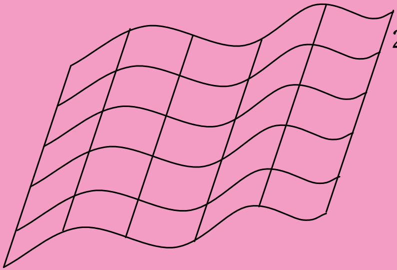
General Relativity in 60 minutes or Less

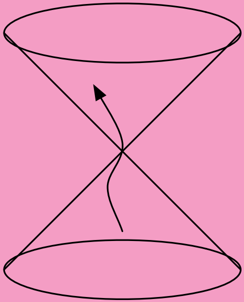
IPTA 2017 Student Workshop

Jeffrey S Hazboun

CIEP



28 June 2017



Introduction: What you won't learn in this talk :-)

- The Einstein summation convention.
- The difference between a contravariant and covariant vector.
- How to calculate the Christoffel connection.
- How to solve for the Schwarzschild solution.
- The Canonical Quantization of the Einstein equation.

Historical Introduction: Before Relativity

- Newton comparing gravity to light in a letter to Charles Boyle:
  "So may the gravitating attraction of the earth be caused by the continual condensation of some other such like ethereal spirit... in such a way... as to cause it [this spirit] from above to descend with great celerity for a supply; in which descent it may bear down with it the bodies it pervades, with force proportional to the superficies of all their parts it acts upon."
- John Michell and Dark Stars
  Philosophical Transactions of the Royal Society of London, 27 November 1783

$$\frac{1}{2}mv^2 = \frac{GMm}{r}$$

- Maxwell's Equations:

$$\epsilon_0 \mu_0 = 1/c^2$$

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$$\frac{1}{2}mv^2 = \frac{GMm}{r}$$

$$v \to c \Rightarrow \boxed{r = \frac{2GM}{c^2}}$$ Schwarzschild Radius !!

- Maxwell's Equations:

$$\epsilon_0 \mu_0 = 1/c^2$$

Historical Introduction: General Relativity

- Special Relativity: Nothing can travel faster than light.
- Minkowski Space (1907): Special Relativity as Geometry of spacetime.
- Can not reconcile Special Relativity with Newtonian gravity.
- Gravity cannot travel faster than light!
- The curvature of spacetime is gravity...
- Differential geometry as the necessary math, with Grossmann

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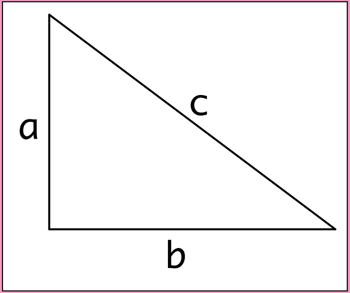
Differential Geometry Primer:

4th Grade Geometry to GR in 3 slides

$$a^2 + b^2 = c^2 \qquad \text{2D}$$

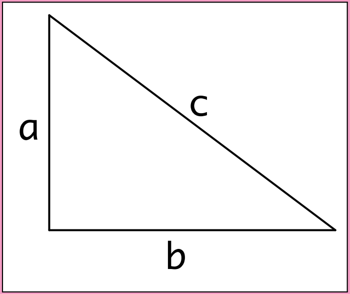


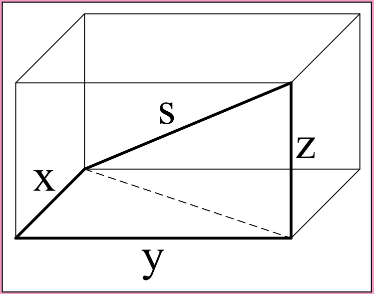
Differential Geometry Primer:

4th Grade Geometry to GR in 3 slides

$a^2 + b^2 = c^2$ 2D

$x^2 + y^2 + z^2 = s^2$ 3D





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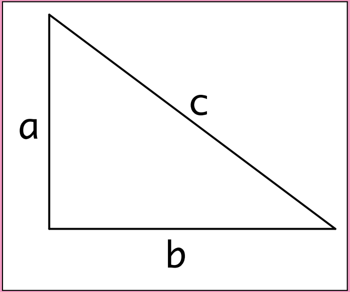

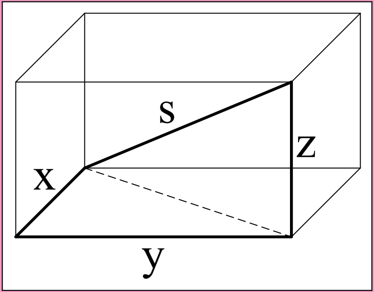

$$w^2 + x^2 + y^2 + z^2 = s^2 \qquad \text{4D}$$

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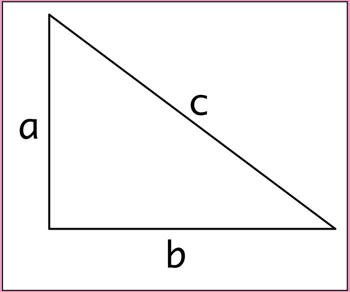
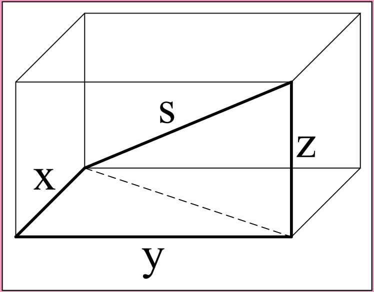
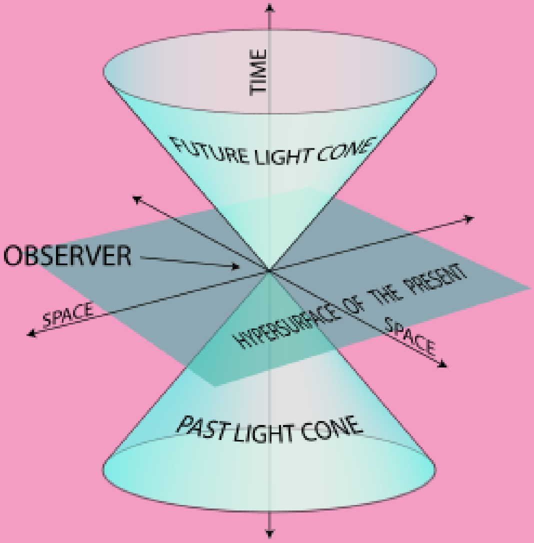




$$w^2 + x^2 + y^2 + z^2 = s^2 \qquad \text{4D}$$

$$-c^2\Delta t^2 + \Delta x^2 + \Delta y^2 + \Delta z^2 = \Delta s^2 \qquad \text{4D Special Relativity}$$

LightCones: Encoded in the metric



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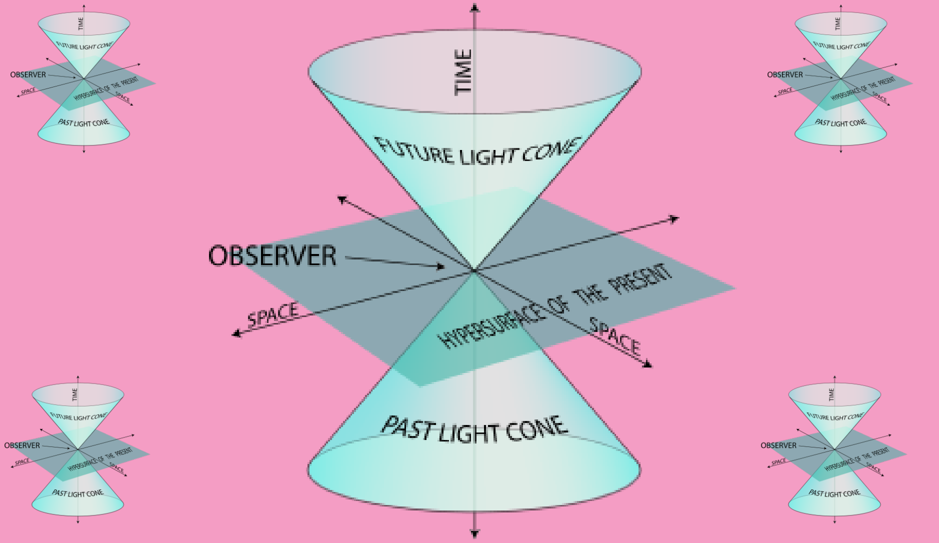

Dot product to $\eta_{\mu\nu}$ to $g_{\mu\nu}$.

$$v^2 = \vec{v} \cdot \vec{v} \quad = \quad \begin{bmatrix} v_x & v_y & v_z \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix}$$

Dot product to $\eta_{\mu\nu}$ to $g_{\mu\nu}$.

$$
\begin{aligned}
v^2 = \vec{v}\cdot\vec{v} &= \begin{bmatrix} v_x & v_y & v_z \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} \\
&= \begin{bmatrix} v_x & v_y & v_z \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix}
\end{aligned}
$$

The Metric
Encoding a generalized Pythagorean Theorem

Euclidean Signature

$$[\delta_{ab}] \equiv \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Lorentzian Signature

$$[\eta_{ab}] \equiv \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Differential Geometry/ General Relativity

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Differential Geometry/ General Relativity

$$\begin{aligned} ds^2 = &- c^2 f_1(x^\mu)\,dt^2 + f_2(x^\mu)\,dt\,dx + f_3(x^\mu)\,dx^2 \\ &+ \ldots + f_8(x^\mu)\,dy^2 + f_9(x^\mu)\,dy\,dz + f_{10}(x^\mu)\,dz^2 \end{aligned}$$

$$[g_{\mu\nu}] \equiv \begin{bmatrix} \text{A} & & & \\ & \text{Bunch} & & \\ & & \text{of} & \\ & & & \text{Functions} \end{bmatrix}$$

Heuristic Einstein Equation

Just a Tensor Differential Equation for the metric

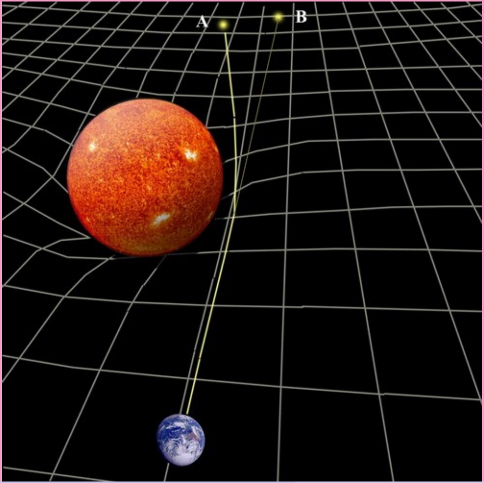
Matrix vs. Tensor?

- Like a generalization of a scalars, vectors and matrices.
- **But** also well behaved under coordinate transformations.

$$G_{\mu\nu}\left(g_{\mu\nu}, \frac{\partial g_{\mu\nu}}{\partial x^{\alpha}}, \frac{\partial^2 g_{\mu\nu}}{\partial x^{\alpha}\partial x^{\beta}}\right) = T_{\mu\nu}\,(\text{Matter Fields})$$

General Relativity & the Einstein Field Equation

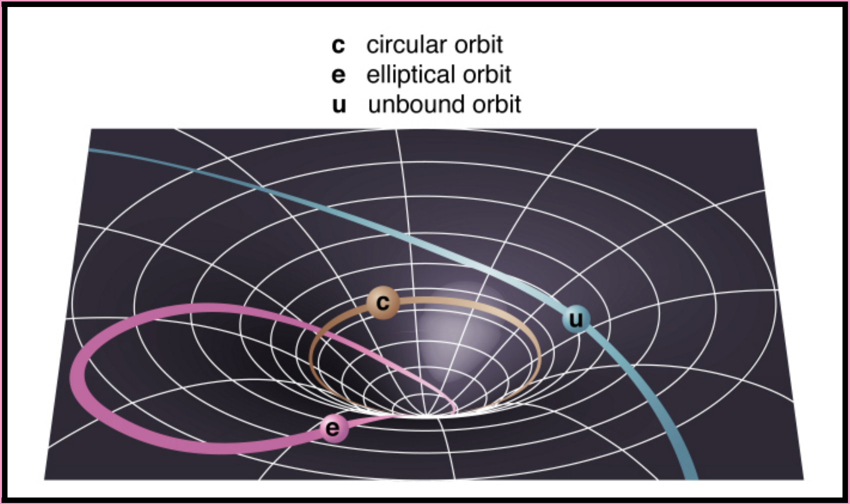
Spacetime Curvature = Matter/Energy Content



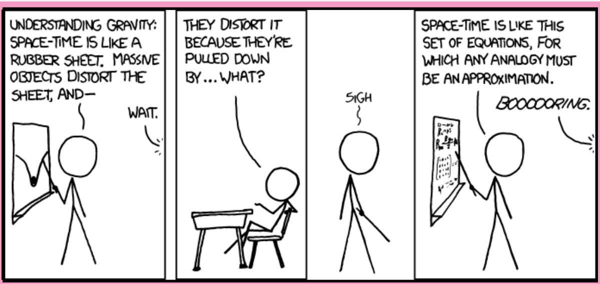
Matter tells spacetime how to *curve*.

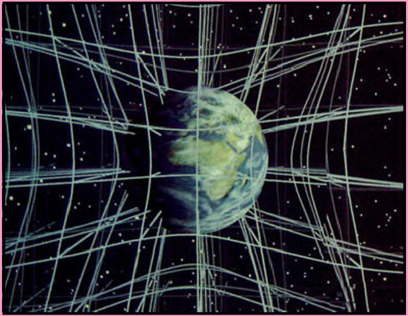
Spacetime tells matter how to *move*.

Particles follow curved lines when spacetime is curved



Just an analogy...



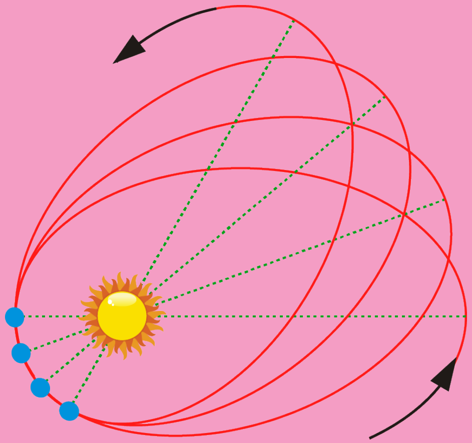


Advance of the perihelion of Mercury

- GR Kepler's Law:

$$\frac{d^2u}{d\phi^2} - 1 + u = \frac{3G^2M^2}{L^2}u^2 \quad \text{GR Term}$$

$$u \equiv \frac{L^2}{GMr}$$



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Credit: WikiCommons

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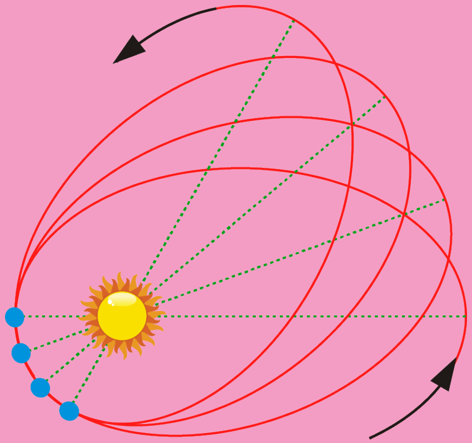
- Advance in (arcseconds / century):

Source	
Precession of Equinox	5025
Sun's Oblateness	0.028
Perturbations of Planets	532
GR	43
Total	5601



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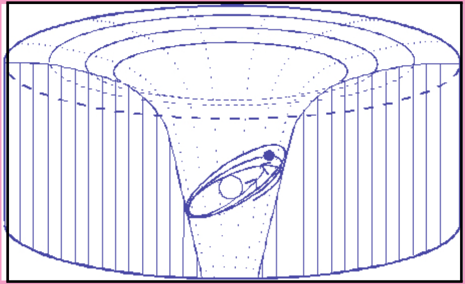


Paul Marmet

General Relativity in our phones.

Special Relativity

Relative speed → Δt slower

7μs slower per day.

General Relativity

Smaller curvature → Δt faster

45μs faster per day.

Net Result

Time passes $38\mu s$

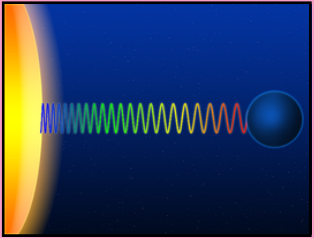
faster per day.

Errors of 10 km per day.

GPS Satellites:

GR in Practice: Moving through curved spacetime

- Gravitational Redshift (Einstein Delay)
- Shapiro Effect (Shapiro Delay)
- Gravitational Lensing



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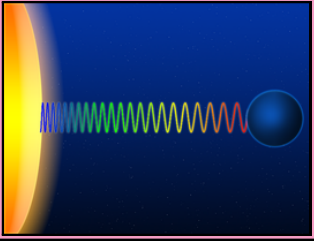

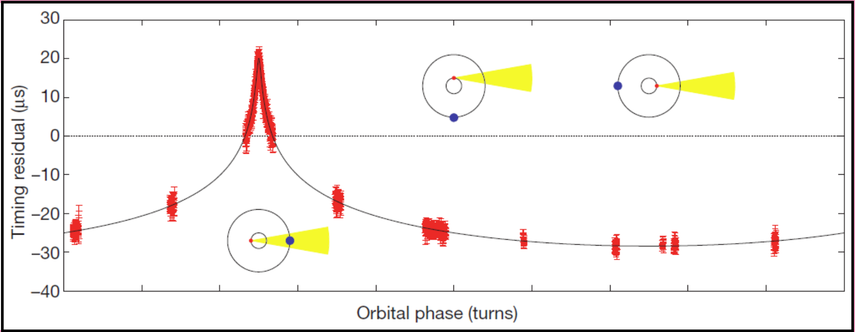

Demorest, et al., Nature. 2010 Oct 28;467(7319):1081-3. doi: 10.1038/nature09466

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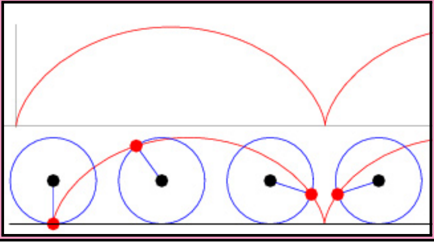

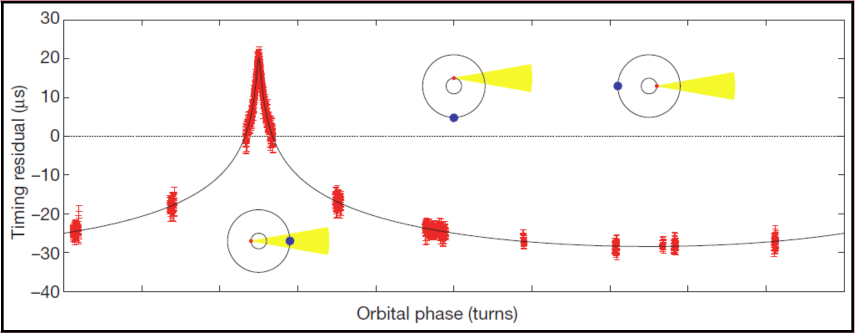

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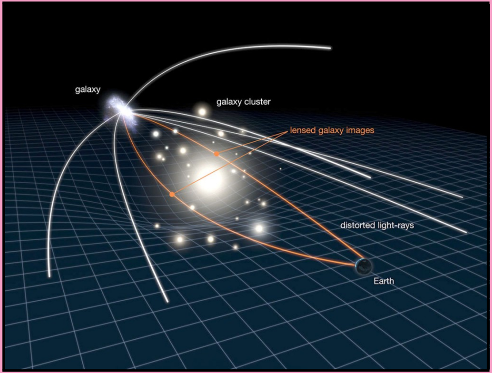

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GR in Practice: Moving through curved spacetime

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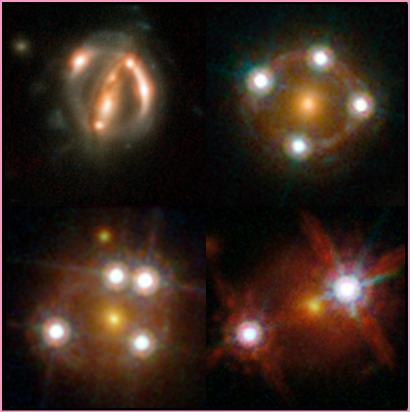

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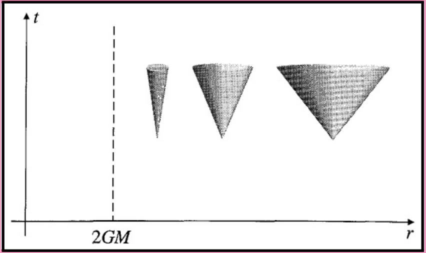
The Schwarzschild Metric and Black Holes

$$ds^2 = -\left(1 - \frac{2Gm}{rc^2}\right) dt^2 + \frac{1}{\left(1 - \frac{2Gm}{rc^2}\right)} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta \, d\phi^2$$



Credit: Sean Carroll

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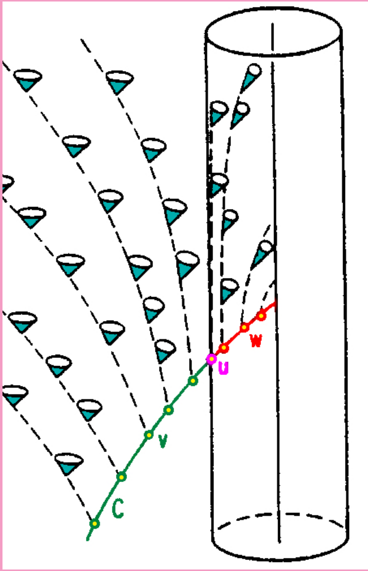

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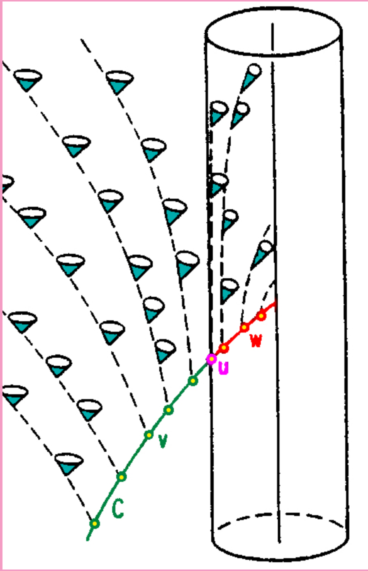
The singularity at $r = 0$ is a different story.



Linear Gravity

Einstein 1916 & 1918

- What do you do when you have complicated DiffEQ?
  Symmetry or Perturbation Theory

- What do small perturbations in the metric look like in the curvature?

$$g_{\nu\mu} = \eta_{\nu\mu} + h_{\nu\mu}$$

  Where $h_{\nu\mu}$ is assumed small.

- The Einstein equation, written in terms of $h_{\nu\mu}$ is a wave equation

$$-\frac{\partial^2}{\partial t^2} h_{\nu\mu} + \frac{\partial^2}{\partial \vec{x}^2} h_{\nu\mu} = -16\pi G T_{\nu\mu}$$

Quadrupolar Signal

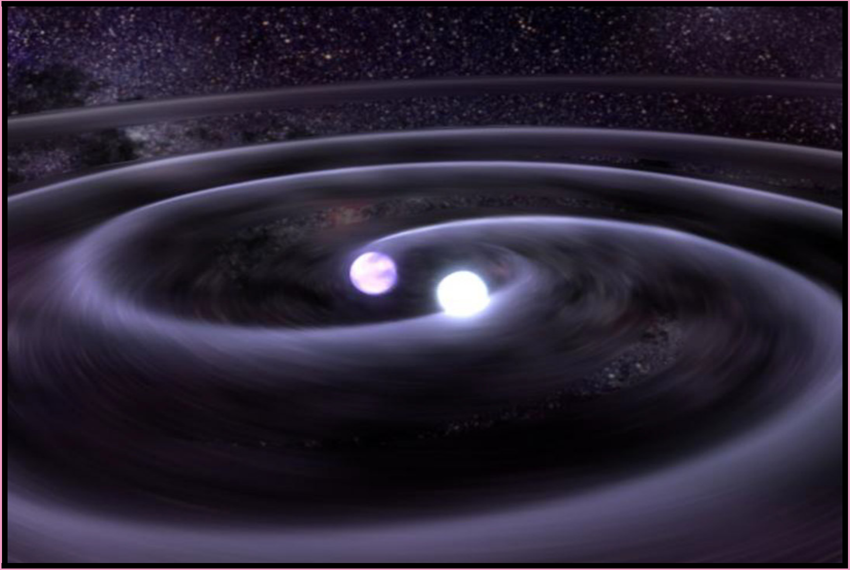
- One of the simpler solutions of this wave equation relates to the second derivative of the quadrupole moment.

$$h_{ij}(t,\vec{x}) = \frac{2G}{r}\frac{d^2\ddot{Q}_{ij}}{dt^2} \qquad Q_{ij} = Q_{ij}\left(t - \frac{r}{c}\right)$$

- Mass (Energy, momentum) curves spacetime, but does not create waves.
- Motion is relative, so a changing position can be “dealt with” using a change of coordinates.
- The quadrupole is related to the moment of inertia tensor.

Multipole	Gravity	Type
Monopole	Total mass	Scalar (0-Tensor)
Dipole	Position of Mass	Vector (1-Tensor)
Quadrupole	Shape of Mass	2-Tensor

Gravitational Waves are Dynamic Curvature



Gravitational Back Reaction and Hulse-Taylor

- Einstein published the quadrupole formula in 1916.
- Einstein published the *correct* quadrupole formula in 1918.

$$P = -\frac{G}{5}\langle \dddot{Q}_{\mu\nu}\dddot{Q}^{\mu\nu}\rangle$$

- Hulse and Taylor discover the binary pulsar, B1913+16, in 1974.
- Taylor and Weisberg publish the energy loss due to gravitational back reaction.

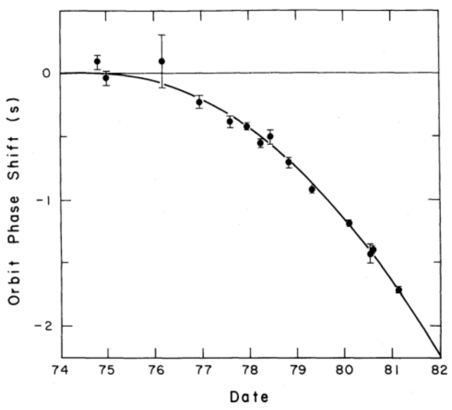
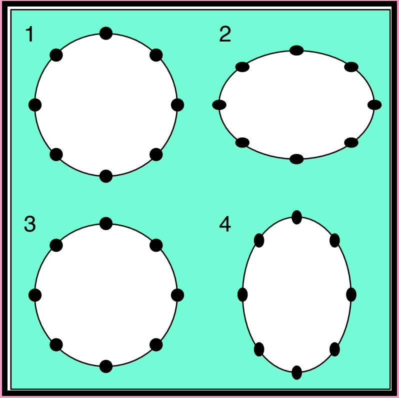


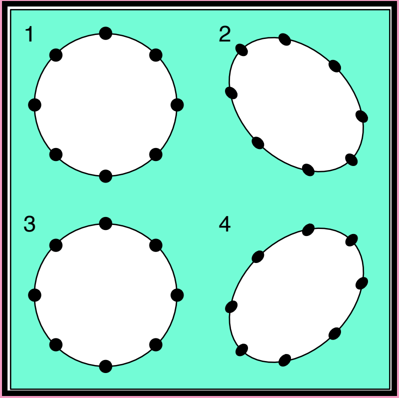
FIG. 6.—Orbital phase residuals, obtained from the data listed in Table 4. If the orbital period had remained constant, the points would be expected to lie on a straight line. The curvature of the parabola drawn through the points corresponds to the general relativistic prediction for loss of energy to gravitational radiation, or $\dot{P}_b = -2.40 \times 10^{-12}$.

Taylor and Weisberg, 1981

How do we detect Gravitational Waves?

- If you want to detect a physical phenomenon, you ask yourself "What does is do to a physical system?"
- Gravitational waves change the proper spacetime distance between points.





How Strong are Gravitational Waves?

$$h = \frac{\Delta L}{L}$$

- Couple Dancing
  $h \sim 2 \times 10^{-54}$

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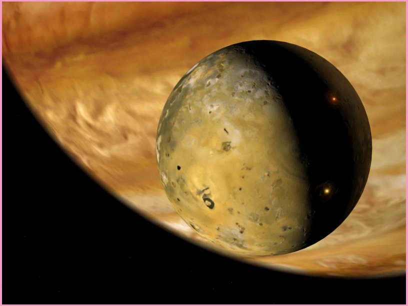

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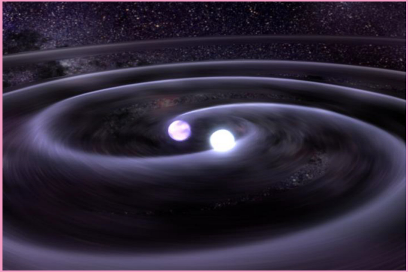
- NS Binary at Galactic Center $h \sim 2 \times 10^{-19}$



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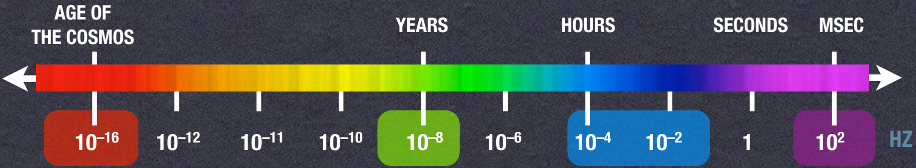
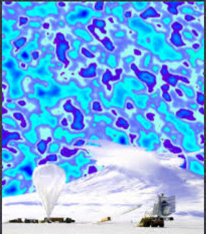
- NS Binary at Galactic Center $h \sim 2 \times 10^{-19}$
- SMBH Binary at Cosmological Distance $h \sim 2 \times 10^{-15}$

Gravitational Wave Spectrum

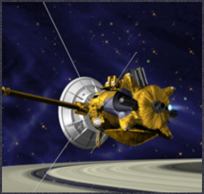


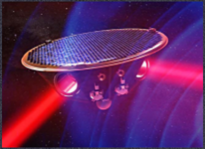


CMB Polarization

Pulsar Timing

Space

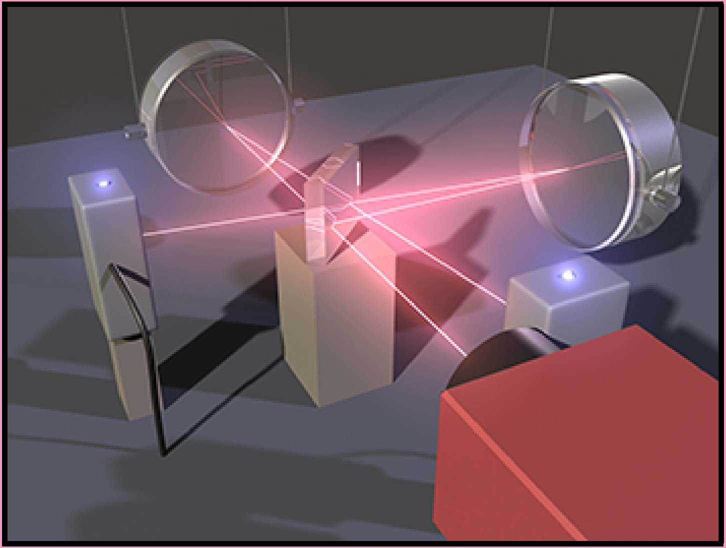




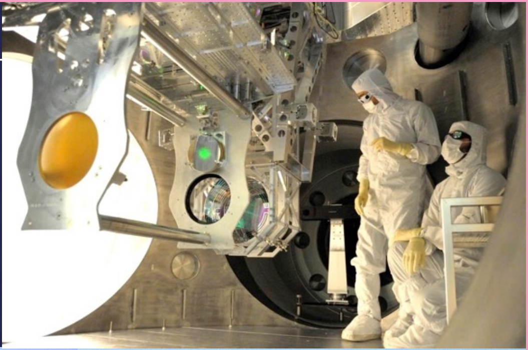
Ground

Gravitational Wave Spectrum

How to detect this motion? Interferometers!

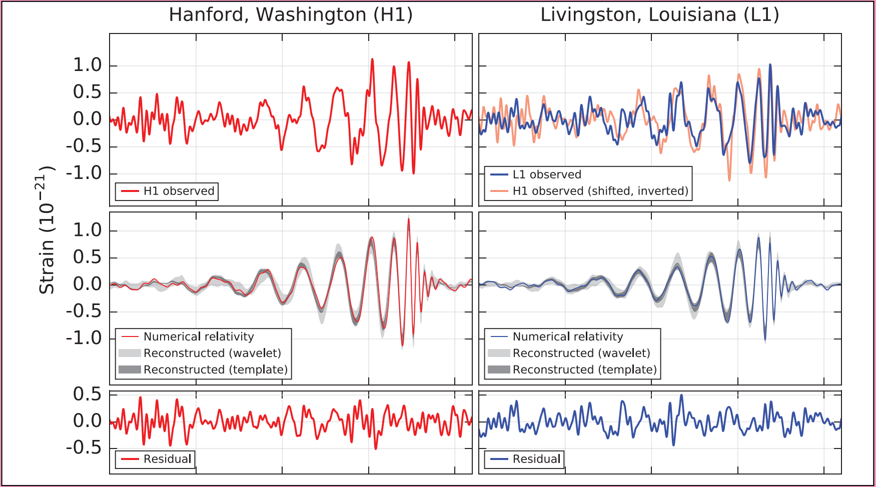


LIGO



LIGO Detected the signal from a Black Hole Binary

On September 14th, 2015 ($\sim 30M_{\odot}$), Phys. Rev. Lett. 116, 061102



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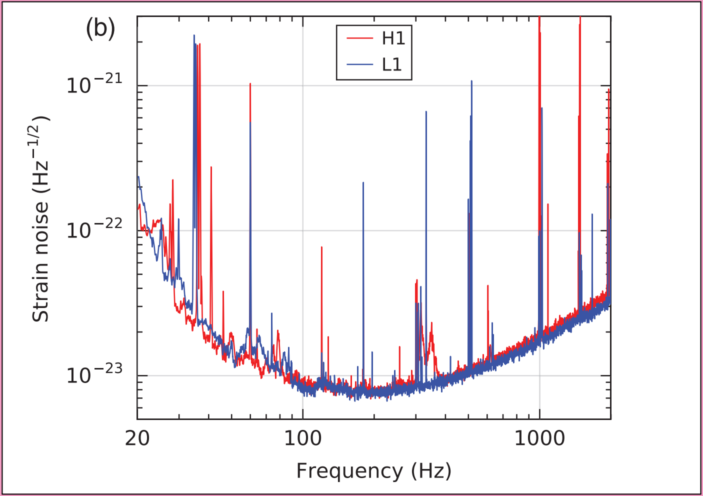
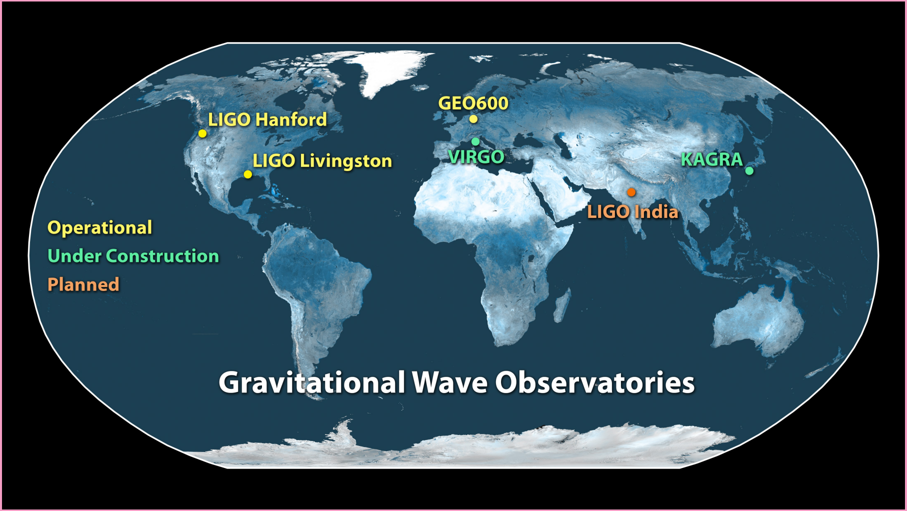
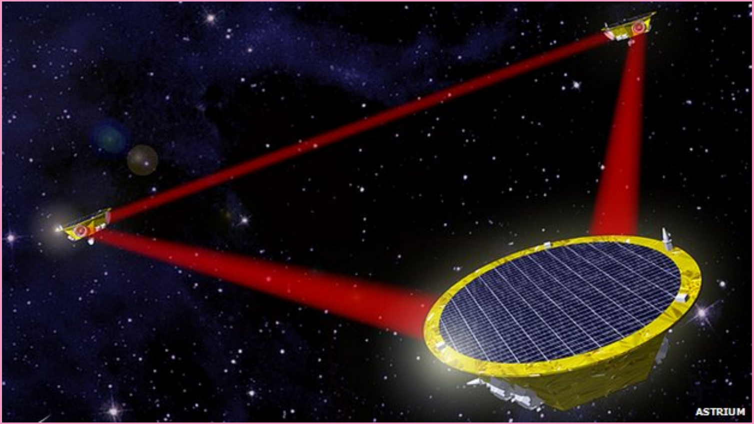


Globe Girdling Network of Gravitational Wave Detectors



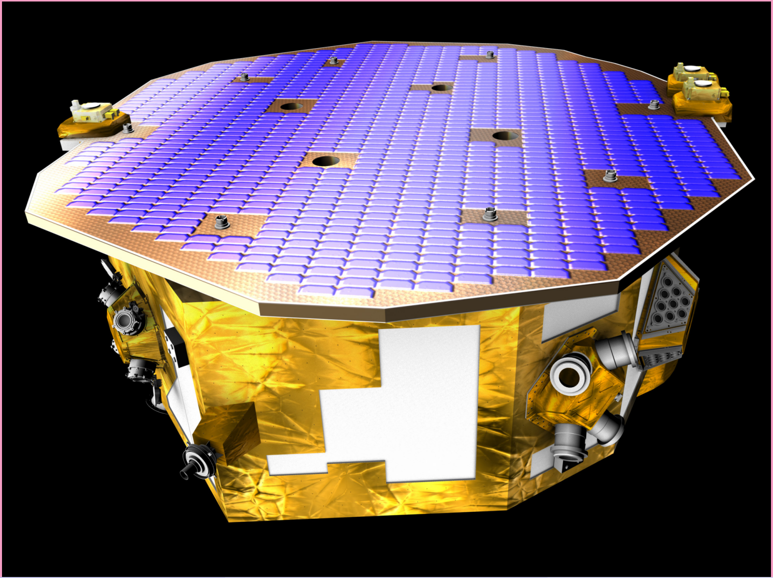
Credit: LIGO

LISA is the L3 ESA Mission



2.5 Million km Arms

LISA Pathfinder has been a huge success



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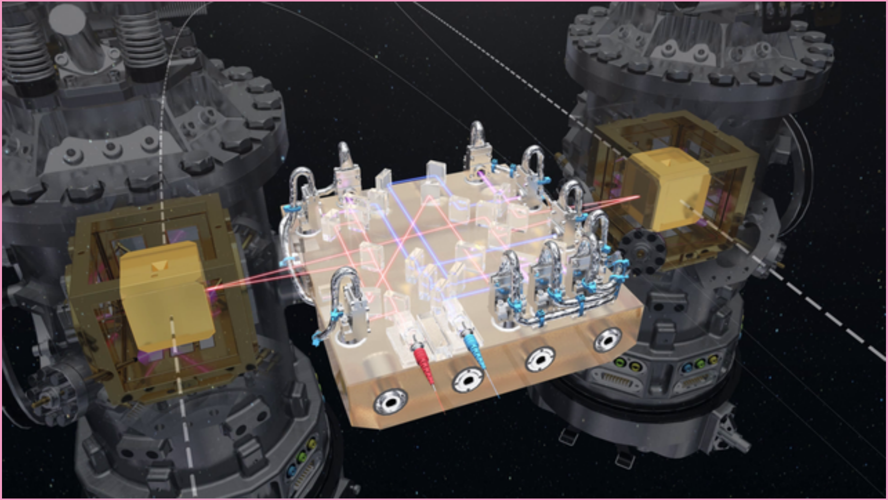

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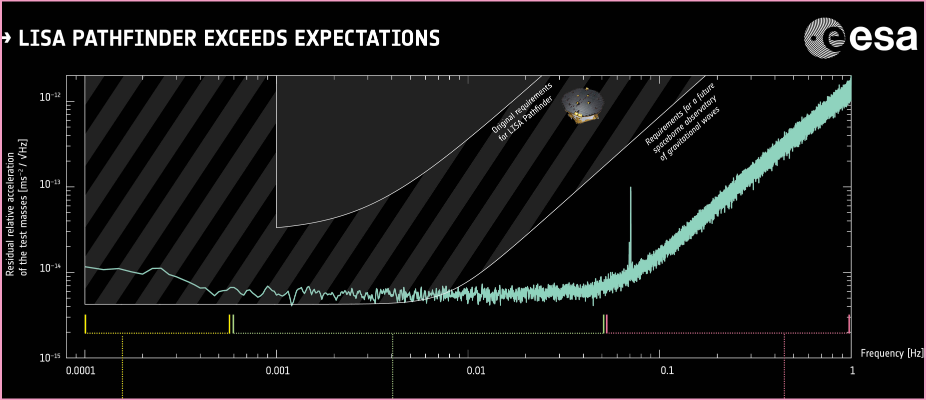
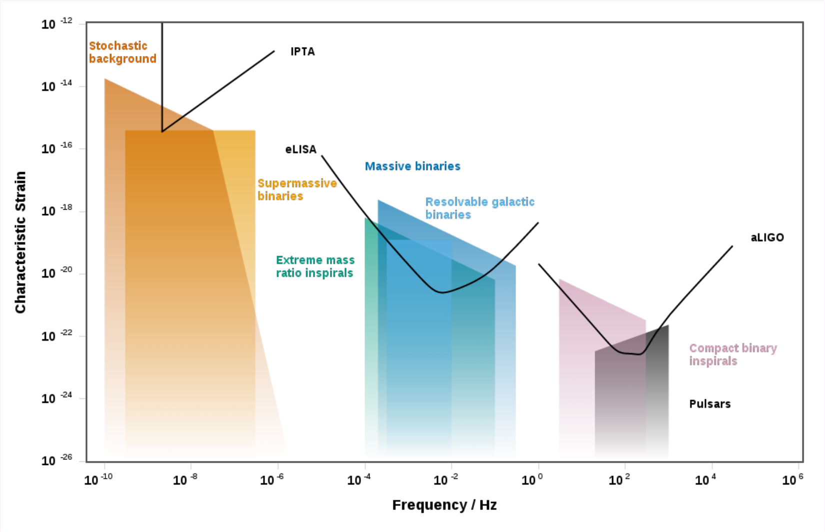


Characteristic Noise Strain



Christopher Berry

Continuous, Burst and Stochastic Signals

Continuous Wave Signal:

- Low to medium strength signal.
- Long lived.
- Few Fourier modes.

Burst Signal:

- Strong to medium strength signal.
- Short lived
- Many Fourier modes.

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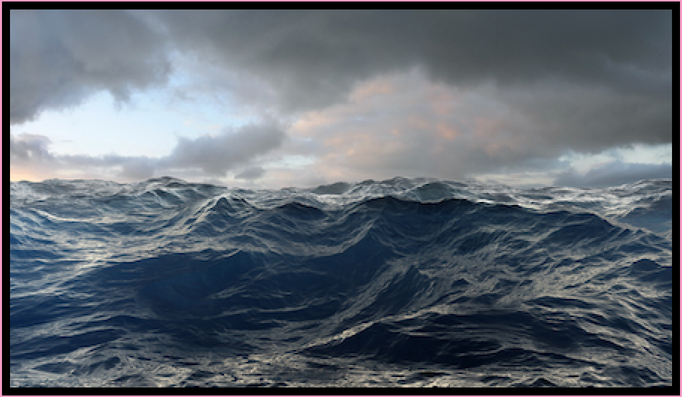
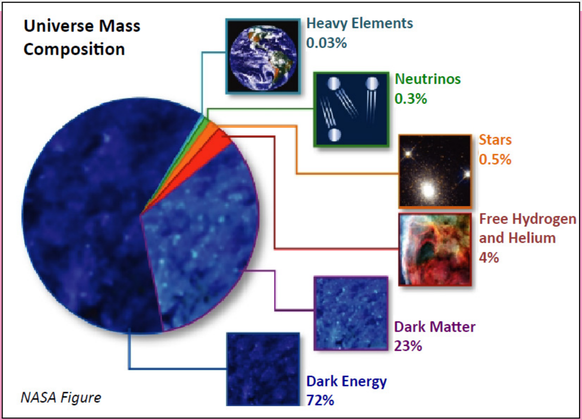
Continuous, Burst and Stochastic Signals

Stochastic Background:

- Individual sources weak, but sum detectable.
- Long lived.
- Many Fourier modes.
  Often following a power-law, or turnover power spectral model.
- Gerhard Mantz, *Rough Seas*



Why Extend General Relativity?



There are two dark clouds over General Relativity...
Dark Energy and Dark Matter.
-Jürgen Ehlers

Until it is solved, the problem of dark energy will be a roadblock on our path to a comprehensive fundamental physical theory.
-Steven Weinberg

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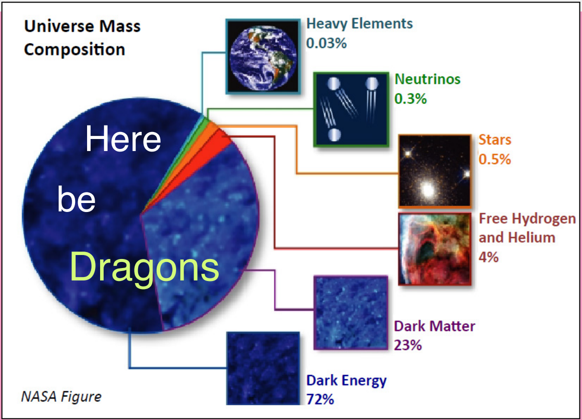

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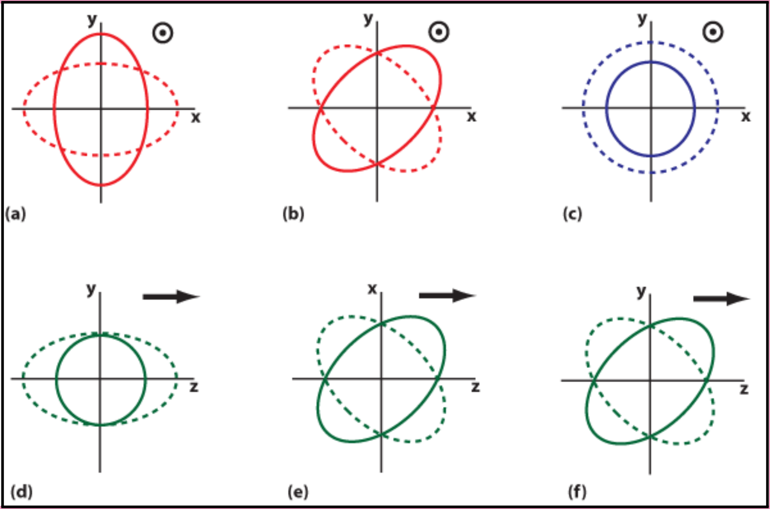
Changes to GR

- MOND
- Massive Graviton Theories (propagation tests)
- Changing the Einstein field equation.
- Bimetric theories
- ...

Other Polarizations

Metric Theories of Gravity



Will, 2006