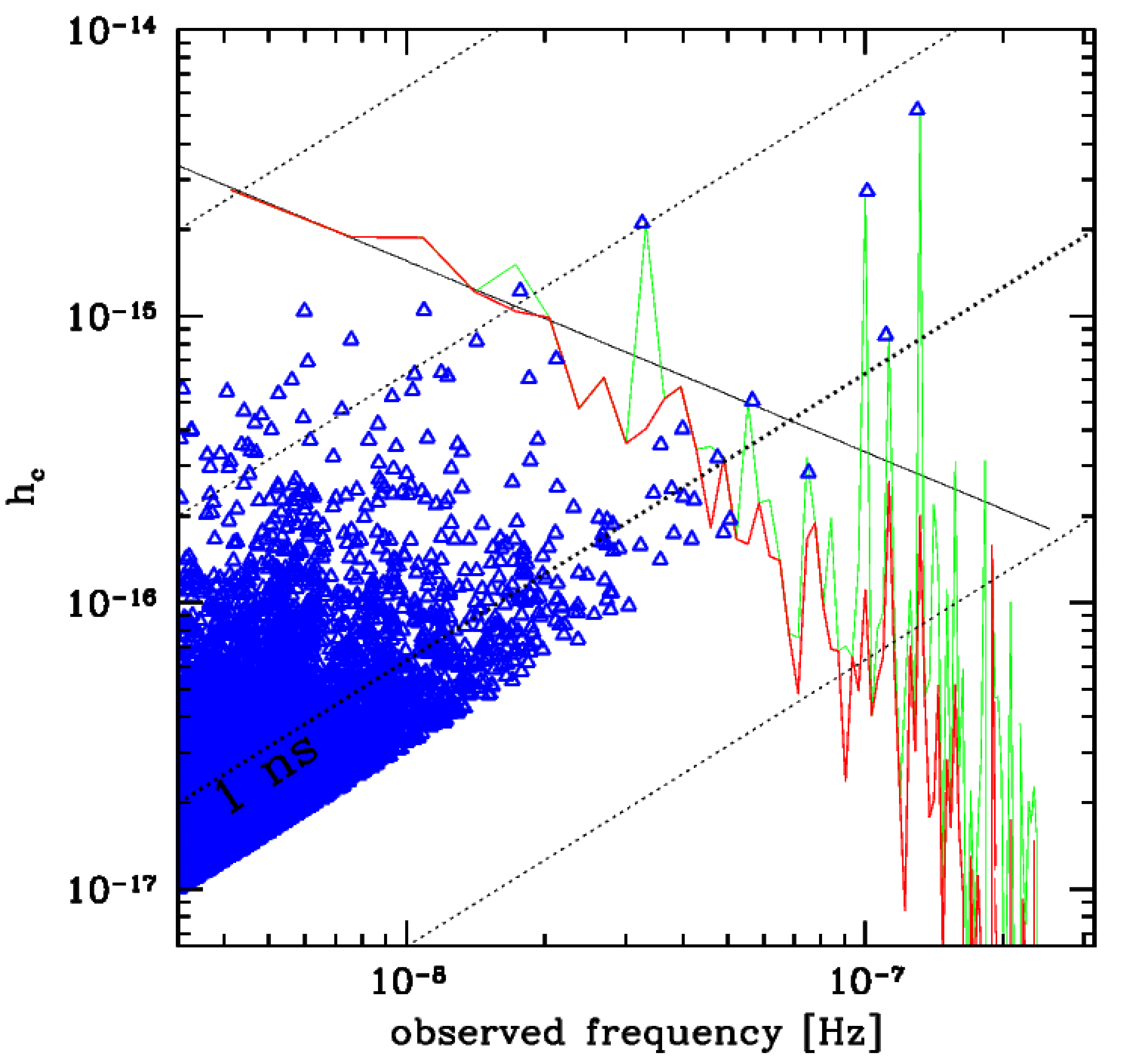
Theoretical spectrum: simple power law

(Phinney 2001)

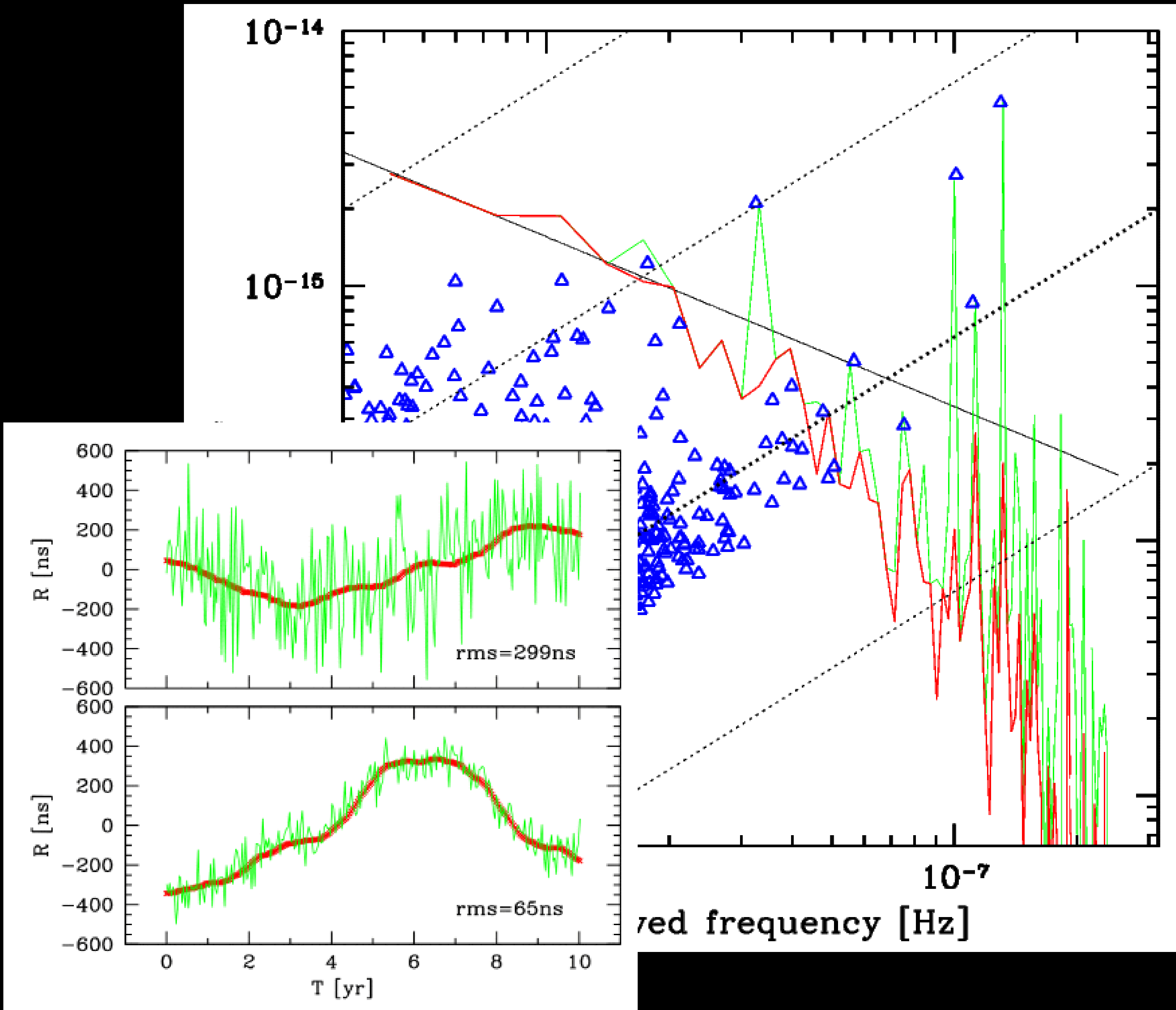
$$h_c(f) = A \left(\frac{f}{\rm yr^{-1}} \right)^{-2/3}$$



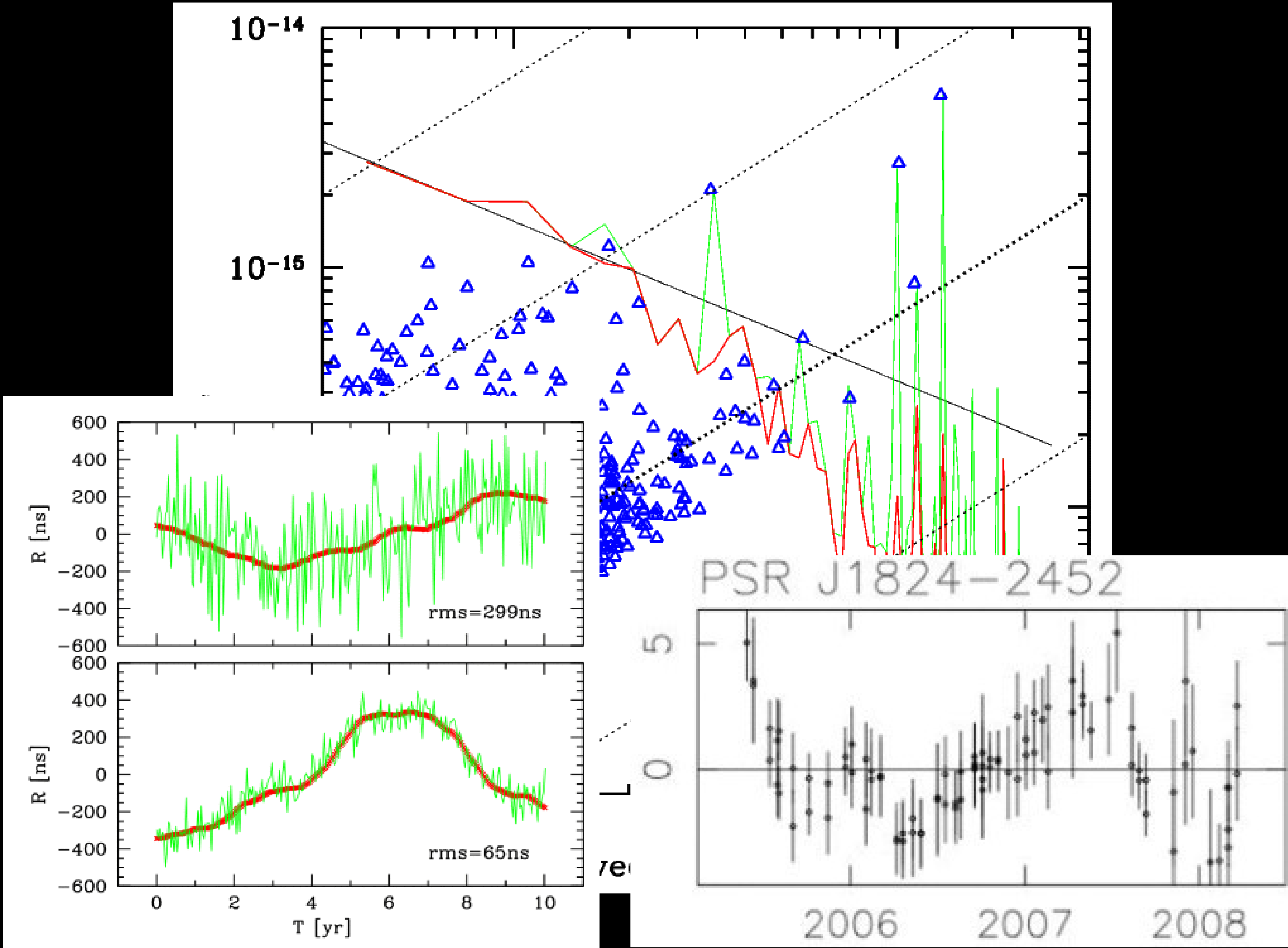
The signal is contributed by extremely massive ($>10^8 M_\odot$) relatively low redshift ($z<1$) MBH binaries (AS et al. 2008, 2012)



10^{-14}
10^{-15}
10^{-16}
10^{-17}
h_c
1 ns
10^{-8}
10^{-7}
observed frequency [Hz]



10−14
10−15
10−7
ved frequency [Hz]
600
400
200
0
−200
−400
−600
R [ns]
rms=299ns
rms=65ns
0
2
4
6
8
10
T [yr]



10^{-14}
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600
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200
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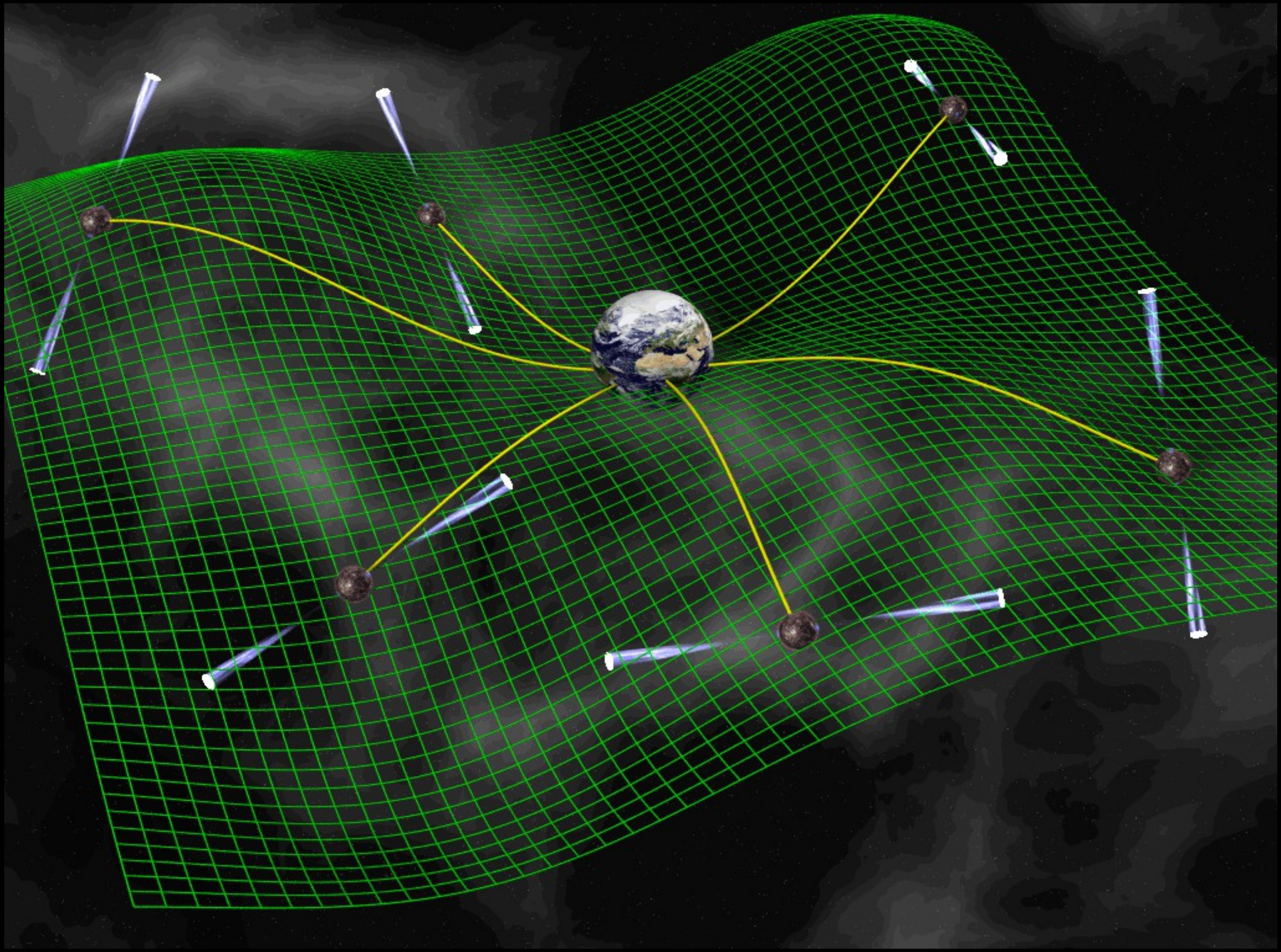
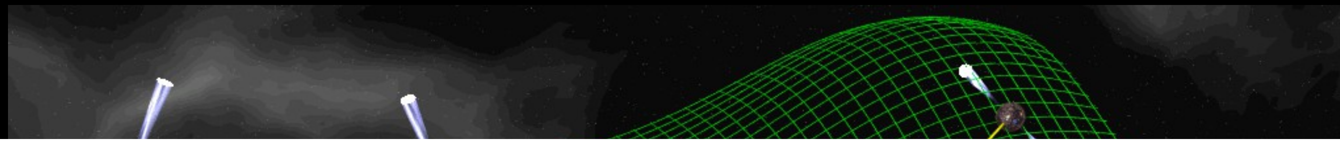
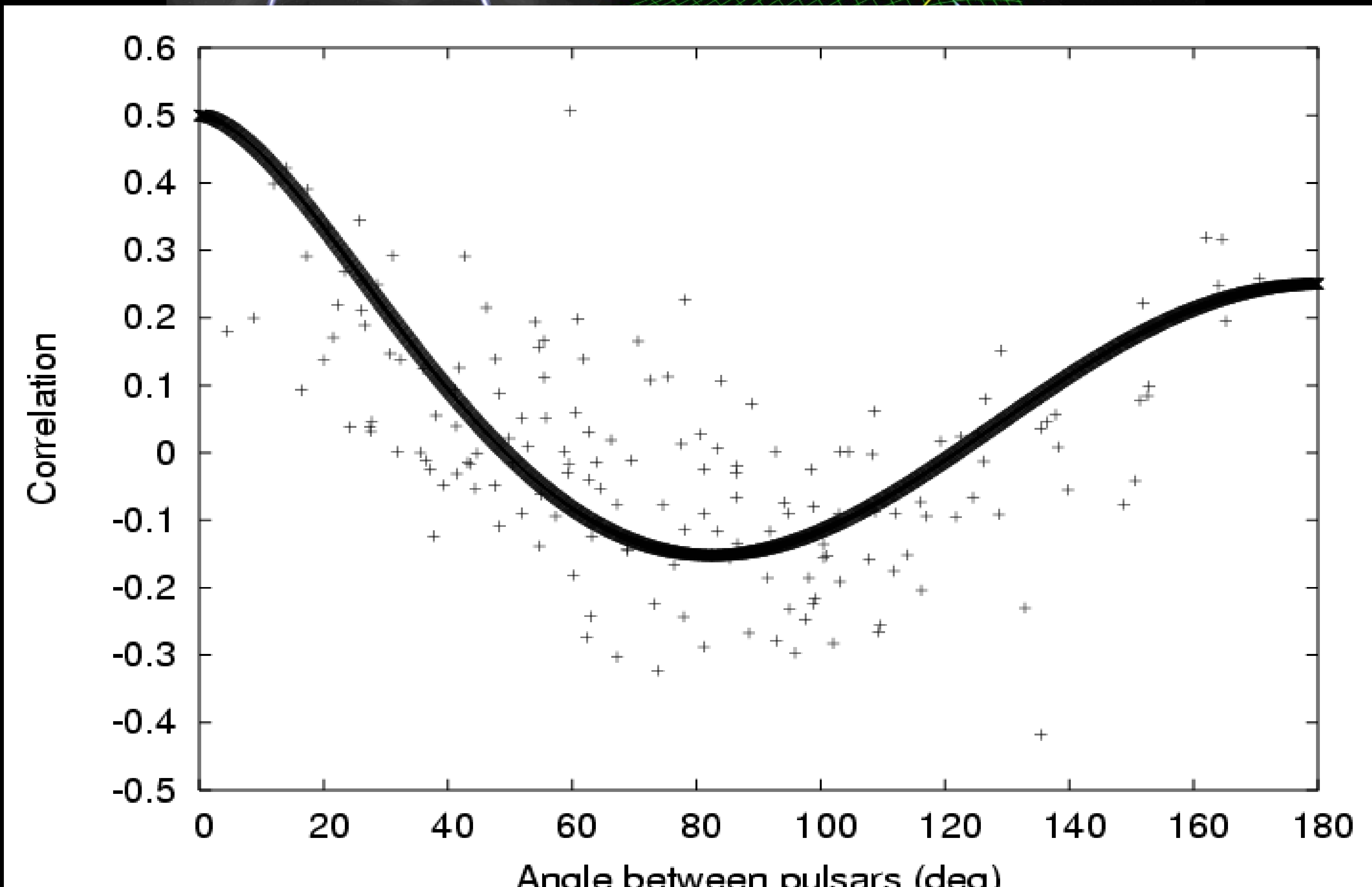
−200
−400
−600
R [ns]
rms=299ns
rms=65ns
0
2
4
6
8
10
T [yr]
PSR J1824−2452
5
0
2006
2007
2008

We are looking for a correlated signal



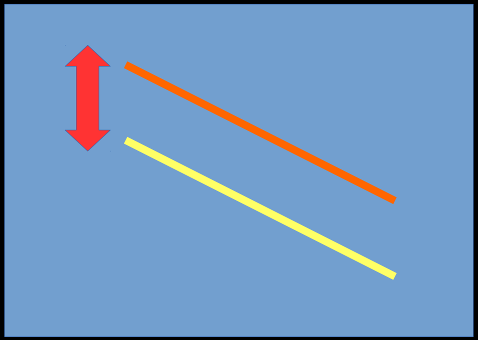
We are looking for a correlated signal





(Hellings & Downs 1983)

The overall GW signal



Population parameters

1-Galaxy merger rate <-----> MBHB merger rate

affects the number of sources at each frequency ---> N_0

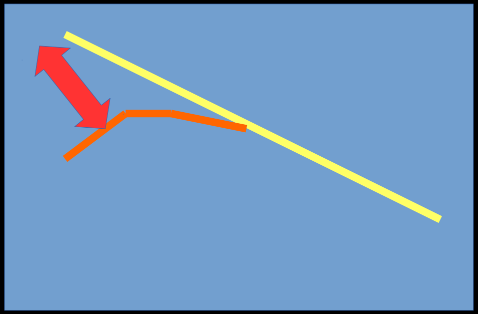
2-MBH mass – merging galaxy relation

affects the mass of the sources ----> M_c

$$h_c^2(f) = \int_0^\infty dz \int_0^\infty dM_1 \int_0^1 dq \frac{d^4N}{dz\,dM_1\,dq\,dt_r} \frac{dt_r}{d\ln f_{K,r}} \times h^2(f_{K,r}) \sum_{n=1}^{\infty} \frac{g[n,e(f_{K,r})]}{(n/2)^2} \delta\left[f - \frac{n f_{K,r}}{1+z}\right].$$

$$h_c(f) \propto n_0^{1/2} f^{-\gamma} M_c^{5/6}$$

Local dynamics



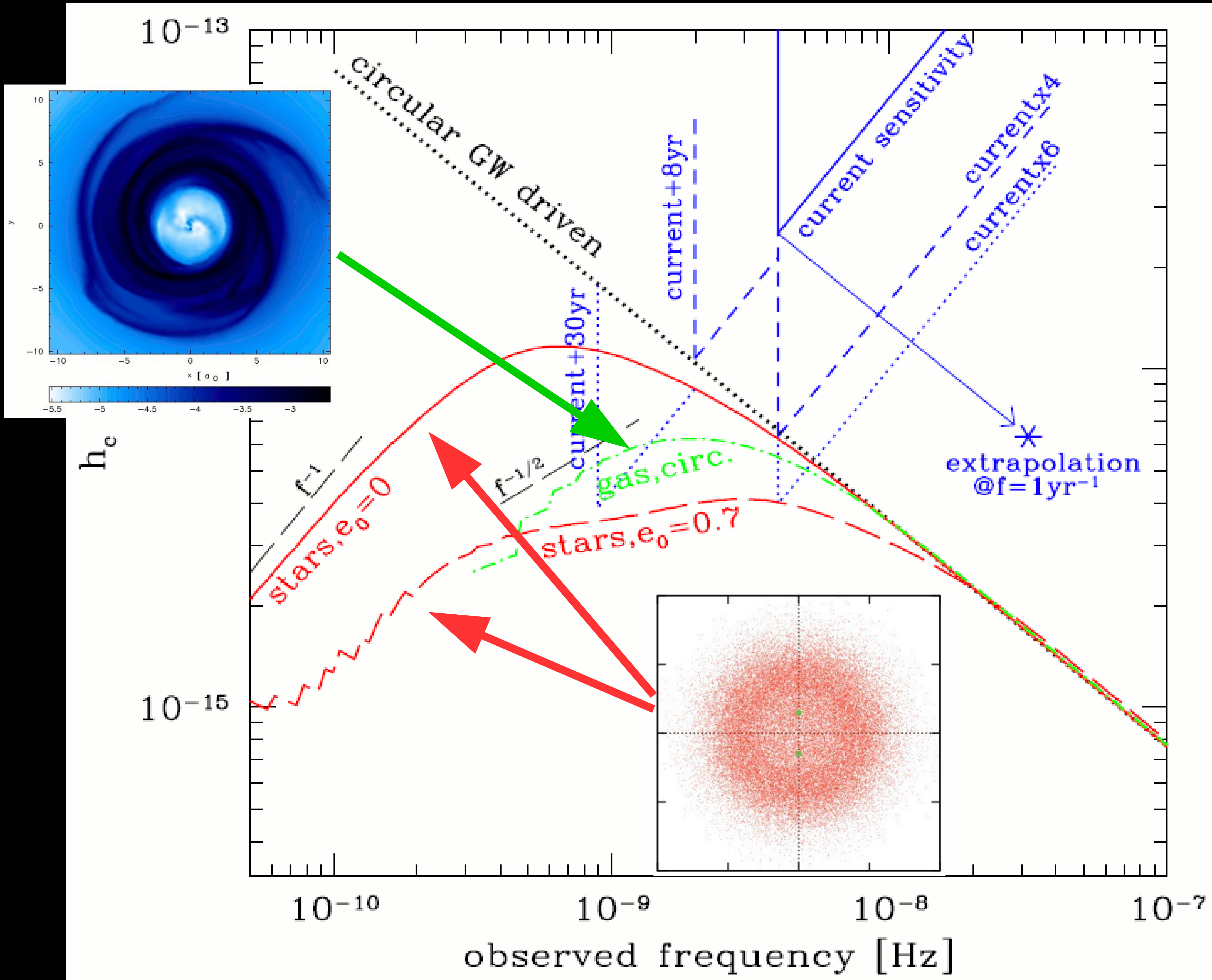
1-Accretion (when? how?)

affects the mass of the sources ---> M_c

2-MBHB – environment coupling (gas & stars)

affects the chirping rate of the binaries ---> γ

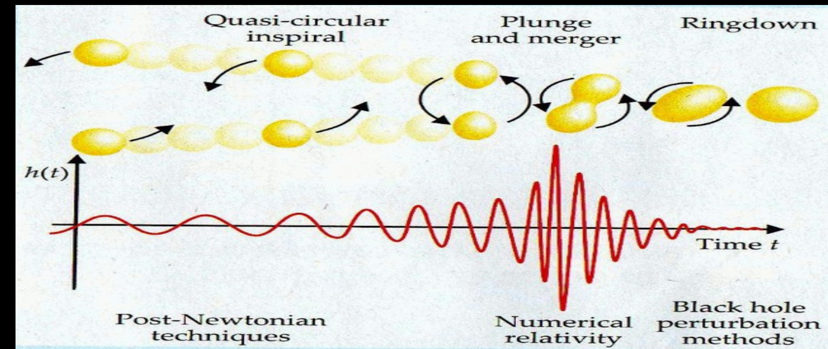
affects the eccentricity ---> chirping rate ----> γ & single source detection

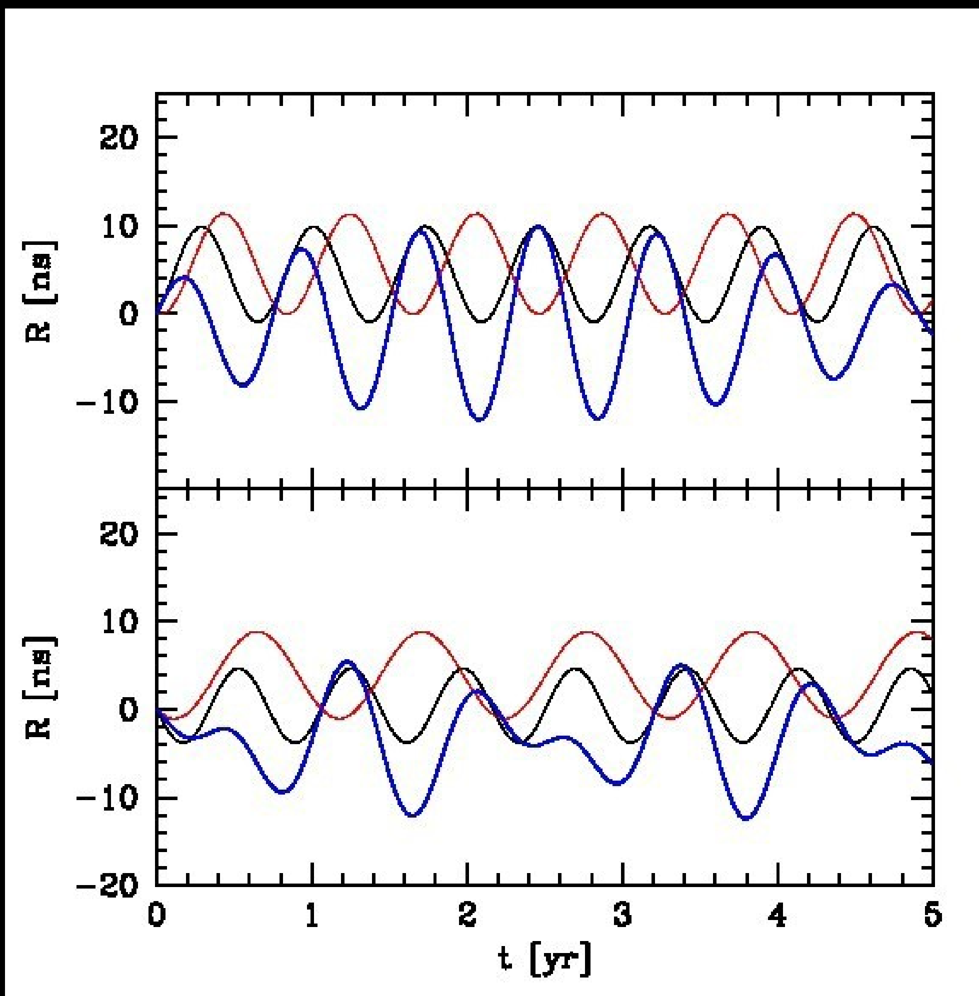


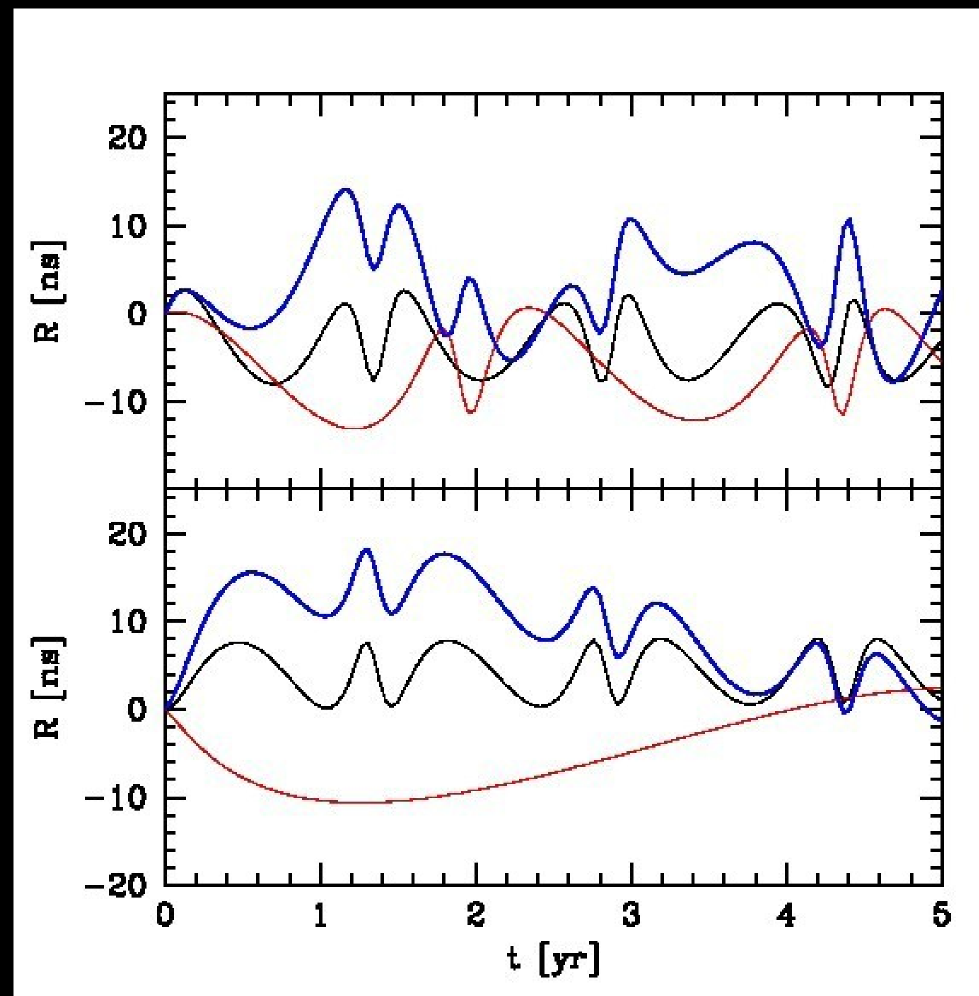
(Kocsis & AS 2011, AS 2013, Ravi et al. 2014, McWilliams et al. 2014)

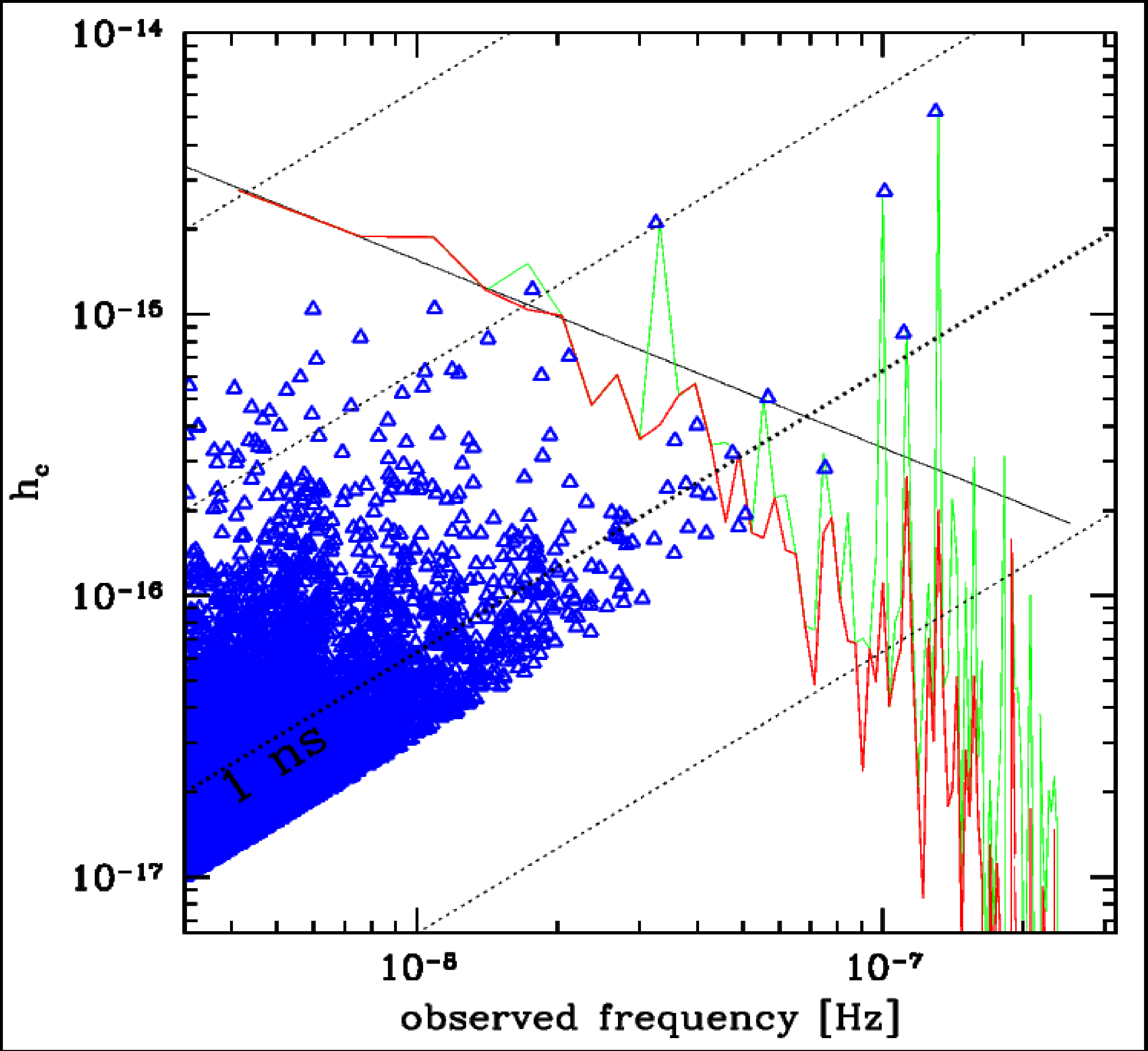
Single MBHB timing residuals

$$\frac{\nu(t)-\nu_0}{\nu_0} = \Delta h_{ab}(t) \equiv h_{ab}(t_{\rm p},\hat{\Omega}) - h_{ab}(t_{\rm ssb},\hat{\Omega})$$









10^{-14}
10^{-15}
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h_c
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observed frequency [Hz]

The overall GW signal

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Local dynamics

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$$h_c(f) \propto n_0^{1/2} f^{-\gamma} M_c^{5/6}$$

1-Population parameters

Minimal assumptions:

-Whenever there is a galaxy merger there is a SMBHB merger (pending a DF timescale that does not affect major mergers)

-SMBH are connected through the properties of galaxies through scaling relations

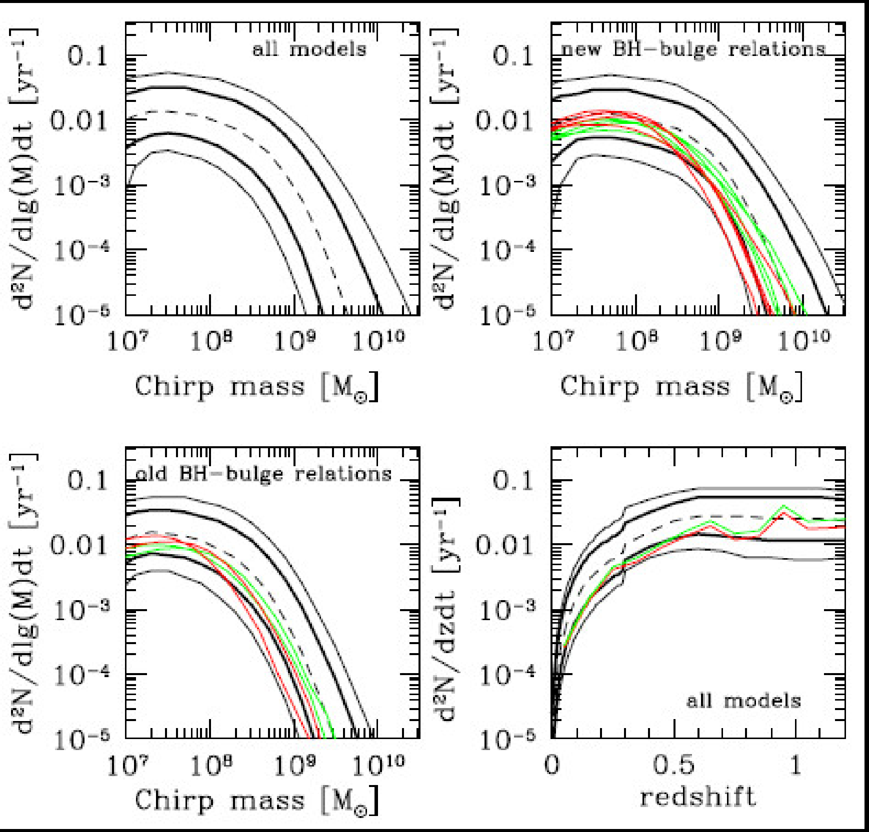
-SMBHB are circular GW driven in the PTA band

Even so....

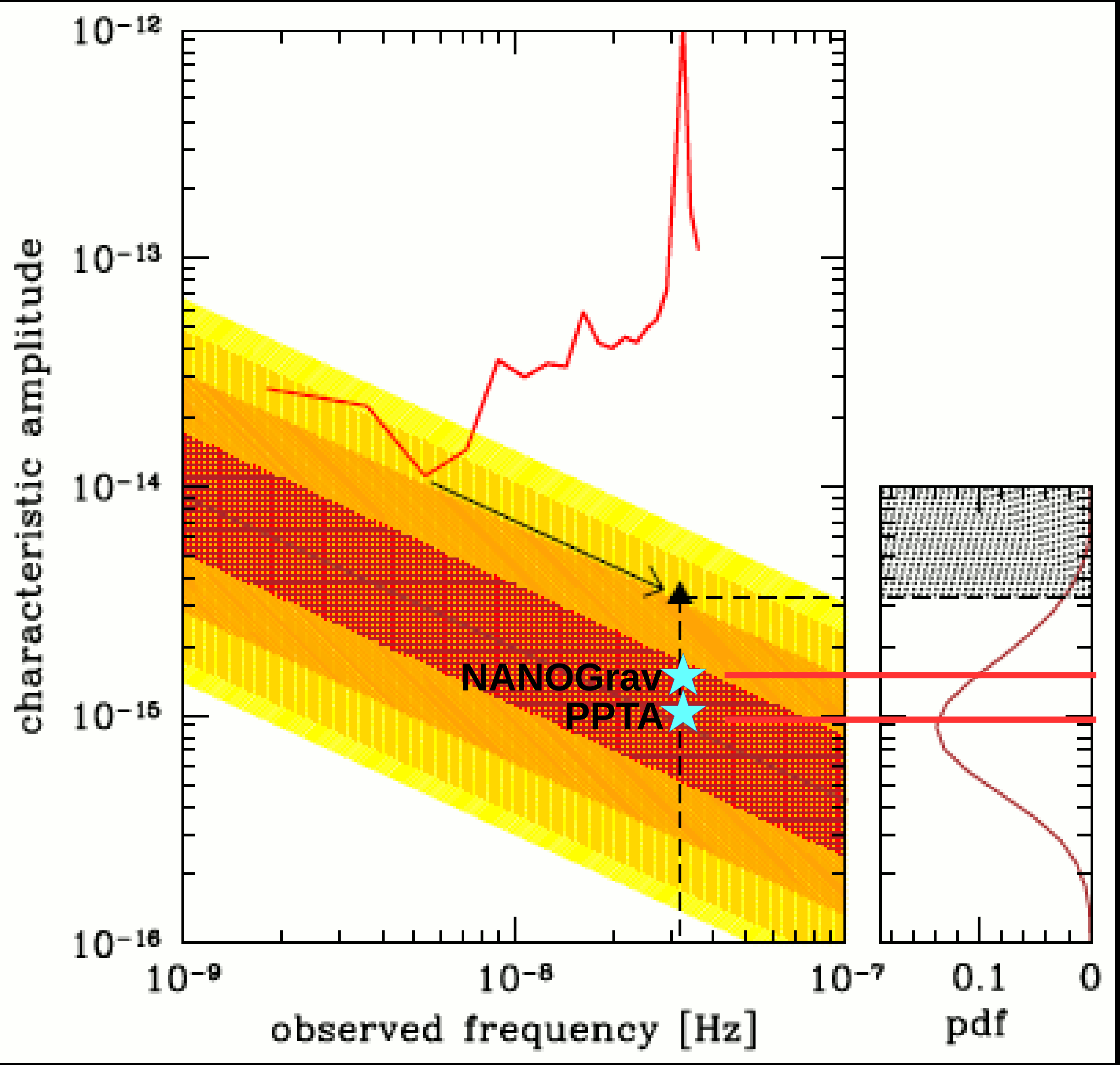
The MBHB merger rate is poorly determined:

-The galaxy merger rate is not know very well observationally

-The MBH-galaxy scaling relations has uncertainties and scatter (MBH measurements are hard)



Uncertainty in the GW background level



(Lentati et al. 2015, Arzoumanian et. 2015, Shannon et al. In press)

Predictions shown here (AS 2013):

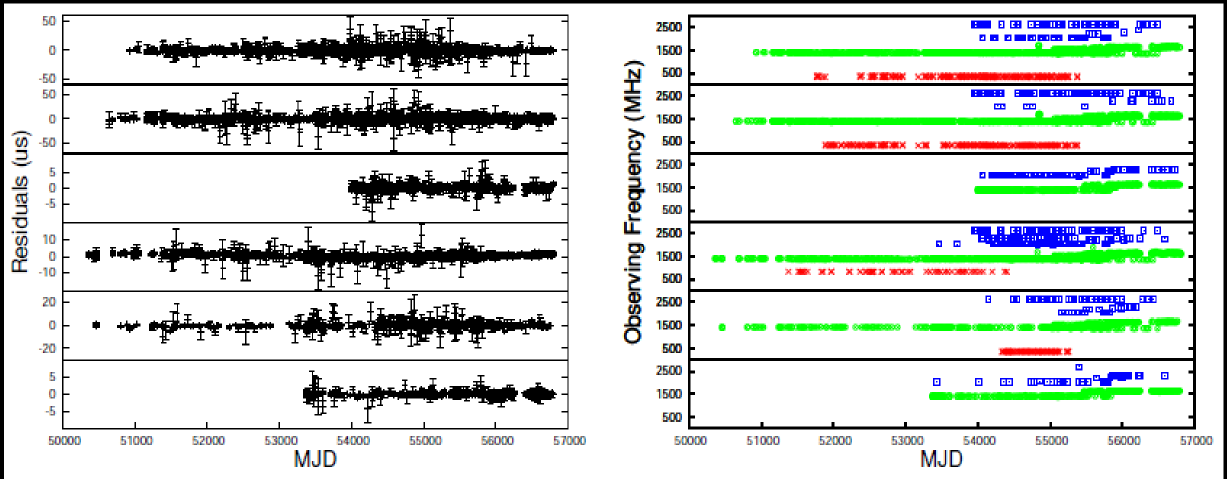
>Assume circular GW driven binaries

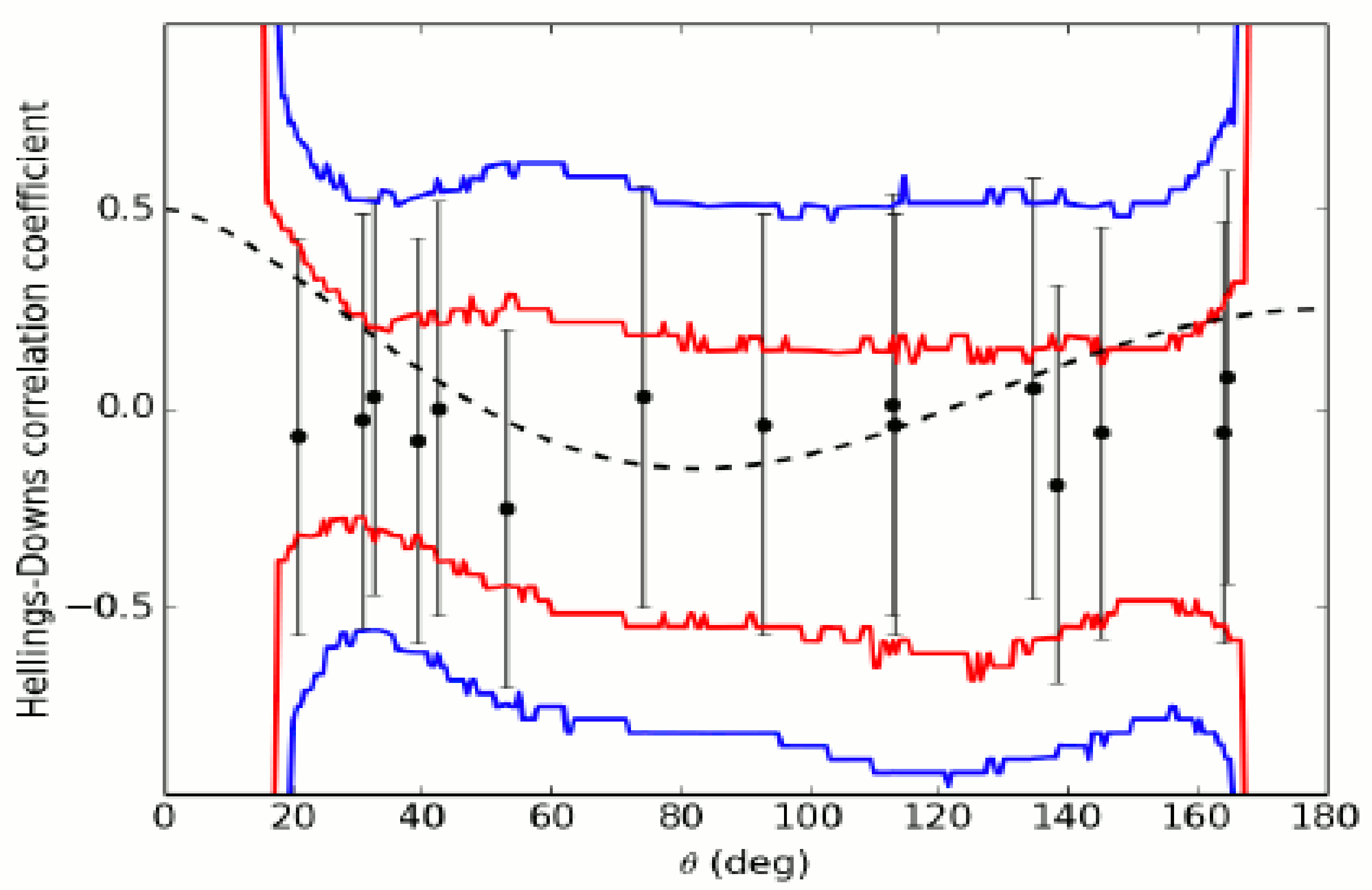
>Efficient MBH binary merger following galaxy mergers

>Uncertainty range takes into account:
- -merger rate
- -MBH-galaxy relation
- -accretion timing

(AS 2008, 2013; Ravi et al. 2012, 2015; Roebber er al. 2015; Kulier et al. 2014; McWilliams et al. 2014)

Pulsar correlations (EPTA, Lentati et al. 2015)





Constrains on GWB anisotropy

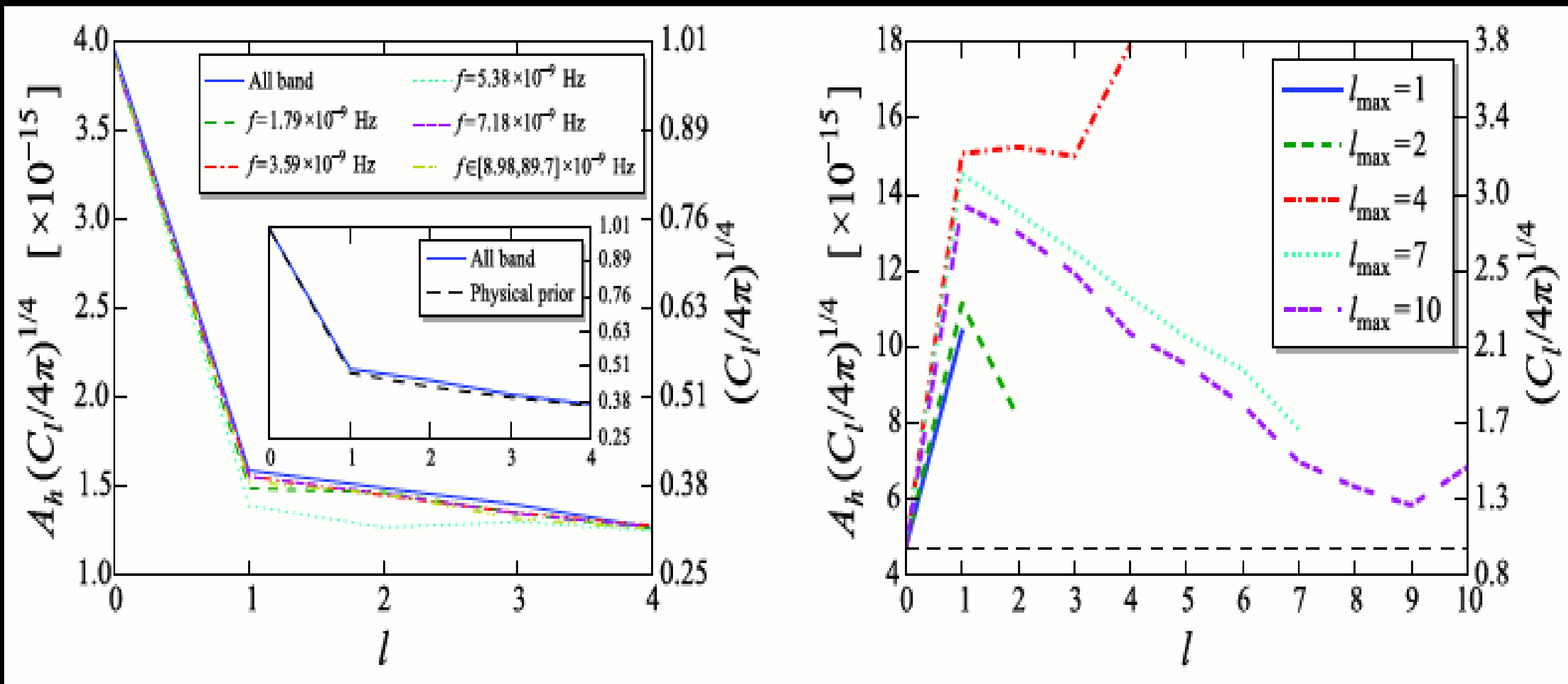
(EPTA, Taylor et al. 2015)

If the GWB is anisotropic, the power across the sky can be decomposed in spherical harmonics:

>To each multiple corresponds a different correlation pattern among pulsars,

>The overall correlation is a weighted sum of the individual correlations

$$\Gamma_{ab} = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} c_{lm} \Gamma_{lm}^{(ab)}$$



2-Local Dynamics:Coupling with the environment

1. **dynamical friction** (Lacey & Cole 1993, Colpi et al. 2000)

- from the interaction between the DM halos to the formation of the BH binary
- determined by the global distribution of matter, driven by stars and/or gas
- efficient only for ***major mergers*** against mass stripping

2. **hardening of the binary** (Quinlan 1996, Miloslavljevic & Merritt 2001, Sesana et al. 2007, Escala et al. 2004, Dotti et al. 2007)

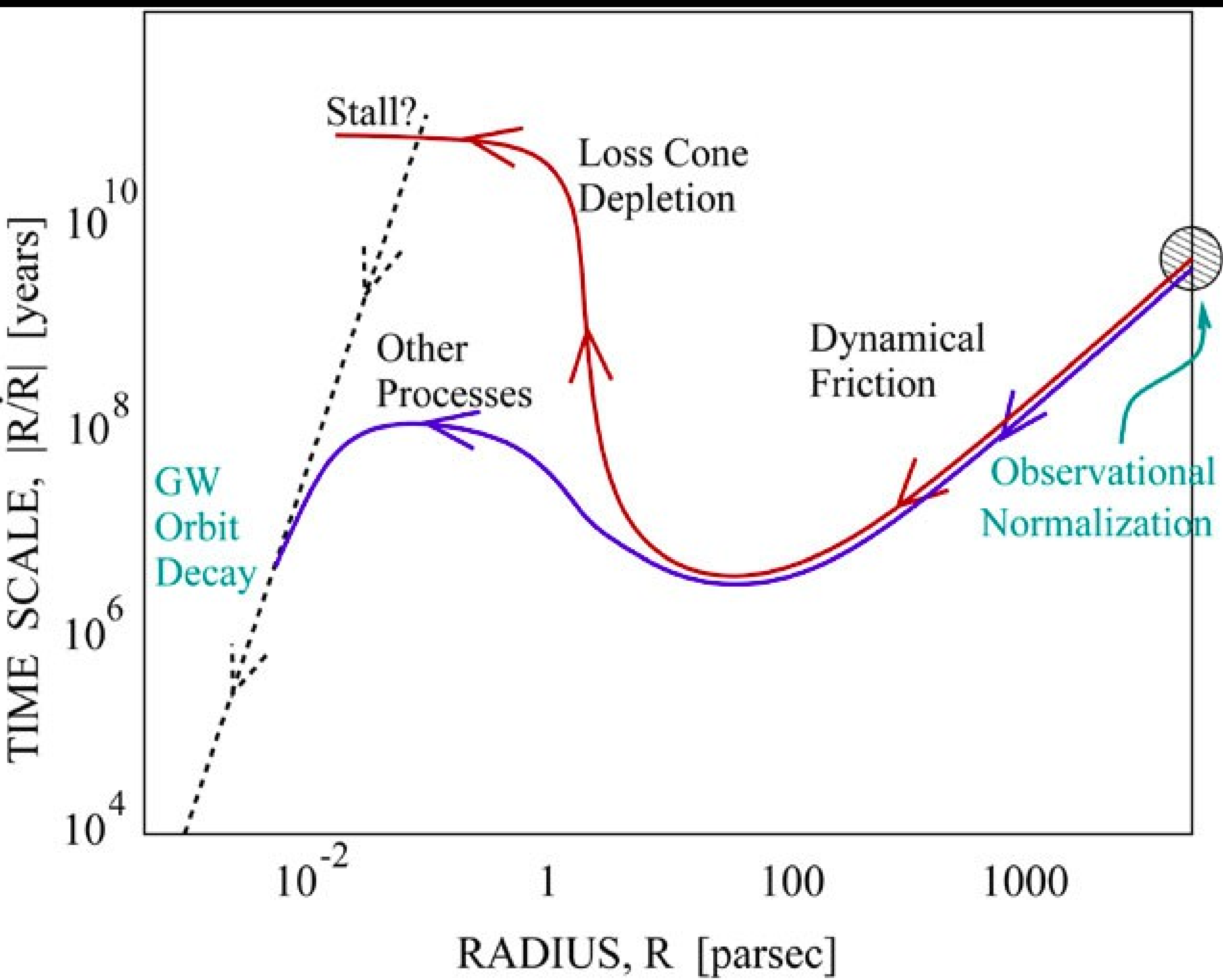
- ***3 bodies interactions*** between the binary and the surrounding stars
- the binding energy of the BHs is larger than the thermal energy of the stars
- the SMBHs create a ***stellar density core ejecting the background stars***
- ***Dynamical drag*** caused by a thick ***circumbinary disk***

3. **emission of gravitational waves** (Peters 1964)

- takes over at subparsec scales
- leads the binary to coalescence

The two MBH separation has to decay from 10 kpc to 10^{-6}pc

DYNAMICAL RANGE OF TEN ORDER OF MAGNITUDE!!!!!

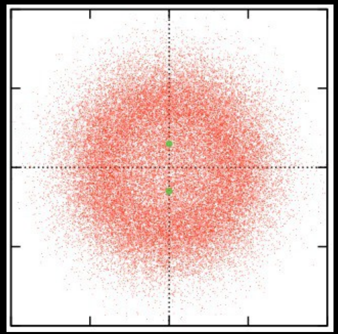


Stall?
Loss Cone
Depletion
Other
Processes
Dynamical
Friction
GW
Orbit
Decay
Observational
Normalization
TIME SCALE, |R/Ṙ| [years]
10^{10}
10^{8}
10^{6}
10^{4}
10^{-2}
1
100
1000
RADIUS, R [parsec]

STELLAR DRIVEN BINARIES

assuming stars are supplied to the binary loss cone at a constant rate:

$$\frac{da}{dt} = \frac{a^2 G\rho}{\sigma} H$$



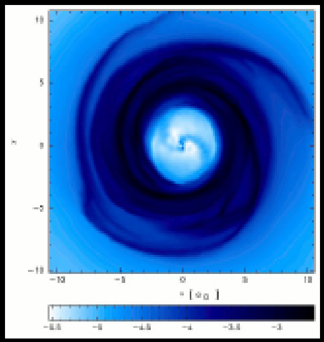
$$dt/d\ln f \propto f^{2/3} M_1^{2/3}$$

$$h_c \propto M_1^2 q f$$

GAS DRIVEN BINARIES

self-consistent solution for the binary-disk interaction with no leakage in the cavity:

$$\frac{da}{dt} = \frac{2\dot{M}}{\mu}(a a_0)^{1/2}$$



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$$f_{\rm star/GW} \approx 5 \times 10^{-9} M_8^{-7/10} q^{-3/10}$$

$$f_{\rm gas/GW} \approx 5 \times 10^{-9} M_8^{-37/49} q^{-69/98}$$

STELLAR DRIVEN BINARIES

assuming stars are supplied

to the bin

constant

$$\frac{da}{dt} =$$

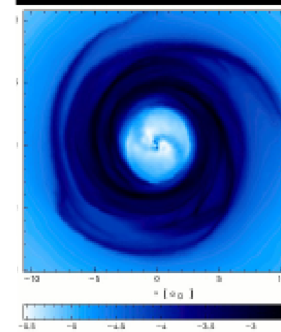
$$dt/d\ln$$

$$h_c \propto M$$

GAS DRIVEN BINARIES

self-consistent solution for the

no



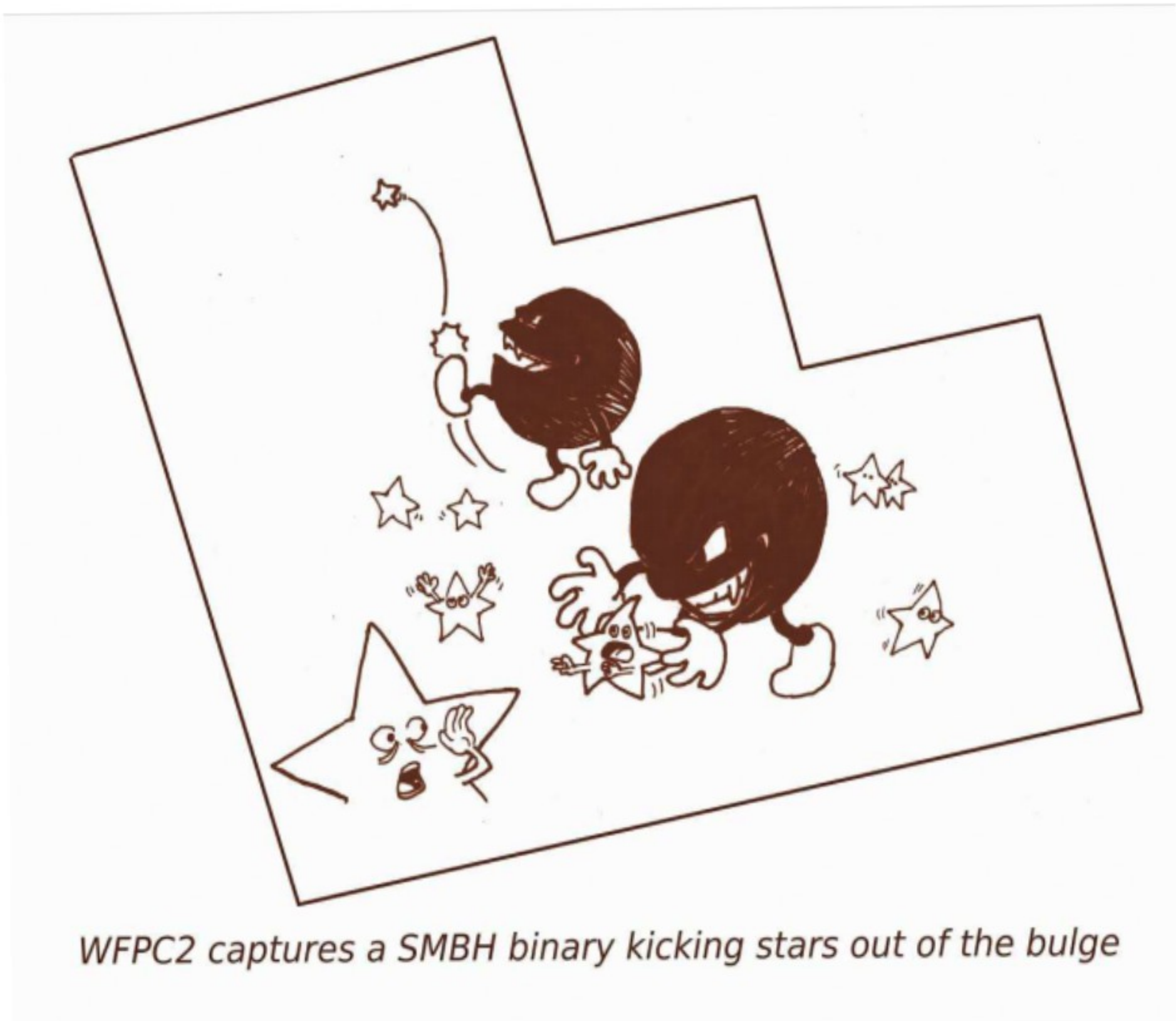
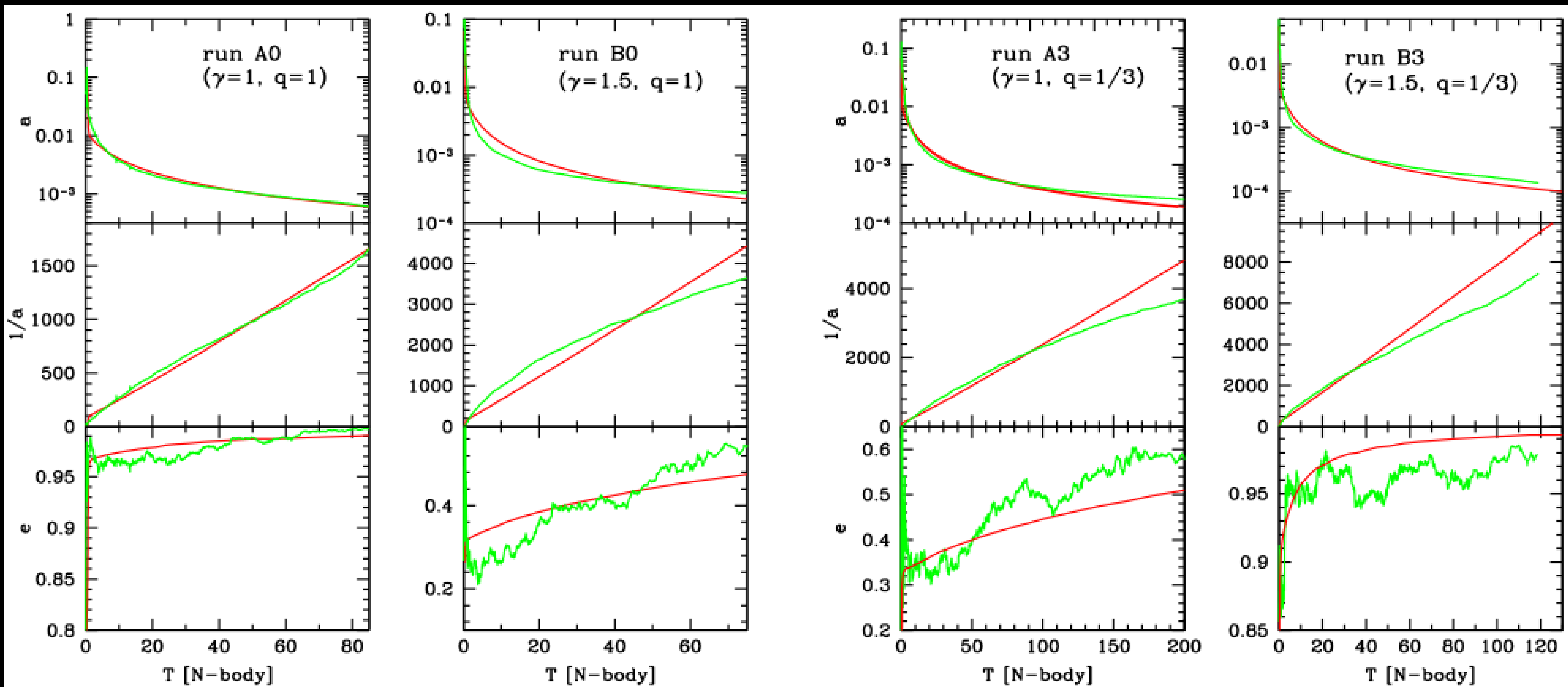


FIG. 7.— Cartoon showing a pair of supermassive black holes kicking stars away as they dance towards coalescence at the centre of a galaxy. Credit: Paolo Bonfini.

AS & Khan 2015 (See also Vasiliev et al. 2015)

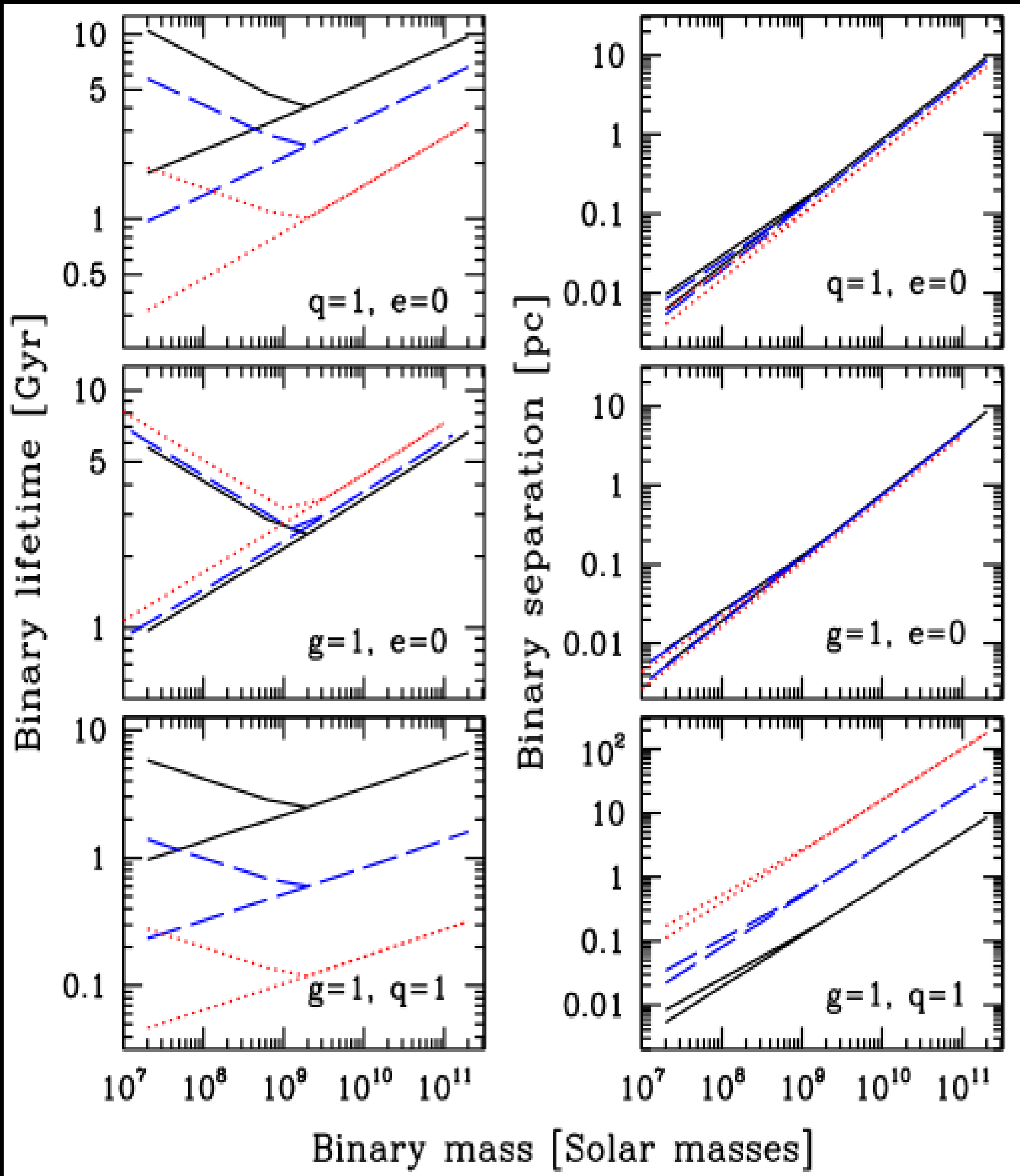
Compare:
-'realistic' mergers with N-body simulations
-semianalytic models including scattering of bound and unbound stars



Reasonable agreement if the evolution is rescaled with ρ and σ at the binary influence radius

$$\frac{d}{dt}\left(\frac{1}{a}\right) = \frac{G\rho}{\sigma} H_{3b}$$

...and compute the coalescence timescale for typical galaxy properties as a function of the MBHB mass



Coalescence timescales are fairly long:

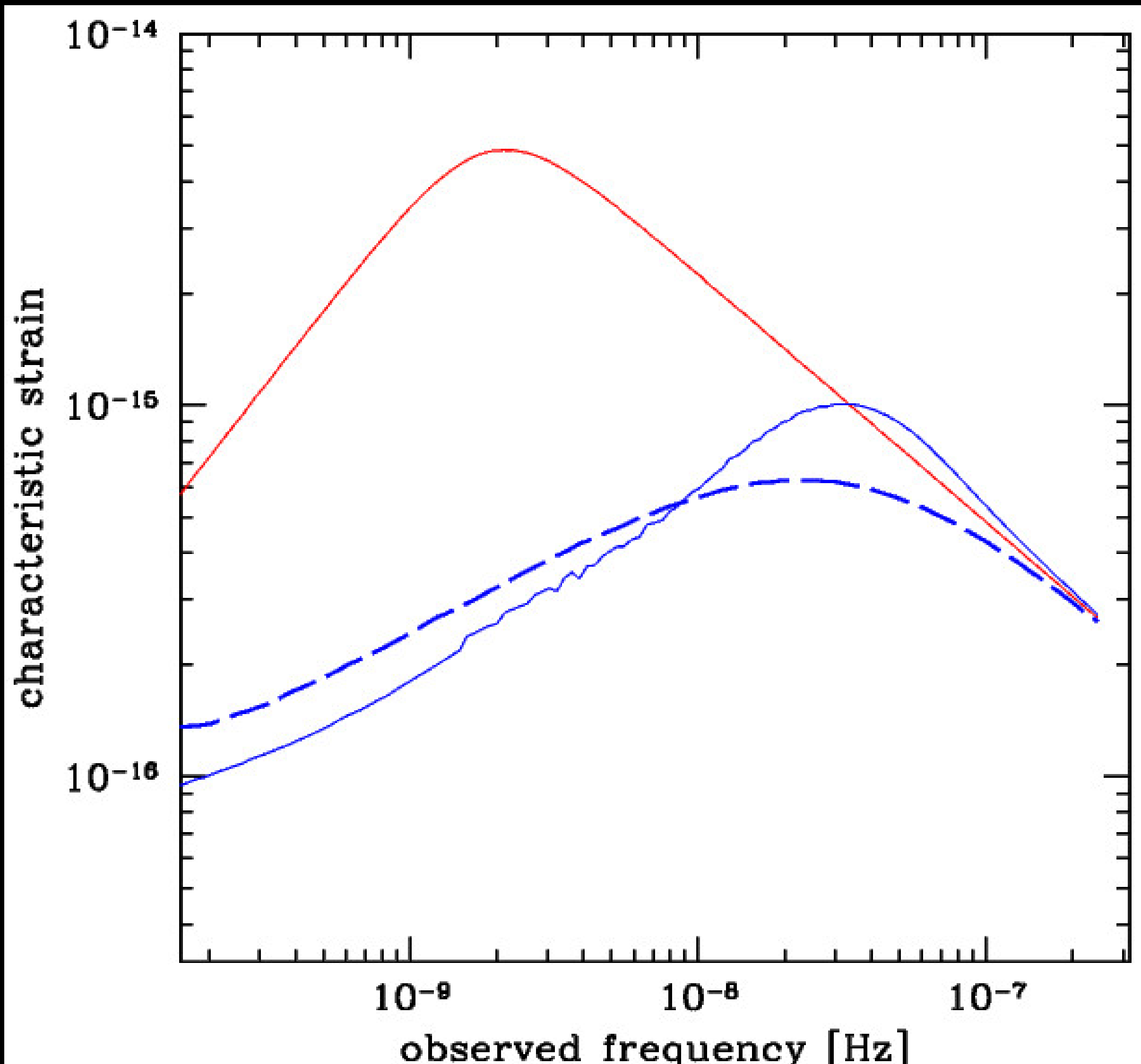
*bending of GW spectrum in the PTA band might not be an issue unless binaries gets very eccentric (might be likely)

*Gyr coalescence timescale open interesting scenarios like triple interactions

Eccentricity

Eccentric binaries emit a whole spectrum of harmonics **(Peters & Mathews 1963)** with the consequence that:

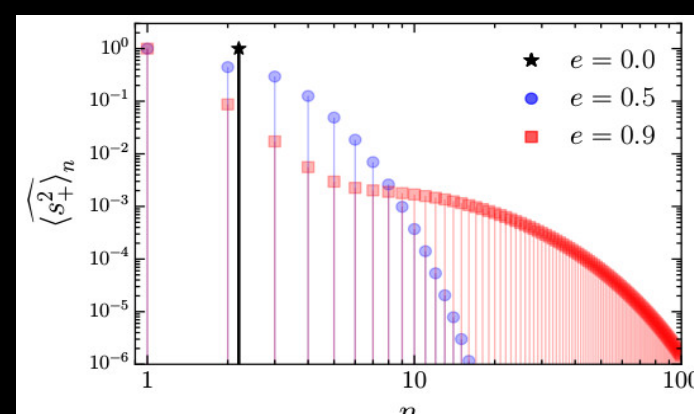
1) they evolve faster (their *dE/dt* Is proportional to $(1-e^2)^{-7/2}$
2) their emission moves toward higher frequency.



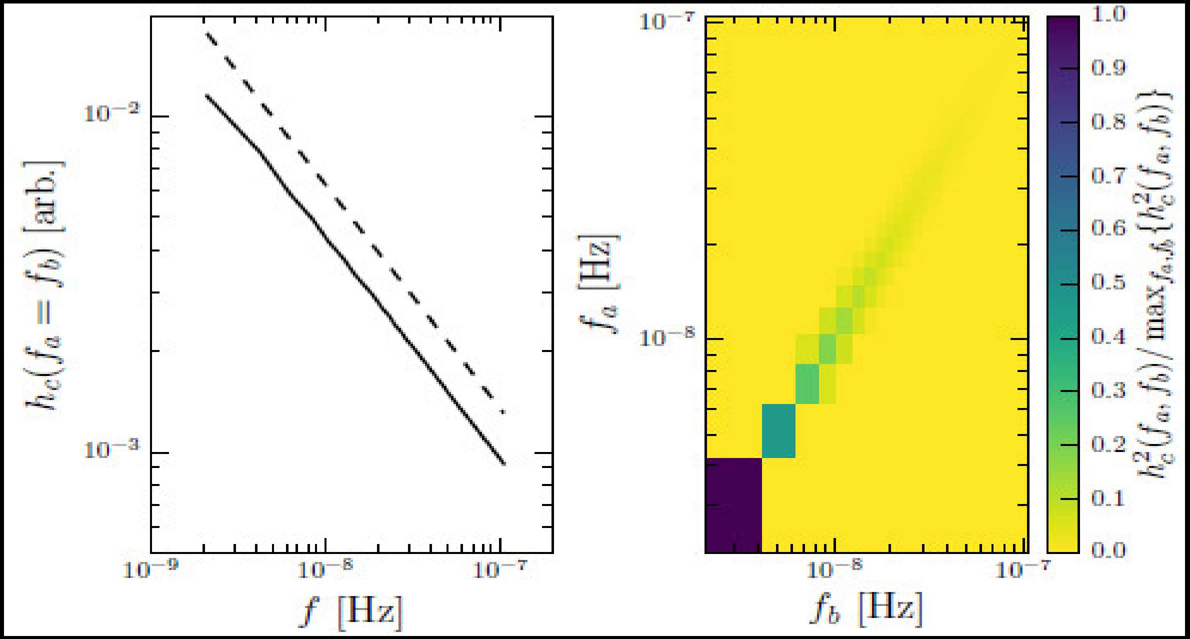
Point 1) causes a drop in the number of sources emitting at each frequency (analogue to environmental coupling)

Point 2) modifies the spectrum of the individual system

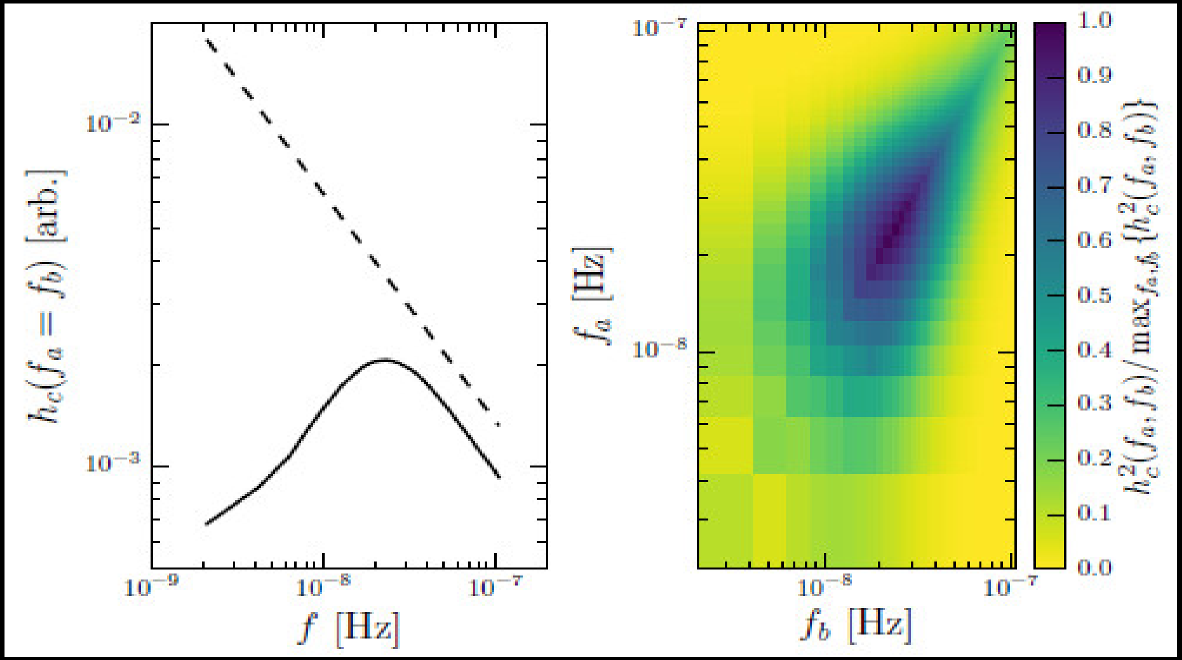
Both effects contribute to the shaping of the spectrum, but 1) is the dominant



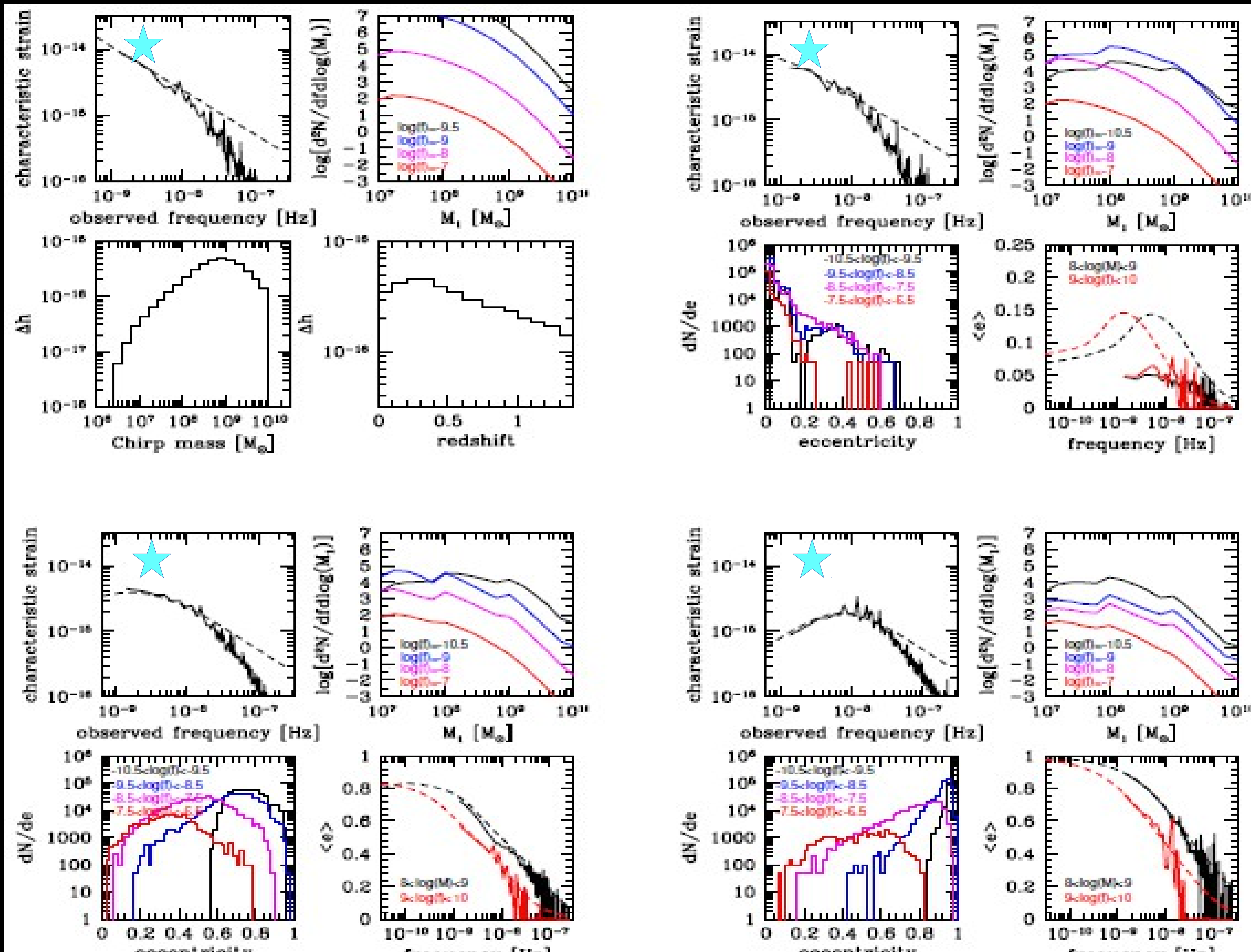
Recognizing eccentricity



If binary are circular, all frequency resolution bin are independent from each other: uncorrelated signal.



Eccentricity induce significant correlation among different frequency bins



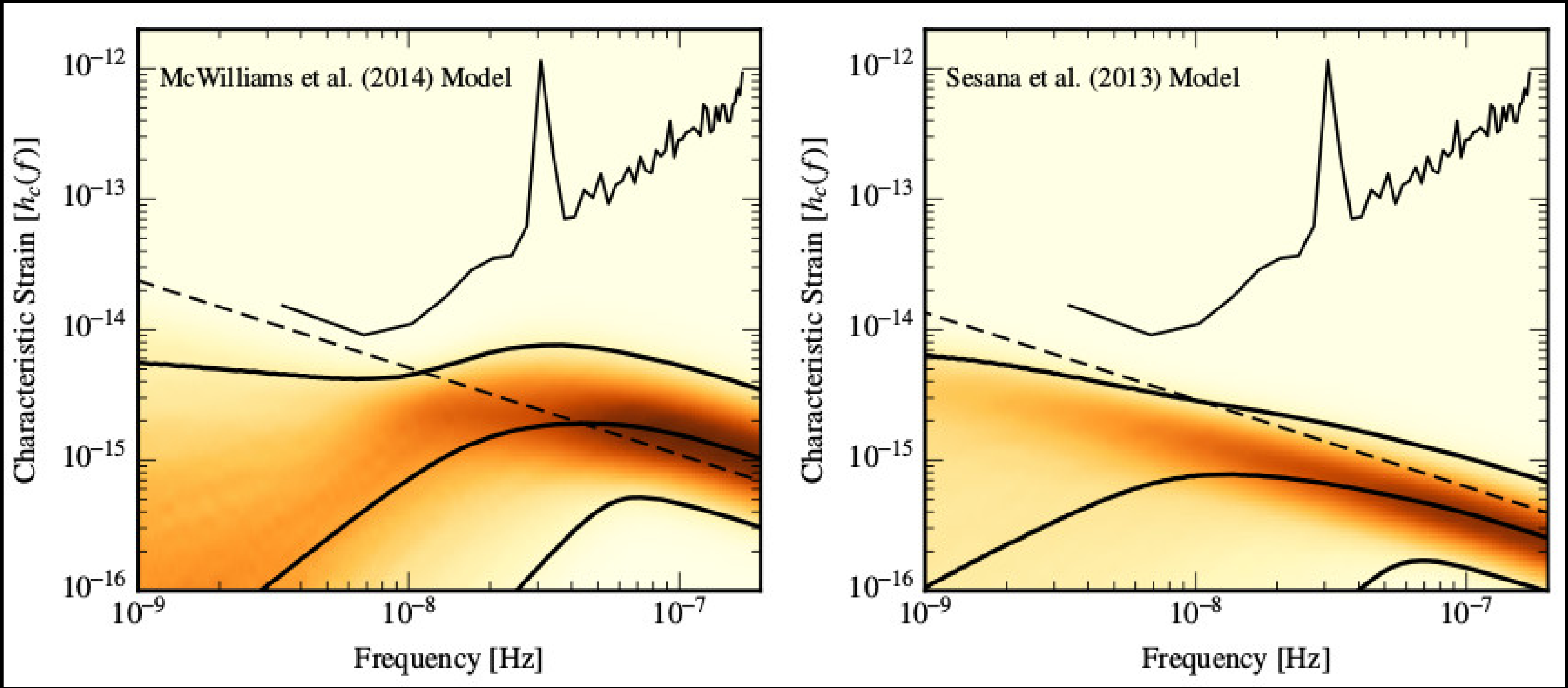
The distribution of initial binary eccentricities is unknown!

Dynamical constraints from PTA

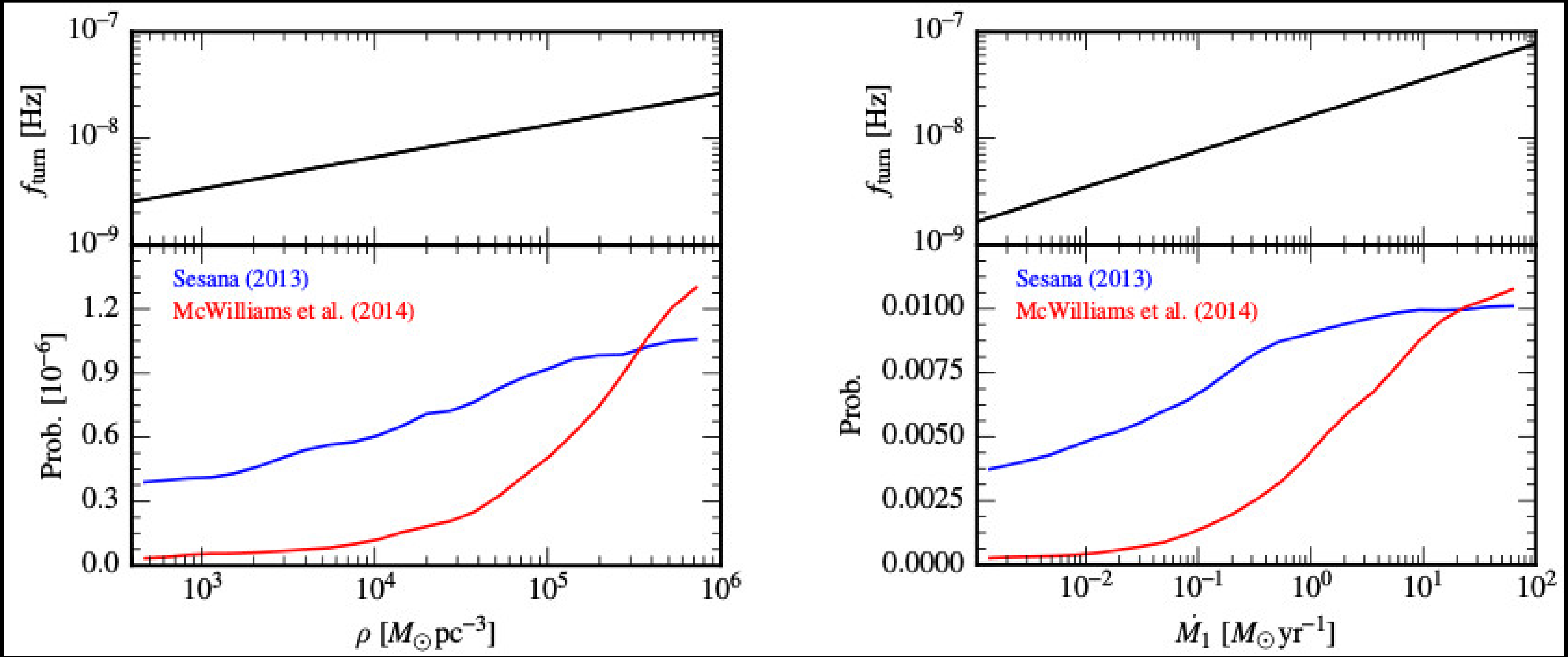
(NANOGrav, Arzoumanian et al. 2015)

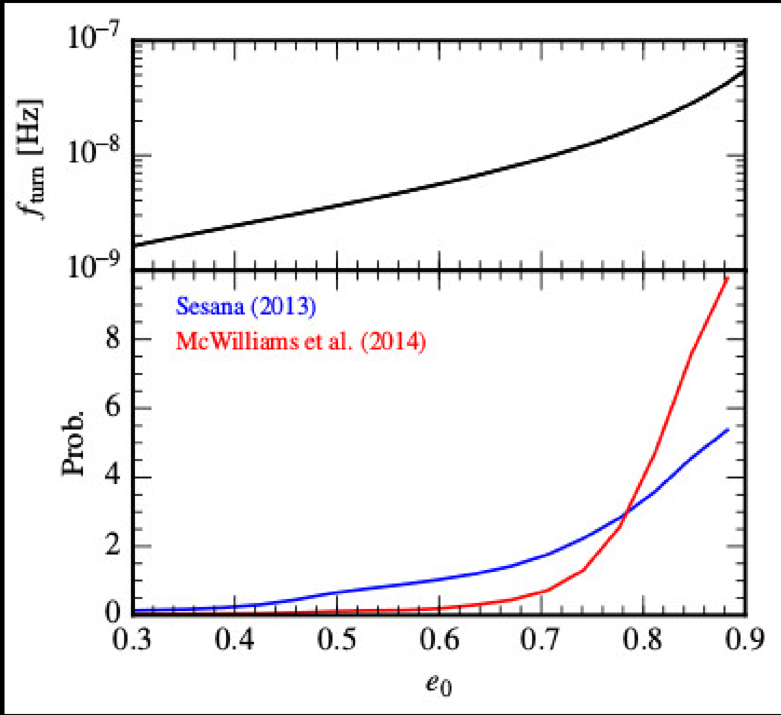
Simple broken-power law model mimicking possible environmental effects (Sampson et al. 2015)

$$h_c(f) = A\frac{(f/f_{\rm year})^{-2/3}}{(1+(f_b/f)^{\kappa})^{1/2}}$$



Depending on the prior on the amplitude, current non detection provide strong/little evidence of a background turnover





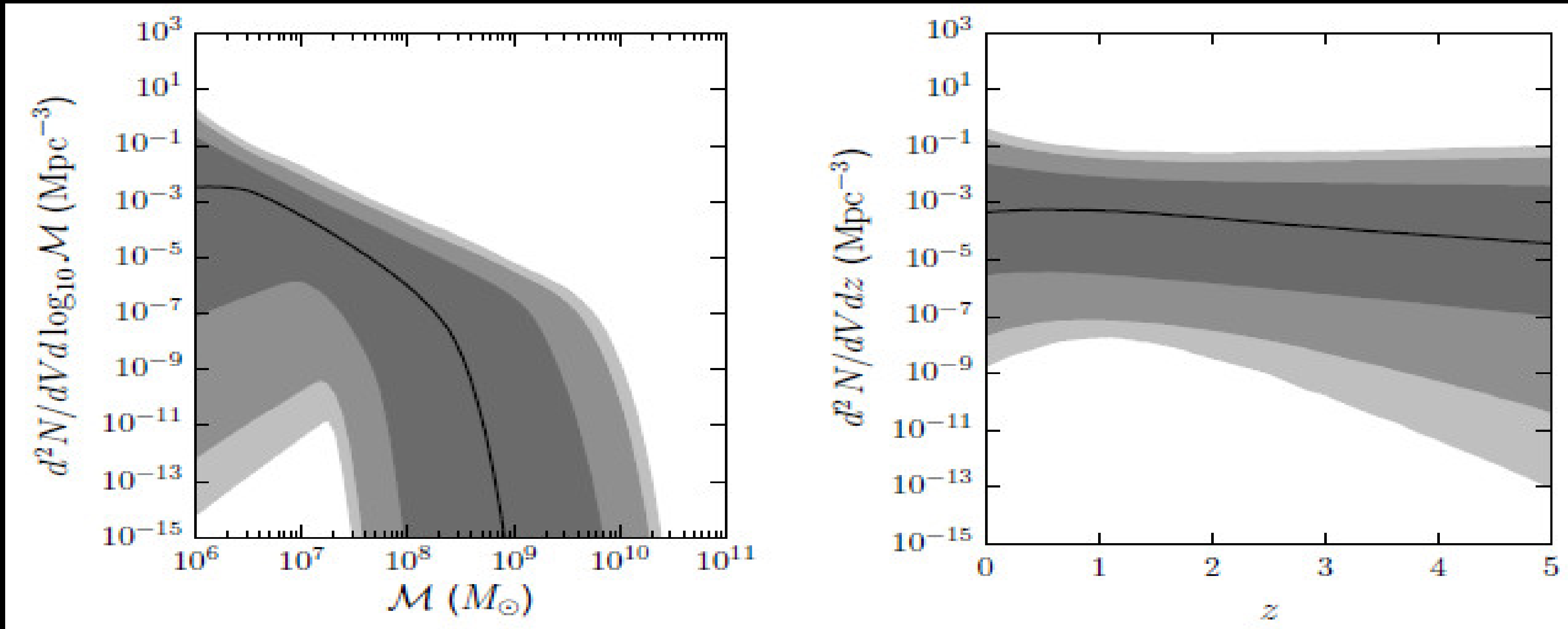
Similarly one can play the game of placing constraints on specific parameters *by keeping everything else fixed*:

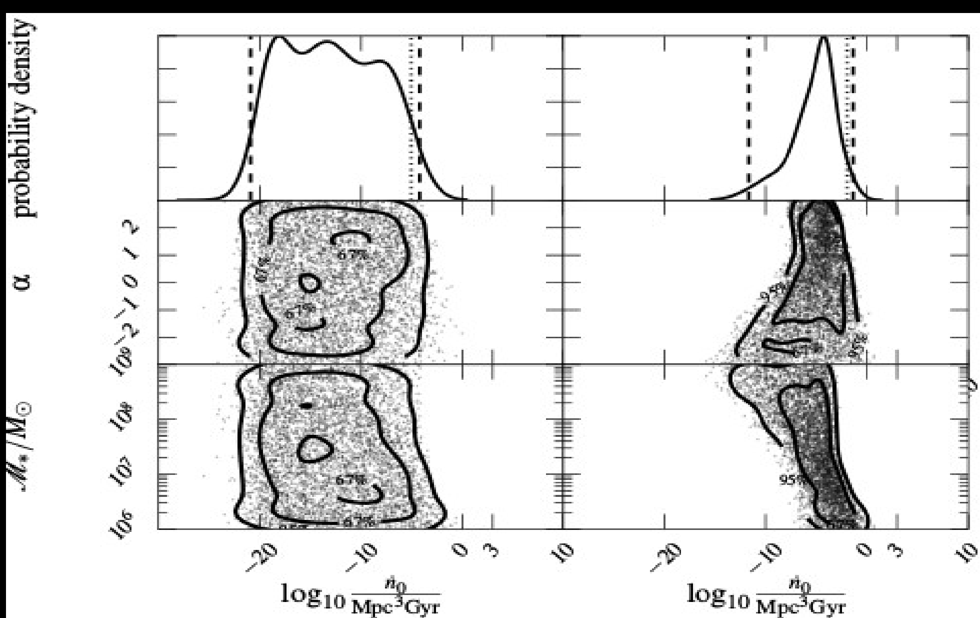
-density of the MBHB environment
-eccentricity

STILL AT THE LEVEL OF TOY MODELLING

What if we don't assume any merger rate prior?

(Middleton et al. 2015)

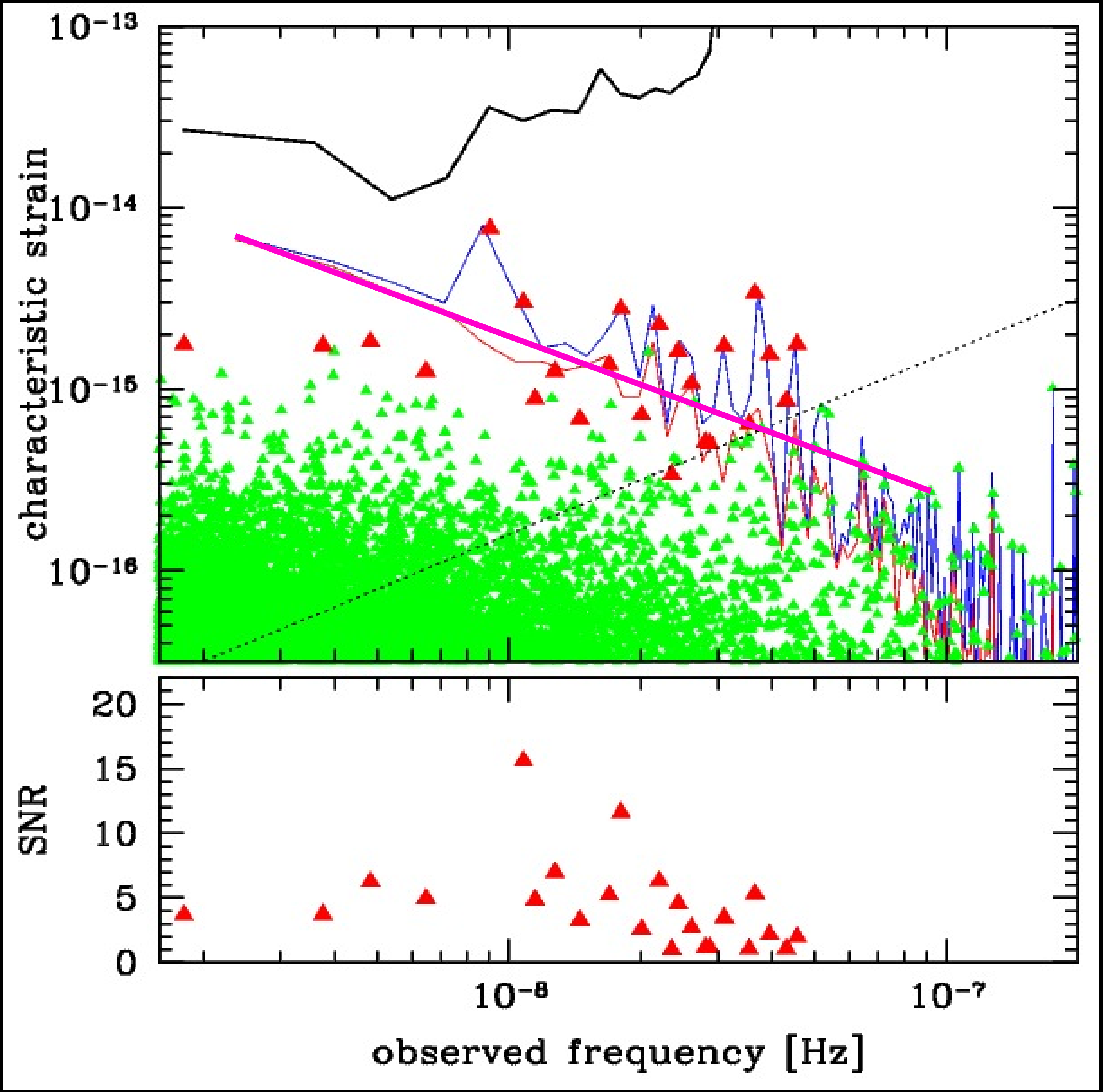




A PTA detection of a stochastic GWB will essentially *only constrain the overall MBHB merger rate*.

Need combination with other observation to be informative

The nature of the signal



*It is not smooth

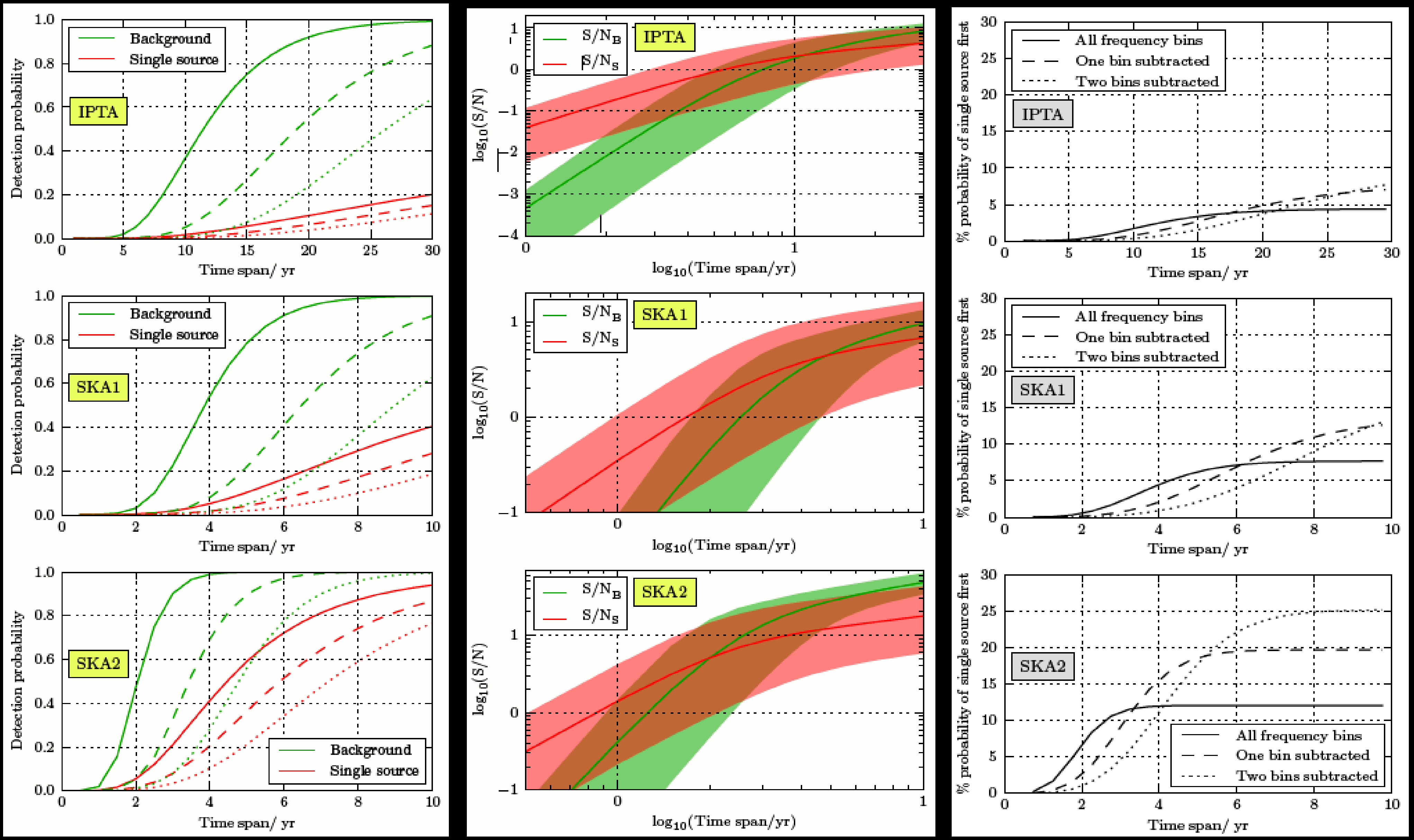
*It is not Gaussian

*Single sources might pop-up

*The distribution of the brightest sources might well be anisotropic

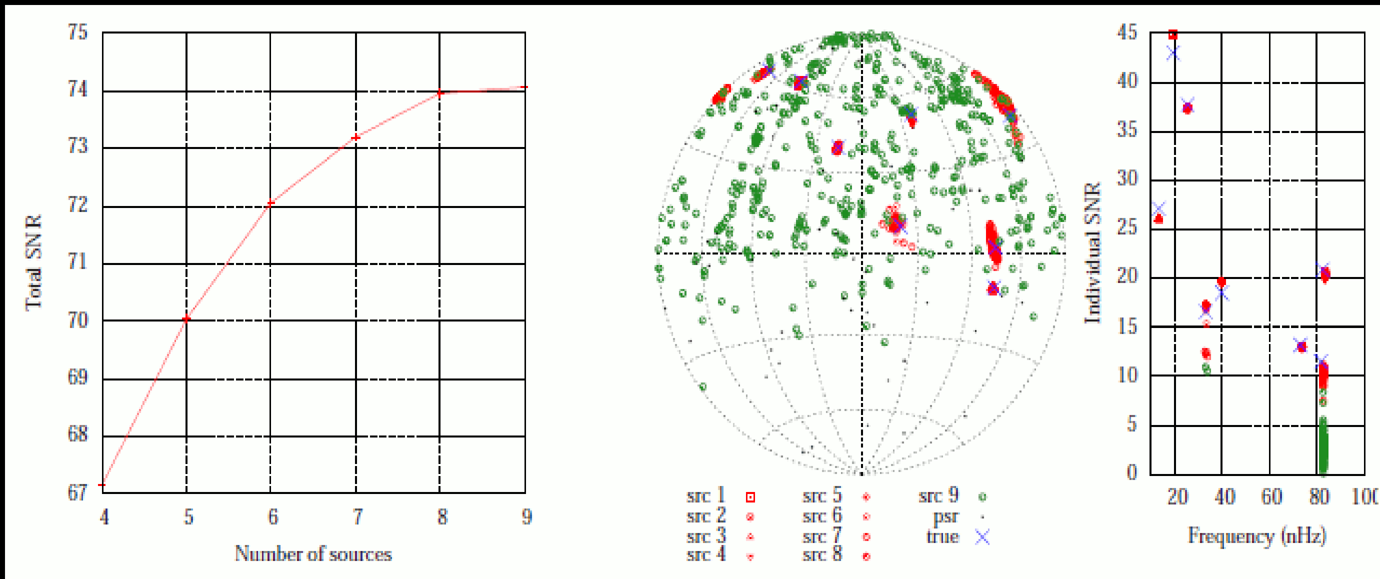
Continuous GW vs stochastic GWB

(Rosado et al. 2015)



- -A stochastic-like signal will be likely detected first (but it can be fairly different from a Gaussian isotropic signal, i.e. dominated by few sources)
- -However single source detection is not ruled out

Identification and sky localization

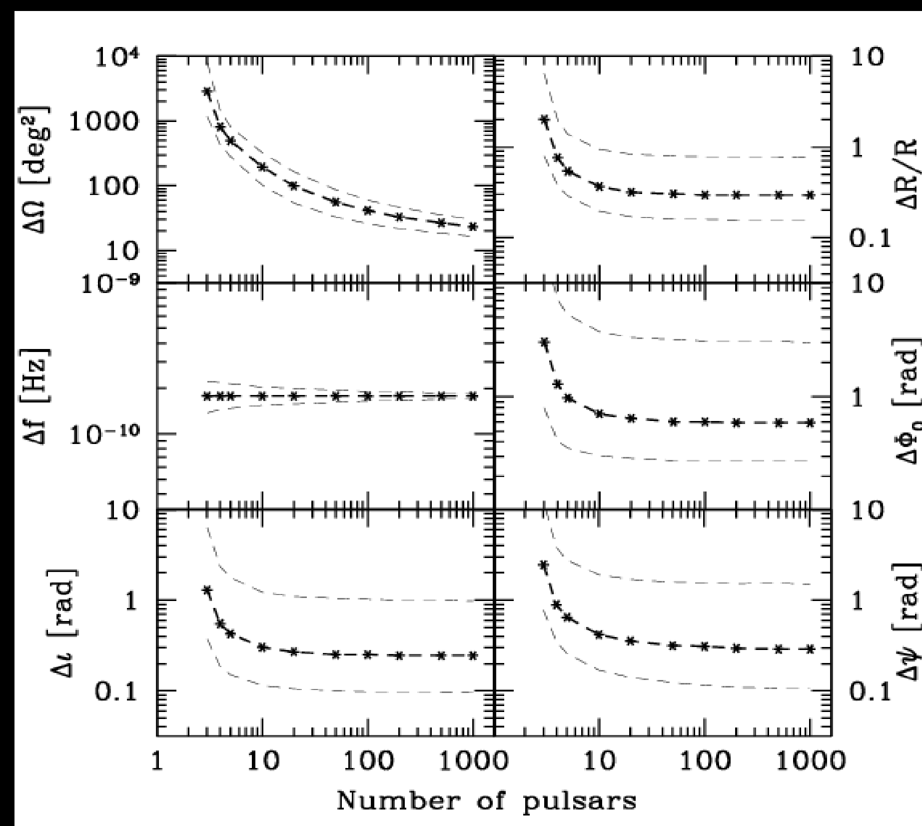


We can recover multiple sources in PTA data

(Babak & AS 2012
Petiteau et al. 2013)

Sources can be localized in the sky
(AS & Vecchio 2010, Ellis et al. 2012).

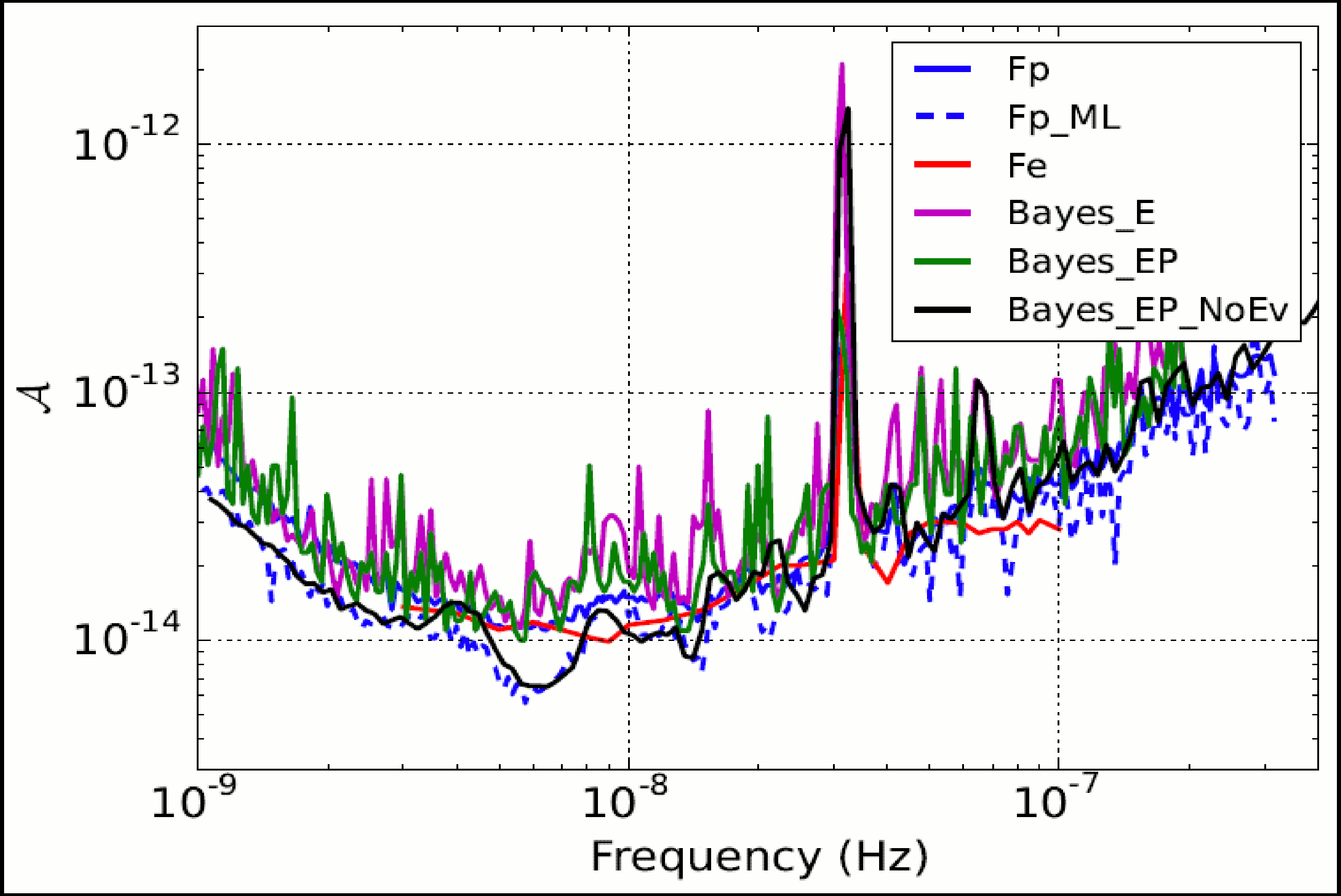
For example, the largest SNR source shown in the previous slide can be located by SKA in the sky with a sky accuracy <10deg^2



Limits on continuous GWs

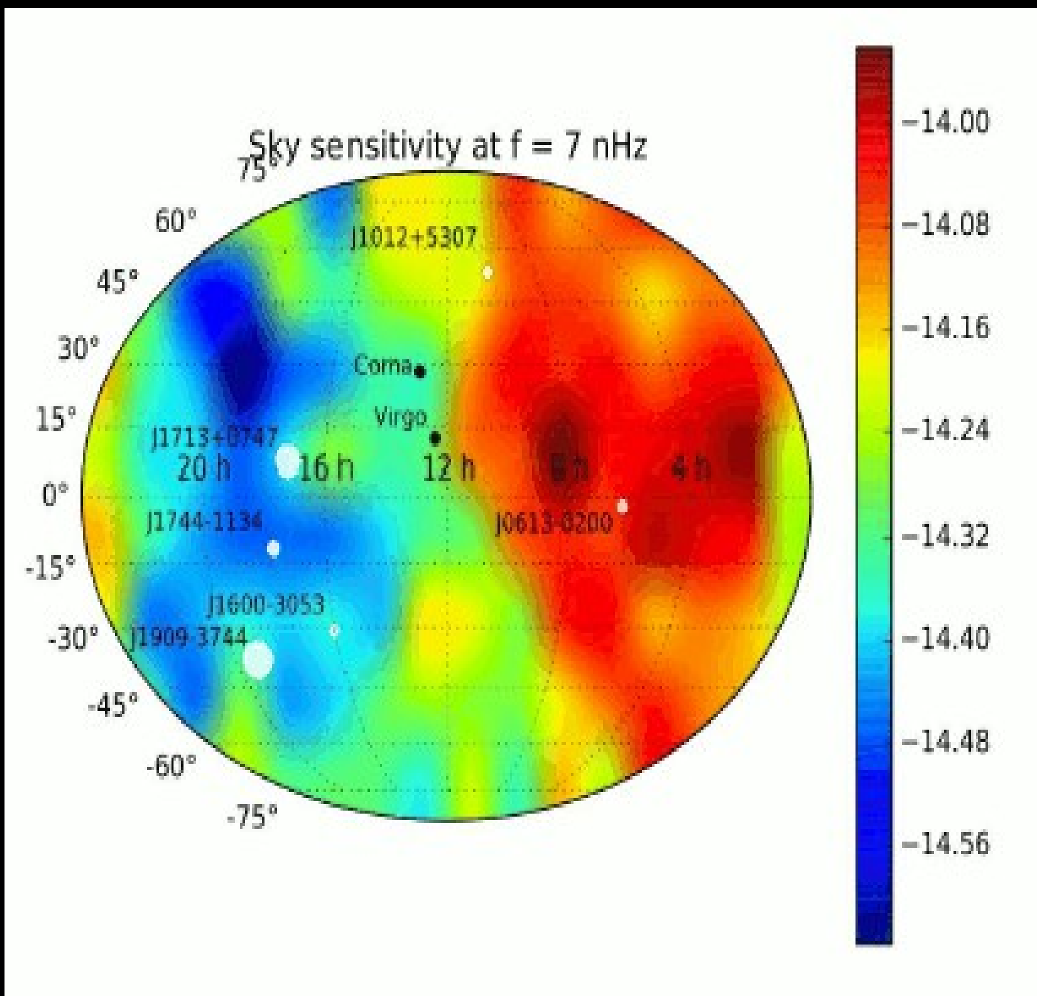
(EPTA, Babak et al. 2015)

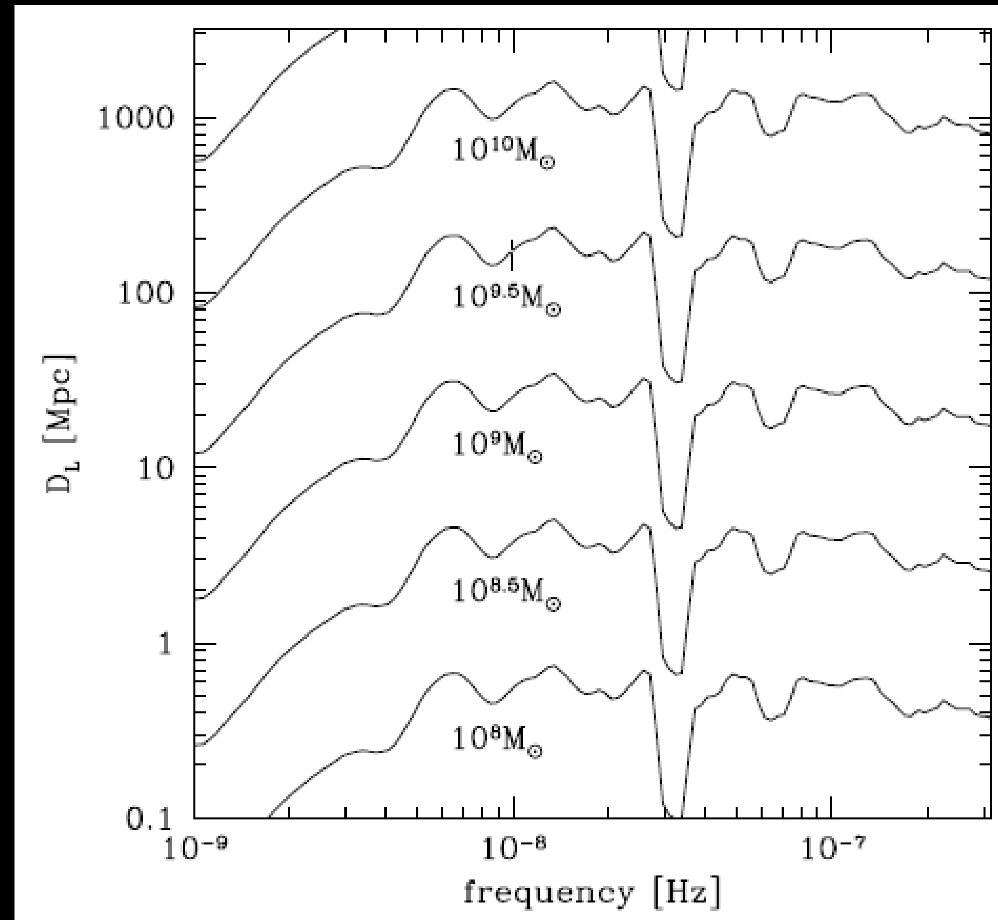
Search ID	Noise treatment	N pulsars	N parameters	Signal model	Likelihood
Fp_ML	Fixed ML	41	1	E+P NoEv	Maximized over 4 constant amplitudes plus pulsar phase
Fp	Sampling posterior	41	1	E+P NoEv	Maximized over 4 constant amplitudes plus pulsar phase
Fe	Fixed ML	41	3	E	Maximized over 4 constant amplitudes
Bayes_E	Fixed ML	41	7	E	Full
Bayes_EP	Fixed ML	6	$7 + 2 \times 6$	E+P Ev	Full
Bayes_EP_NoEv	Fixed ML	41	7	E+P NoEv	Pulsar phase marginalization
Bayes_EP_NoEv_noise	Searched over	6	$7 + 5 \times 6$	E+P NoEv	Pulsar phase marginalization



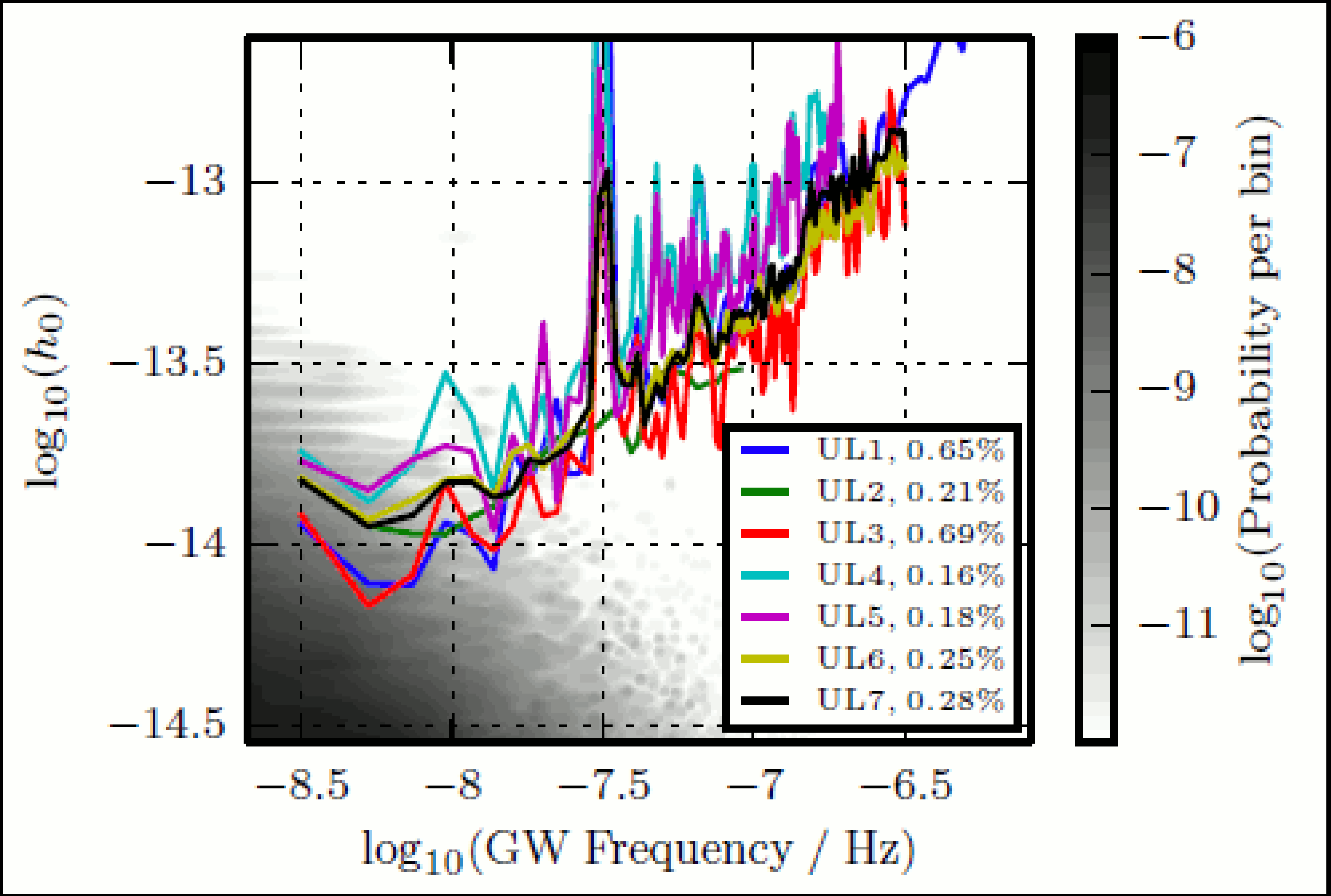
Astrophysical implications

The array sensitivity is function of the sky location, we can build sensitivity skymaps





Data are not yet very constraining, we can rule out very massive systems to ~200Mpc, well beyond Coma

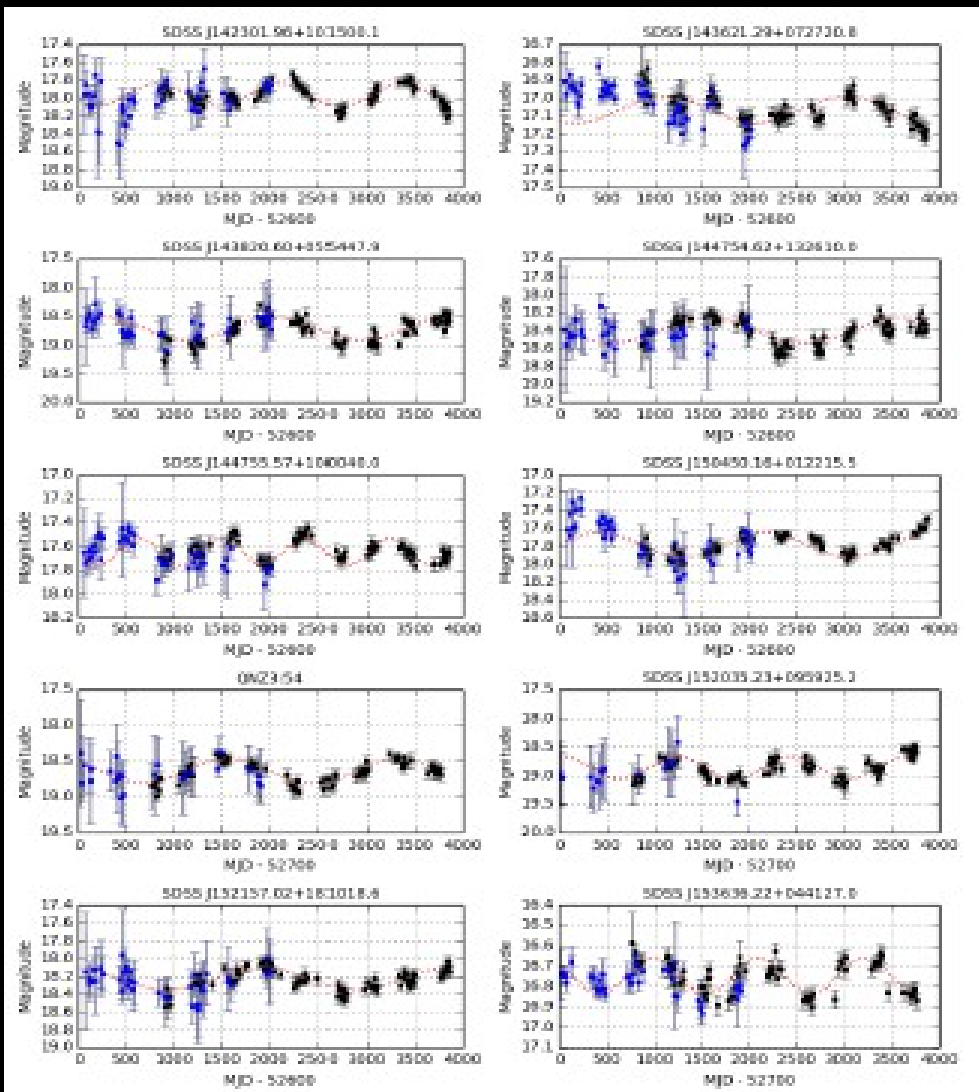


Current astrophysical models predict a ~1% detection probability only at current EPTA sensitivity

An interesting PTA case study

A systematic search for close supermassive black hole binaries in the Catalina Real-Time Transient Survey

Matthew J. Graham,[1*] S. G. Djorgovski,[1] Daniel Stern,[2] Andrew J. Drake,[1] Ashish A. Mahabal,[1] Ciro Donalek,[1] Eilat Glikman[3], Steve Larson[4], Eric Christensen[4]



Catilina survey:

9yr baseline, 250000 QSO

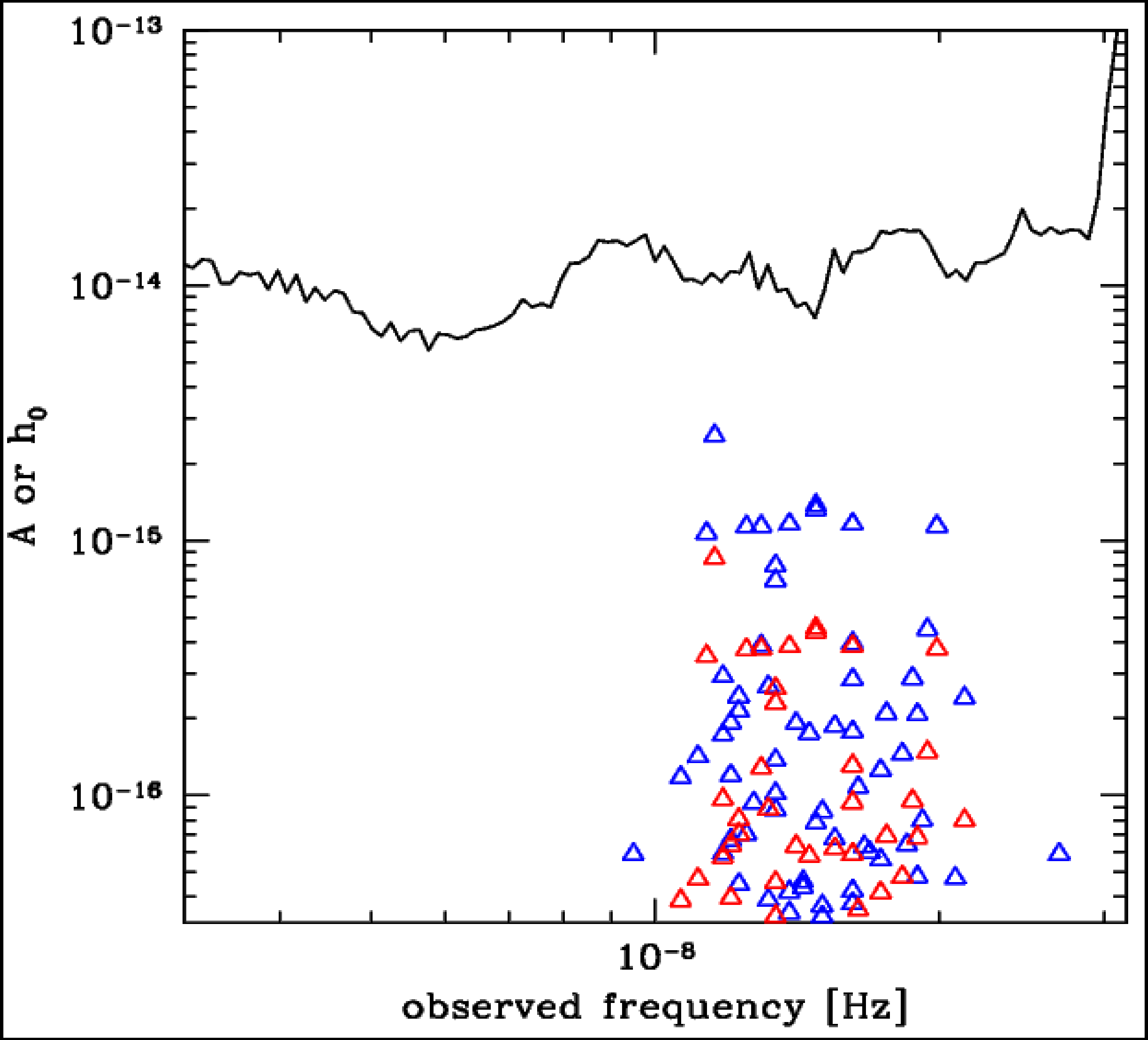
-required 1.5 cycles for periodicity identification.

-111 lightcurves showing periodic behaviour

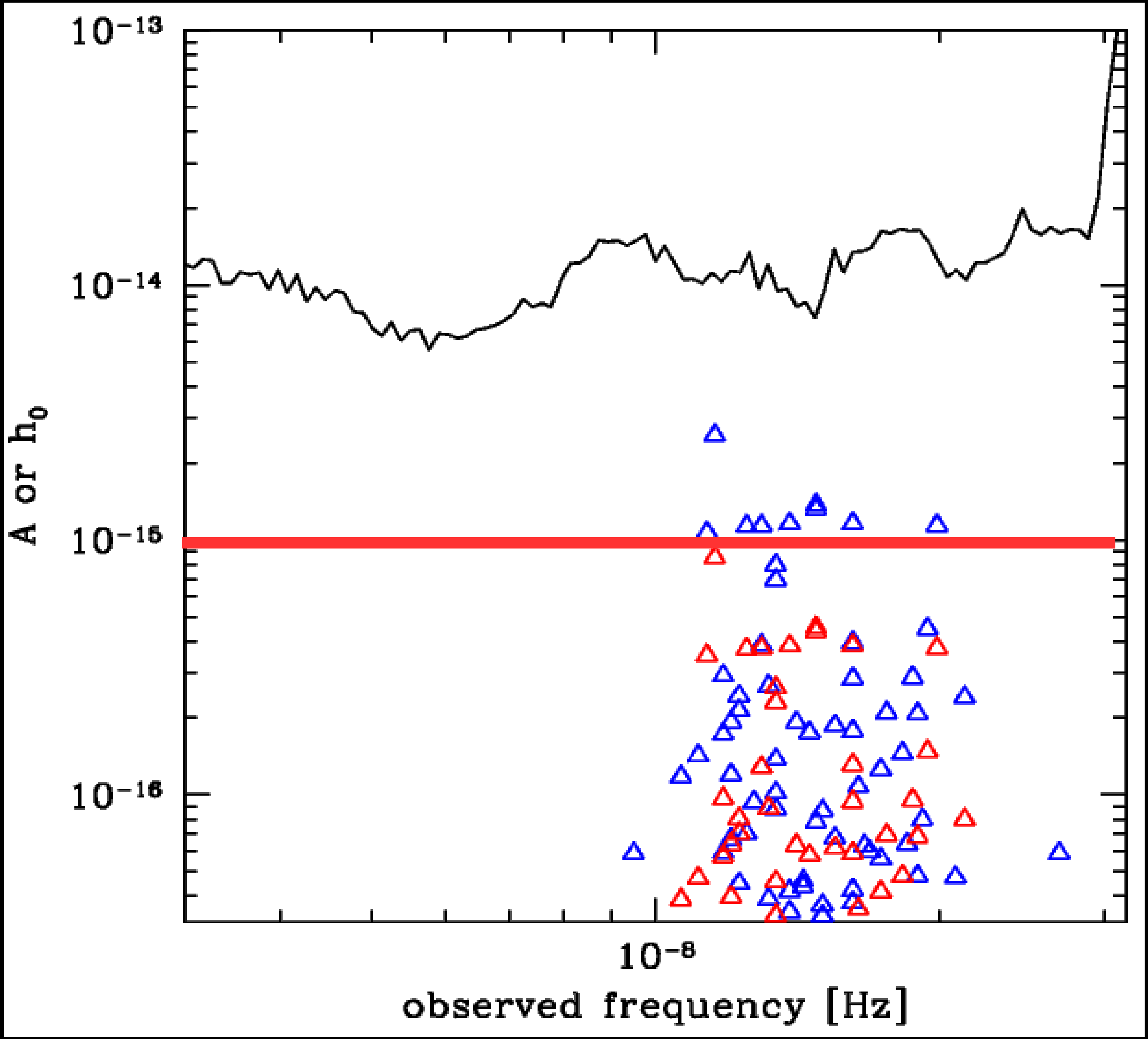
-For most of the systems we have: period, redshift, total mass, sky location, etc etc...

…not that I believe any of them, but...

Strain amplitude of individual sources

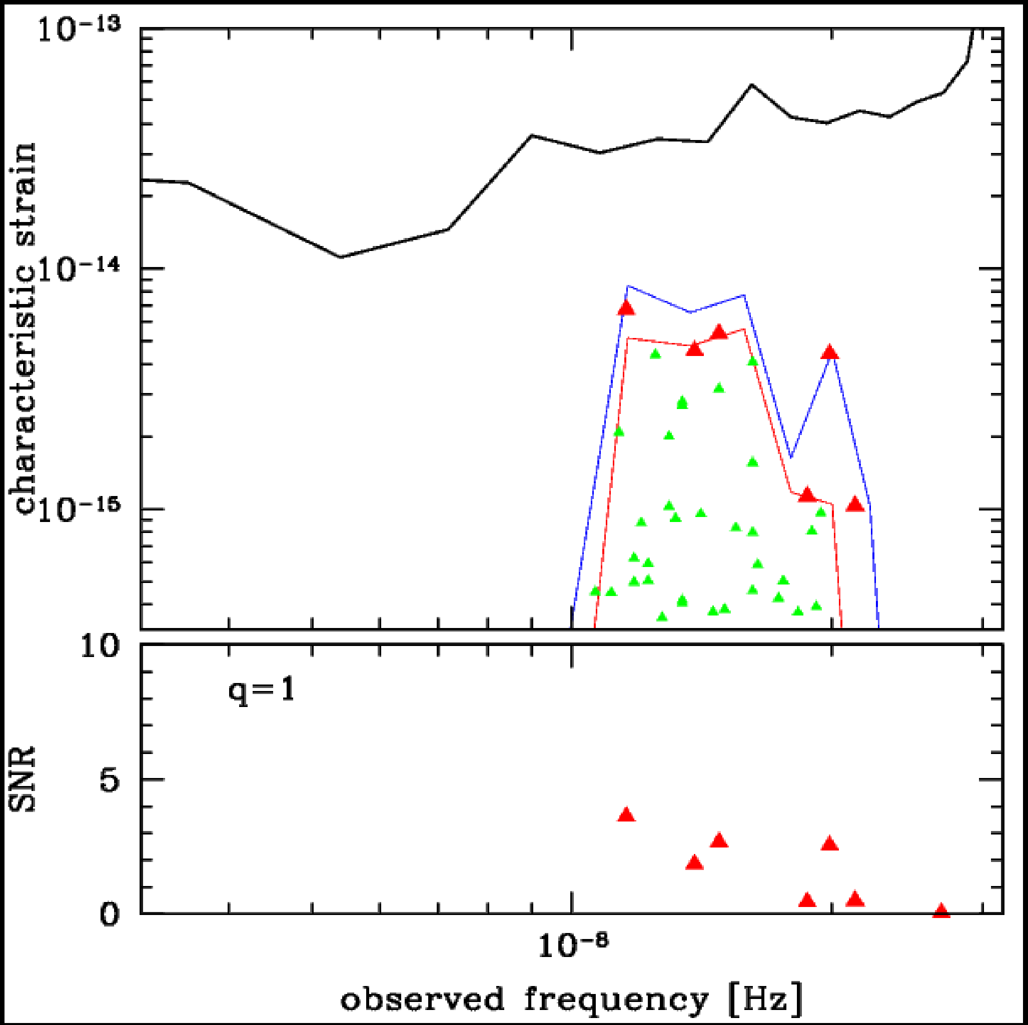


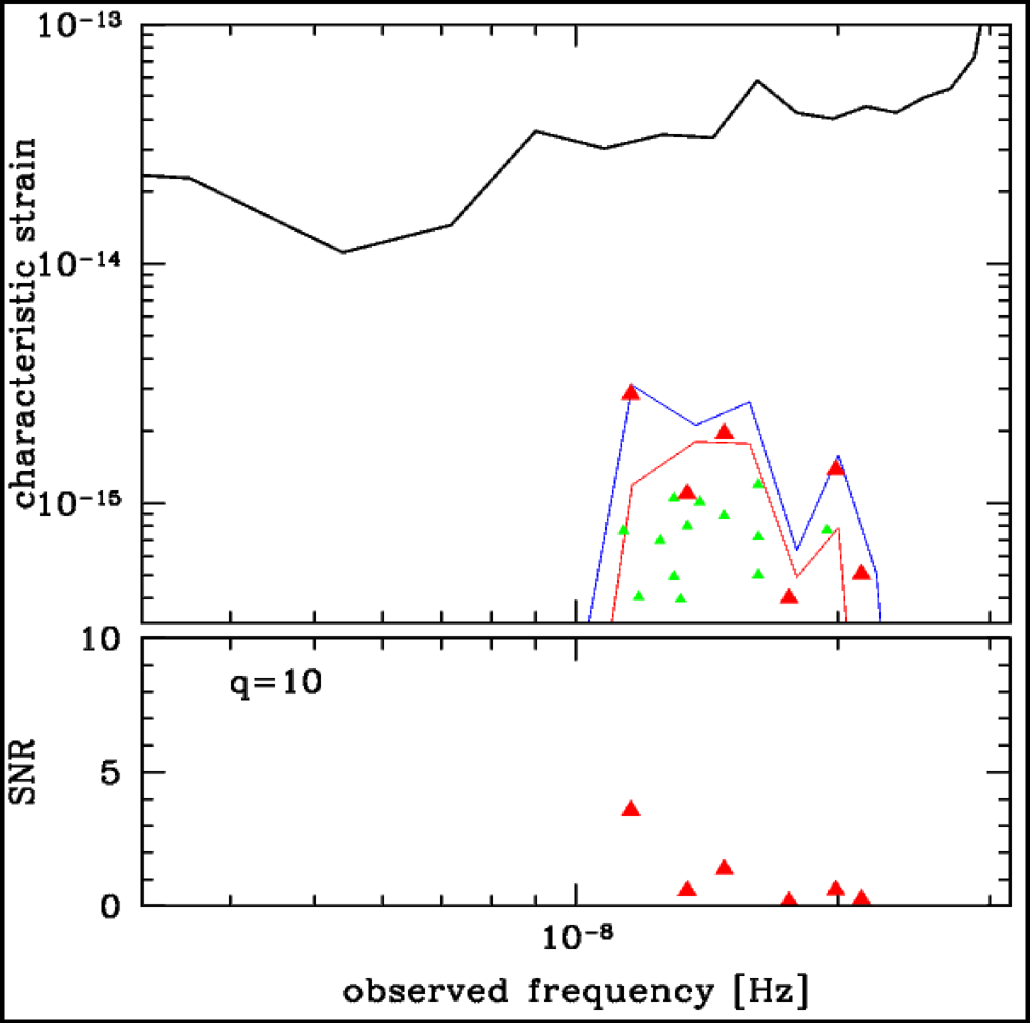
Strain amplitude of individual sources



Collective characteristic strain

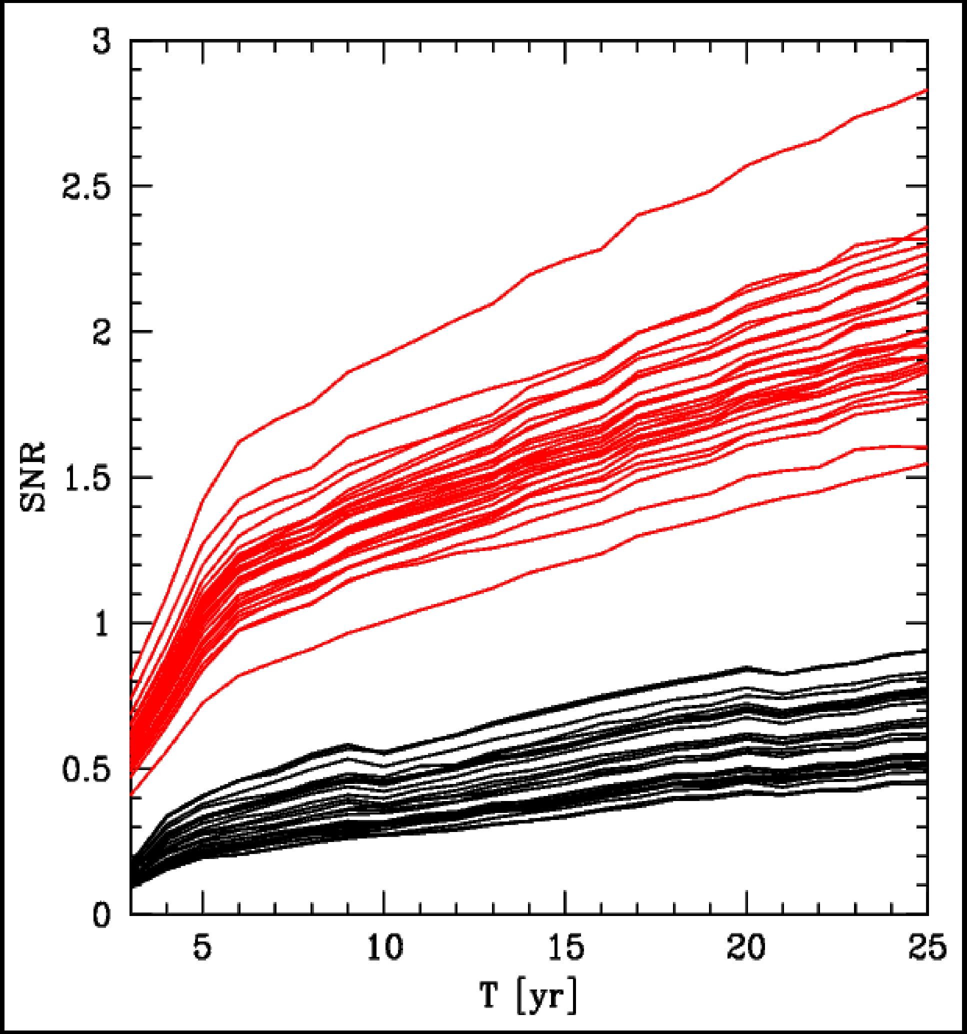
- -Take their systems, assign either q=1 or q=0.1 to all of them.
- -Randomize over inclination, polarization, etc
- -Compute the collective characteristic strain



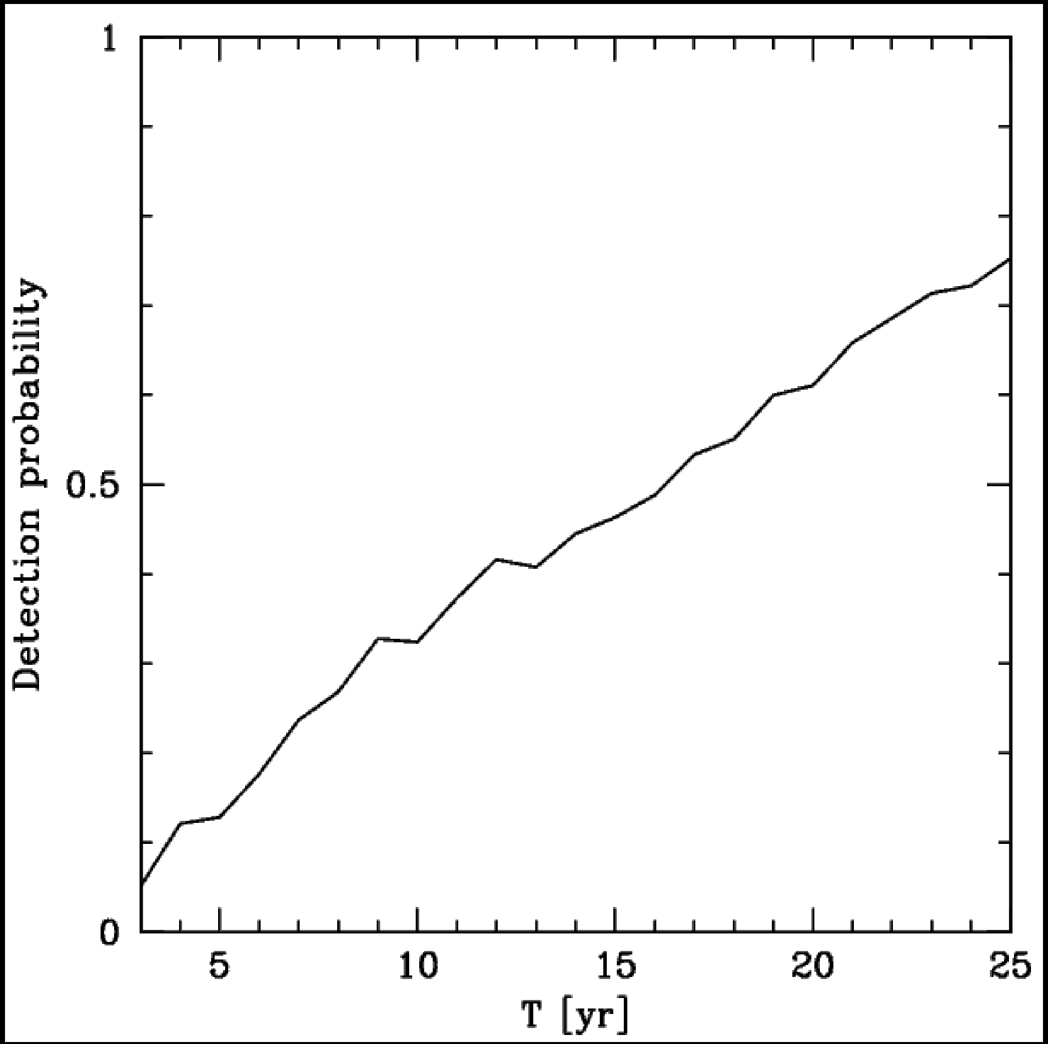


Detection prospects

I took 10 pulsars with 200ns rms randomly located in the sky, white noise only



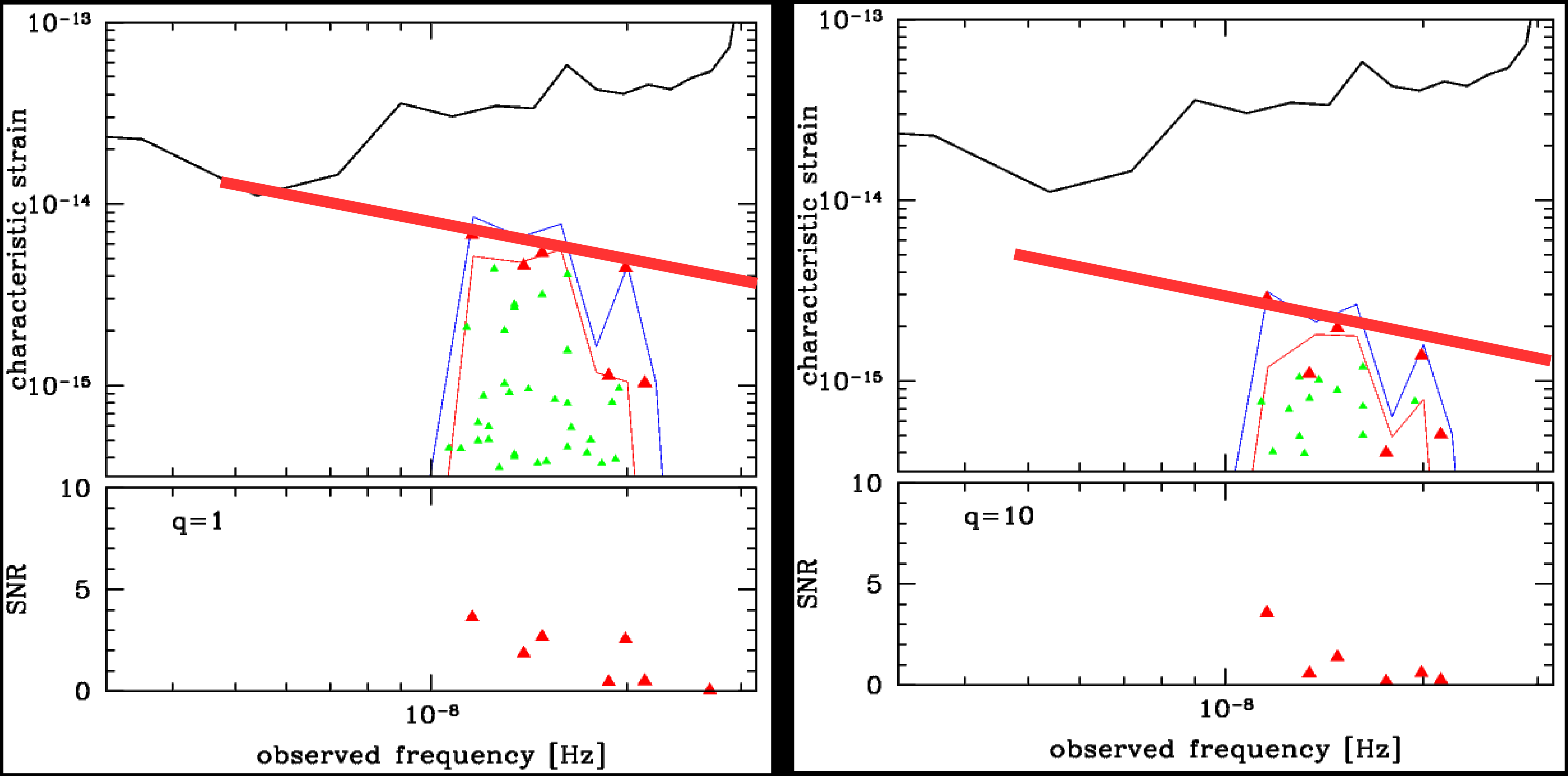
Cross correlation SNR of the overall characteristic amplitude



Probability of detecting an individual source (q=1)

Collective characteristic strain

- -Take their systems, assign either q=1 or q=0.1 to all of them.
- -Randomize over inclination, polarization, etc
- -Compute the collective characteristic strain



WE CAN ALREADY RULE OUT A VANILLA EXTRAPOLATION OF THESE SYSTEMS!

Doggybag

Current limits are getting extremely interesting, showing some tension with vanilla models for the cosmic SMBHB population

PTAs can in principle provide unique information about the dynamics and merger history of MBHBs (e.g. merger rate density, environmental coupling, eccentricity, etc.)

However:

> considering current observational uncertainties, there might be tension, but even vanilla models cannot be confidently ruled out

> detection statistics: is the signal stochastic?

> basically any step towards a more realistic modelling tend to make the signal dimmer:

***coupling with the environment (but how efficient?)**

***eccentricity (critical ingredient)**

> stalling might be an issue in the most massive low density ellipticals

*** time delays?**

*** triple interactions common?**

OUTLINE

>massive black hole (MBH) hierarchical assembly and gravitational wave (GW) detection

>using PTA limits to constrain the MBHB population (stochastic background)

>limits on individually resolvable sources

>Interesting study case: the Catilina survey

Gravitational wave basics

Every accelerating mass with non-zero quadrupole mass moment emits gravitational waves

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \qquad h_{\mu\nu} \ll 1$$

Gws are transverse, have 2 polarizations (in GR) and travel at the speed of light

$$g_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 + h_{+}^{TT} & h_{\times}^{TT} \\ 0 & 0 & h_{\times}^{TT} & 1 - h_{+}^{TT} \end{pmatrix}$$