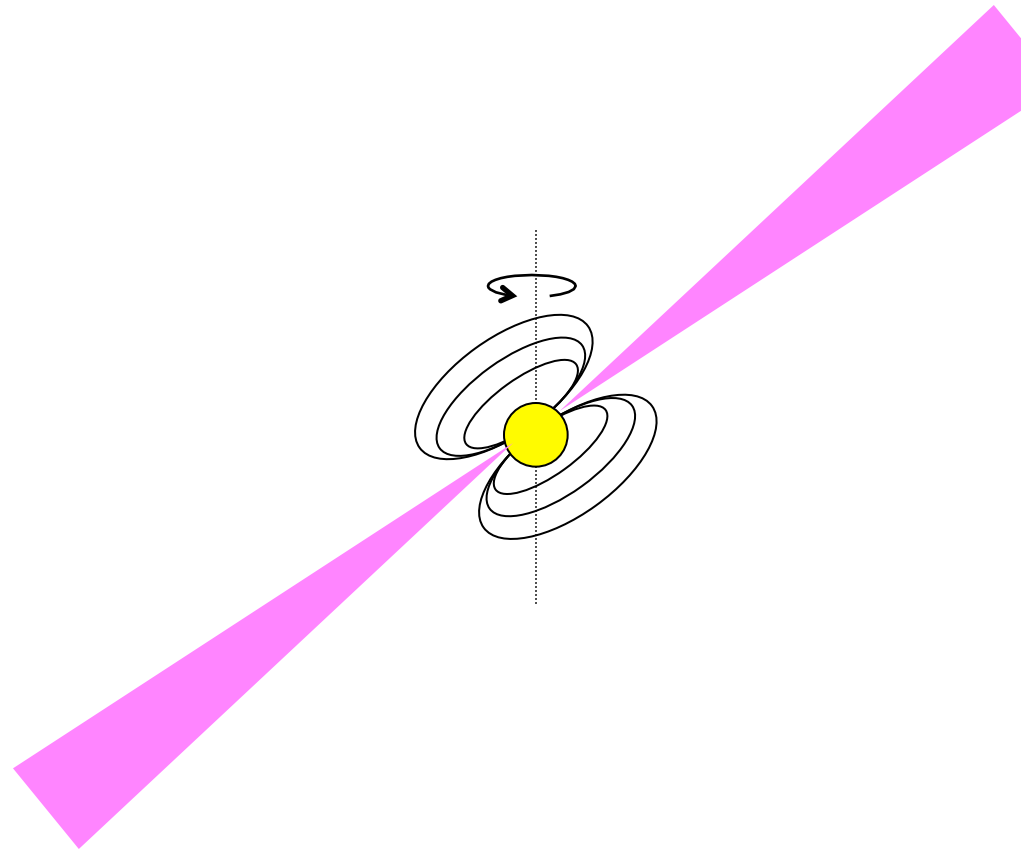


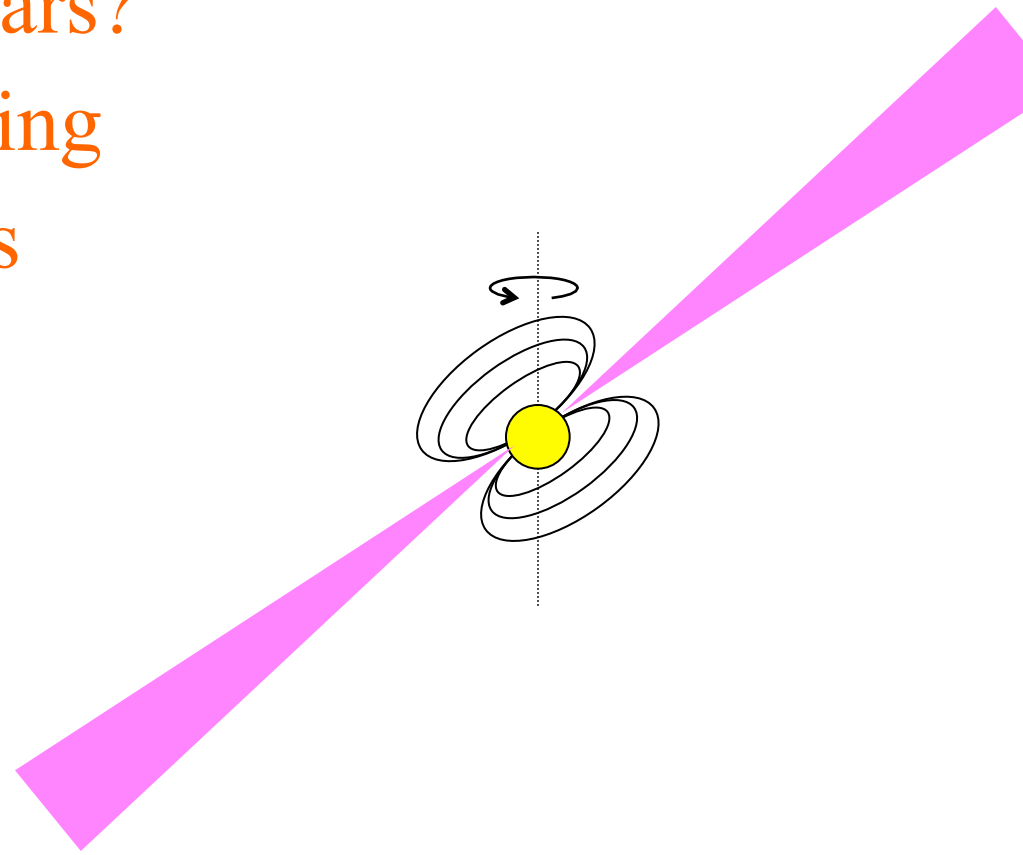
Introduction to Pulsar Timing

David Nice
Lafayette College

International Pulsar Timing Array Student Workshop
Banff, Alberta
16 June 2014



1. Extremely short overview
2. Measuring TOAs
3. What's in a timing model?
4. Uh oh, timing noise
5. Why millisecond pulsars?
6. Tempo; pulse numbering
7. Show me the residuals



1. Extremely short overview

2. Measuring TOAs

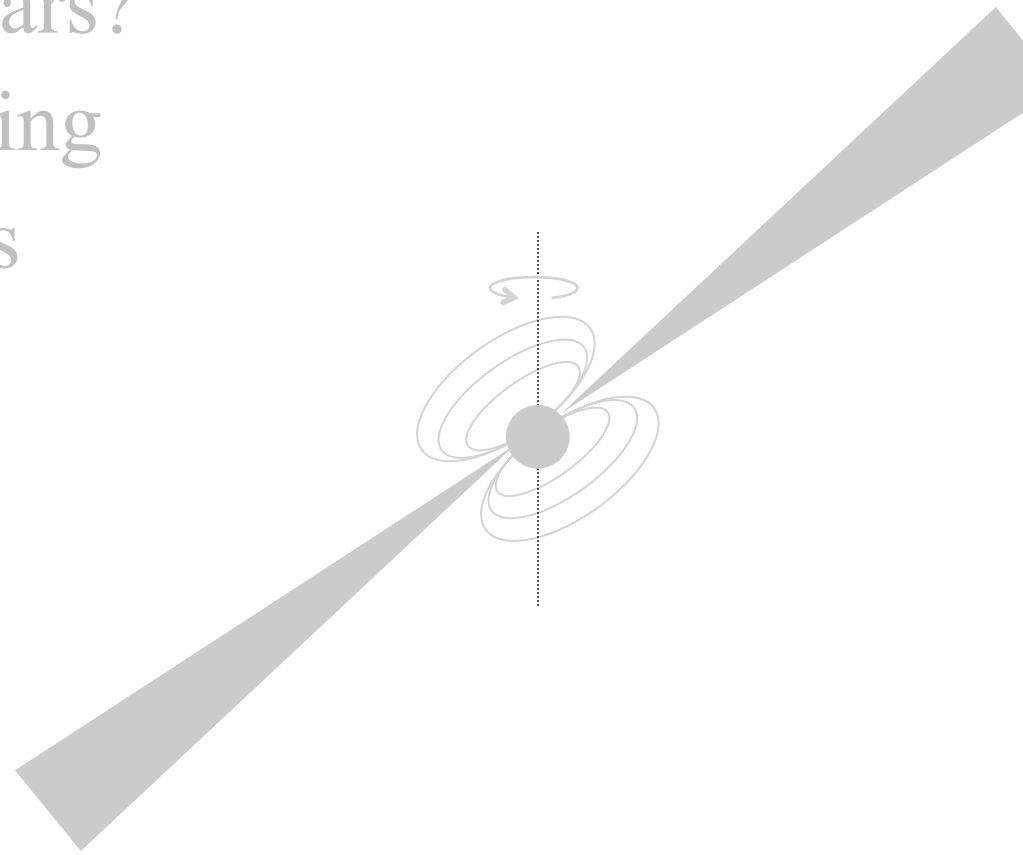
3. What's in a timing model?

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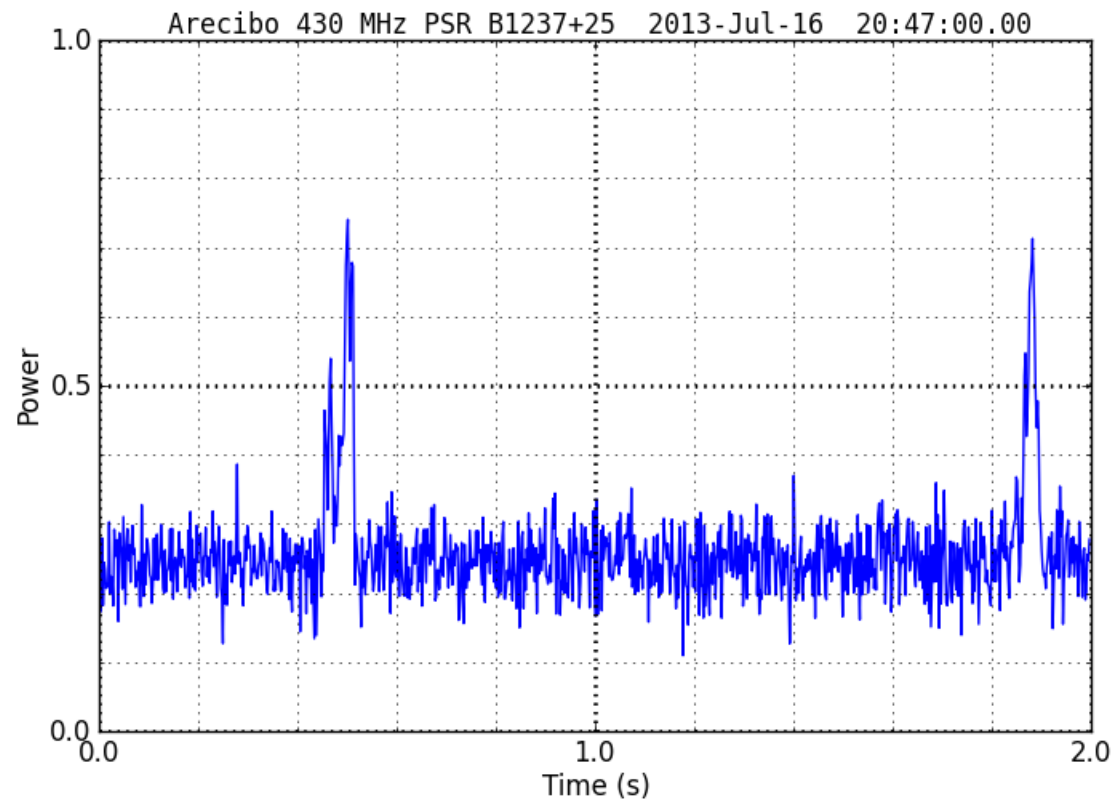
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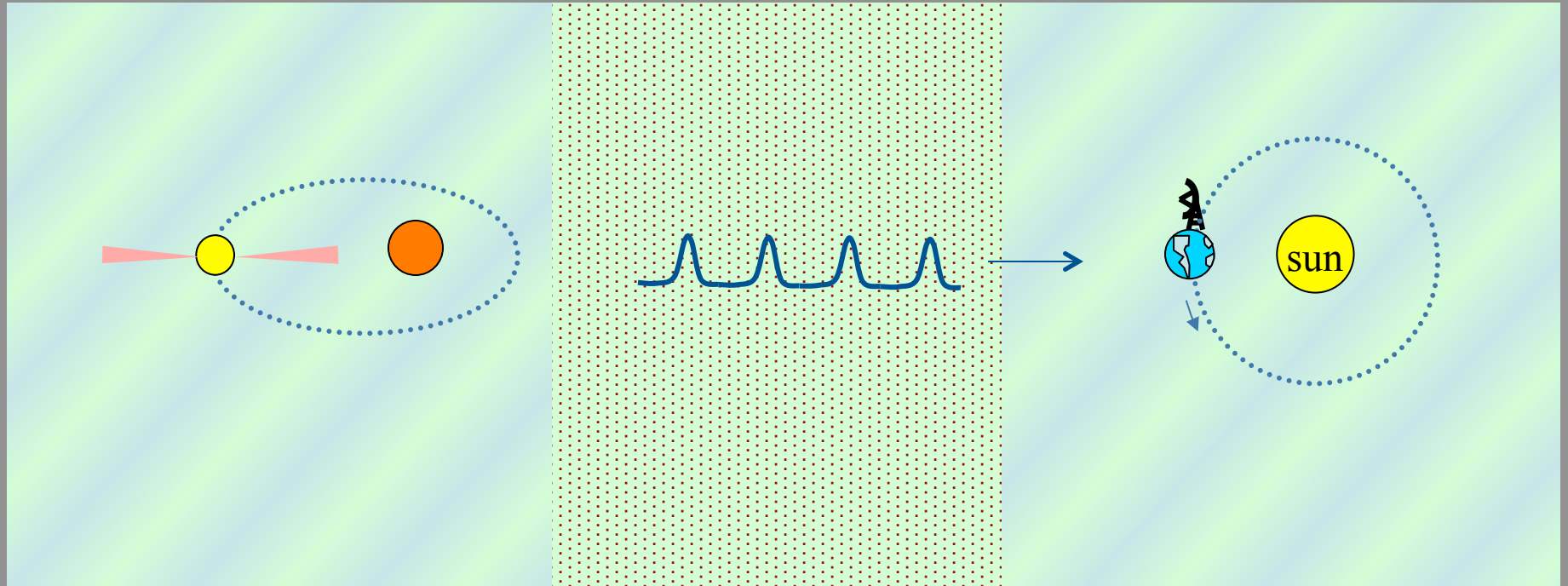
Movie Showing Measured Pulsar Time Series



David Nice/Lafayette College

[Click here to watch the movie.](#)

Observing The Pulsar Signal



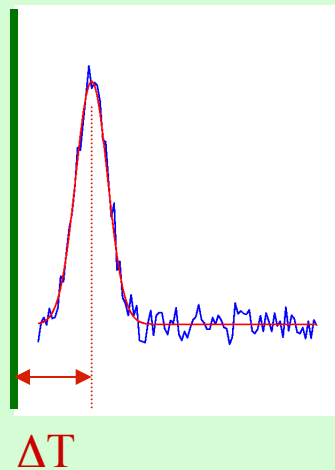
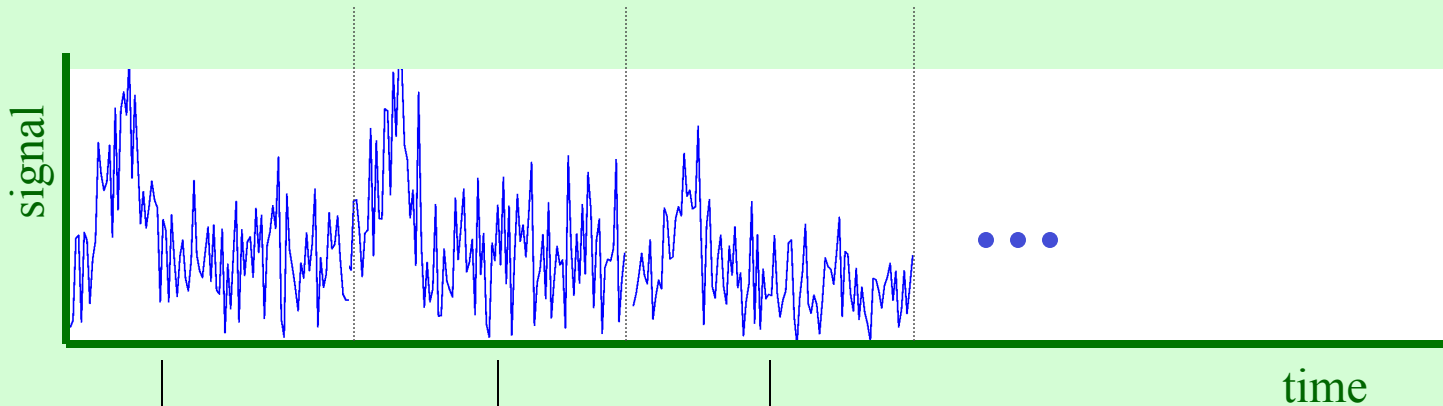
Basic idea:

1. A pulsar emits pulses. These pulses travel to our telescope, where we measure their times of arrival (**TOAs**).
2. The passage of a gravitational wave perturbs the TOAs. We hope to measure these perturbations and thereby detect gravitational waves.
3. Many other phenomena influence measured TOAs. **Pulse timing** is the process of measuring TOAs and disentangling the phenomena affect them.

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Measuring a Time of Arrival

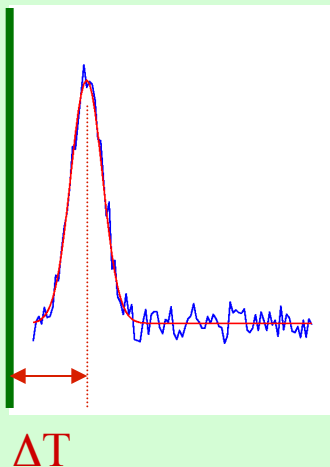


Most pulsar signals are very weak, and individual pulses cannot be detected. (The movie I showed earlier was deceptive.)

Fold the incoming signal at the pulse period to form a **pulse profile** (signal averaging). Then calculate a time of arrival from the **profile**:

Pulse Time of Arrival:
 $\text{TOA} = \text{scan start time} + \Delta T$

Precision of a Time of Arrival Measurement



Pulse Time of Arrival:

TOA = scan start time + ΔT

$$\sigma_{TOA} = \left(\frac{T_{sys}}{G} \right) \left(\frac{\eta}{S} \right) \frac{1}{\sqrt{\eta t B n_p}} \eta P$$

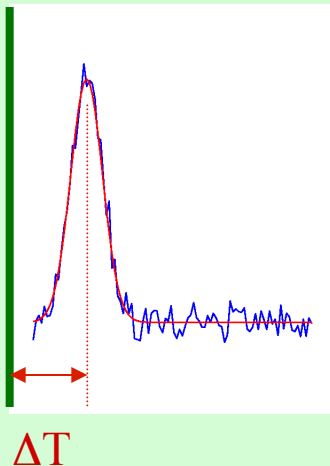
Typical* Values

P	= 0.004 s	Pulse period
η	= 0.05	Duty cycle
T_{sys}	= 20K	System temperature
G	= 2 K/Jy	Telescope gain
S	= 0.001 Jy	Pulsar flux density
t	= 1000 s	Observation time
B	= 10^8 Hz	Bandwidth
n_p	= 2	Number of polarizations

$$\rightarrow \sigma_{TOA} = 200 \text{ ns}$$

*Not really typical. This would be a good strong pulsar observed with 100m telescope.

Precision of a Time of Arrival Measurement



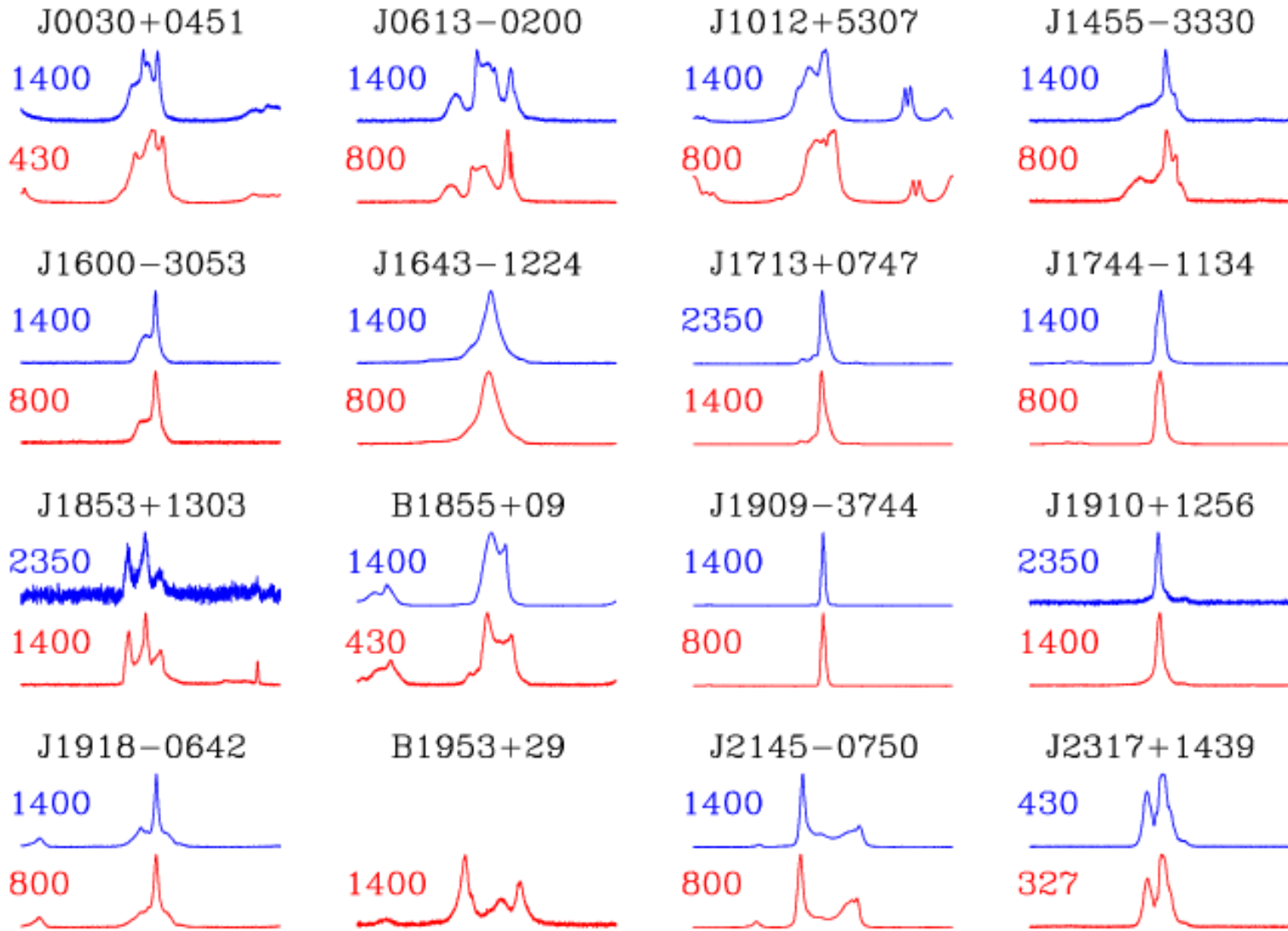
Pulse Time of Arrival:

$$\text{TOA} = \text{scan start time} + \Delta T$$

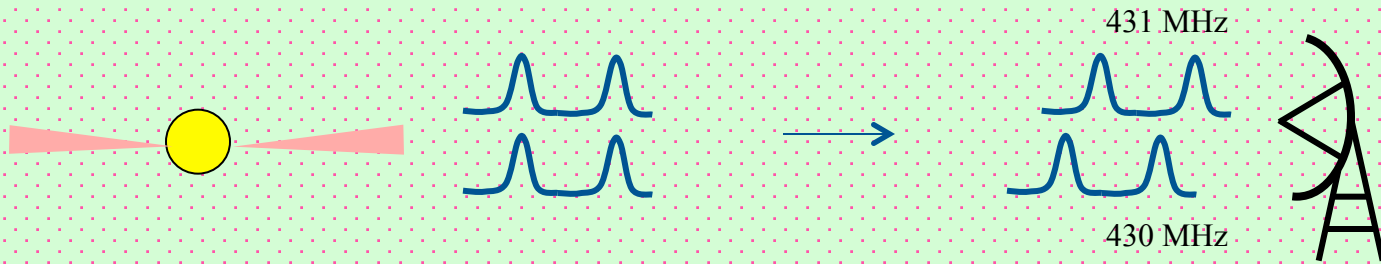
Timing Details

- Pulsars exhibit a wide variety of emission patterns (see next slide).
- The location of the “peak” in any given observation is measured by cross-correlating the data profile and a standard profile representative of the shape of the pulsar.
- Actually this measurement is often done in the Fourier domain. Let’s not discuss this now.
- A typical observation might have 100000’s of pulses. A TOA is a single “average arrival time” representing this entire ensemble of pulses. It is actually a highly precise measurement of when one of those pulses arrived at the telescope.
- Which “one of those pulses” do we choose? Usually we pick one in the middle of the observation. Thus the TOA is really equal to “scan start time + ΔT ” plus an integer number of periods equal to about half the duration of the observation.

Some Millisecond Pulsar Profiles



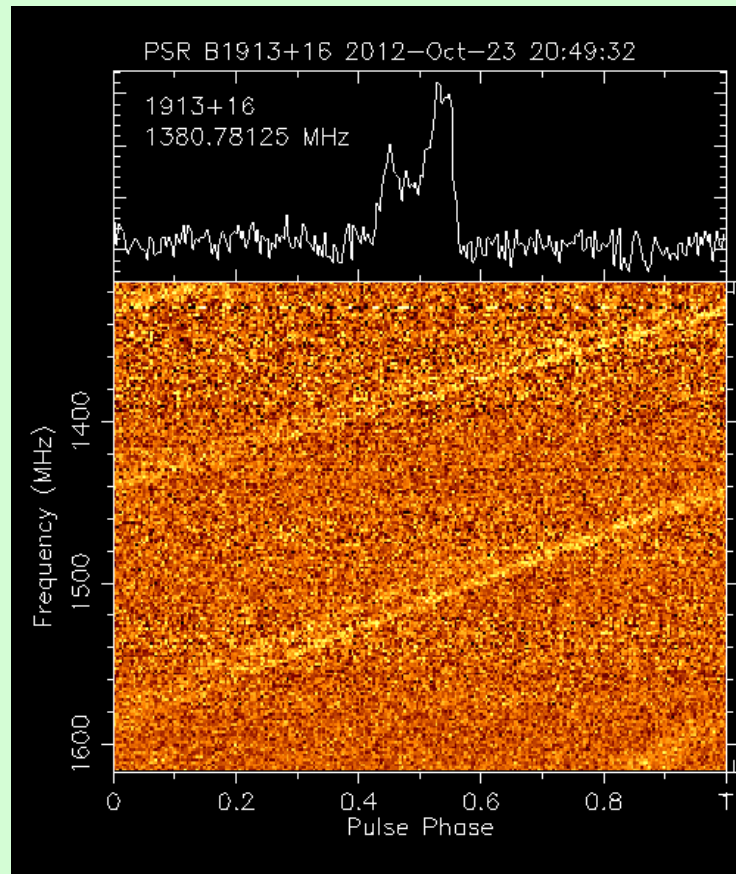
Interstellar Dispersion



column density of electrons: $DM = \int n_e(l) dl$ units of pc cm^{-3}

excess propagation time: $t \text{ (sec)} = DM / 2.41 \times 10^{-4} [f(\text{MHz})]^2$

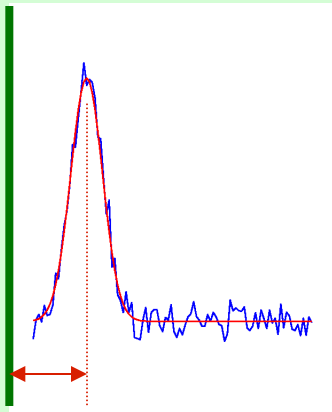
Interstellar Dispersion



Dealing with dispersion

- Use a filterbank or autocorrelator to split the radio spectrum band into narrow channels. Independently detect and fold the signal in each channel. Shift the channels relative to one another, then sum them to get a single dedispersed profile. Calculate a TOA from this profile.
- Instead of adding the channels together, you could calculate a separate TOA for each channel.
- It may appear desirable to make the channels as small as possible, but this can limit the precision of the TOA measurement (“sampling theorem”). A clever way around this limit is coherent dedispersion: the incoming radio signal is Fourier transformed; phases of the Fourier-domain sinusoids are shifted to remove dispersion; and the de-dispersed signal is inverse-transformed back to the time domain, where it is folded in the usual way. This is necessary for the highest timing precision, but it is computationally intense.

What does a Time of Arrival Look Like?



ΔT

Example:

PSR J1518+4904 was observed using the Green Bank 140 Foot telescope at 327 MHz on May 9, 1998. A pulse was measured at time 00:34:07.664279 UT

May 9, 1998 = MJD 50942

00:34:07.664279 = 0.02369981804596 (fraction of day)

TOA = 50942.02369981804596

Observatory	Radio Frequency	Pulse Time of Arrival	Measurement Uncertainty	
a	3751 1518+49	370.000	50942.02369981804596	69.1 9-May-98

Pulse Times of Arrival

Observatory	Radio Frequency	Pulse Time of Arrival	Measurement Uncertainty	
a 3751 1518+49	370.000	50942.02369981804596	69.1	9-May-98
a 3751 1518+49	370.000	50942.02508871578912	74.9	9-May-98
a 3752 1518+49	370.000	50942.02710263928441	107.8	9-May-98
a 3752 1518+49	370.000	50942.02849153928888	68.4	9-May-98
a 3753 1518+49	370.000	50942.03050309034722	63.0	9-May-98
a 3753 1518+49	370.000	50942.03189199466585	71.4	9-May-98
a 3754 1518+49	370.000	50942.03389643284537	64.2	9-May-98
a 3754 1518+49	370.000	50942.03528532340819	57.4	9-May-98
a 3755 1518+49	370.000	50942.03728740139970	74.4	9-May-98
a 3755 1518+49	370.000	50942.03867629785610	65.1	9-May-98
a 3756 1518+49	370.000	50942.04067884384616	54.2	9-May-98
a 3756 1518+49	370.000	50942.04206774860490	87.3	9-May-98
a 3757 1518+49	370.000	50942.04406981298474	88.9	9-May-98
a 3757 1518+49	370.000	50942.04545870833792	71.8	9-May-98
a 3758 1518+49	370.000	50942.04748447411745	110.3	9-May-98
a 3758 1518+49	370.000	50942.04887336536594	78.6	9-May-98
a 3759 1518+49	370.000	50942.05089865820880	60.2	9-May-98
a 3759 1518+49	370.000	50942.05228755033977	131.1	9-May-98
a 3760 1518+49	370.000	50942.05428961858992	63.4	9-May-98
a 3760 1518+49	370.000	50942.05567851214494	93.2	9-May-98
a 3761 1518+49	370.000	50942.05768105475176	116.2	9-May-98
a 3761 1518+49	370.000	50942.05906994776154	75.0	9-May-98
a 3762 1518+49	370.000	50942.06108244410689	72.2	9-May-98
a 3762 1518+49	370.000	50942.06247133259781	76.9	9-May-98
a 3763 1518+49	370.000	50942.06450988581265	86.1	9-May-98
a 3763 1518+49	370.000	50942.06589877480622	61.9	9-May-98
a 3764 1518+49	370.000	50942.06790794988299	90.1	9-May-98
a 3764 1518+49	370.000	50942.06929683956486	67.2	9-May-98
a 3765 1518+49	370.000	50942.07129227137214	63.5	9-May-98
a 3765 1518+49	370.000	50942.07268116130441	139.5	9-May-98

Pulse Times of Arrival

Observatory	Radio Frequency	Pulse Time of Arrival	Measurement Uncertainty	
a 3751 1518+49	370.000	50942.02369981804596	69.1	9-May-98
a 3751 1518+49	370.000	50942.02508871578912	74.9	9-May-98
a 3752 1518+49	370.000	50942.02710263928441	107.8	9-May-98
a 3752 1518+49	370.000	50942.02849153928888	68.4	9-May-98

Many modern TOA files contain more details about the observation, as shown in the example below. The most critical parts are still the observatory, radio frequency, time of arrival, and measurement uncertainty.

```
puppi_56599_J2317+1439_0107.8y.x.ff 1153.437 56599.031436860011304 0.932 ao
-fe L-wide -be PUPPI -f L-wide_PUPPI -bw 12.5 -tobs 1200.8 -tmpl J2317+1439.L-
wide.PUPPI.8y.x.sum.sm -gof 0.999 -nbin 2048 -nch 8 -chan 50 -subint 0 -snr 100.96 -
wt 5629 -proc 8y -pta NANOGrav
```

a 3764 1518+49	370.000	50942.06929683956486	67.2	9-May-98
a 3765 1518+49	370.000	50942.07129227137214	63.5	9-May-98
a 3765 1518+49	370.000	50942.07268116130441	139.5	9-May-98

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Pulse Timing

Pulsar rotation is extremely stable.

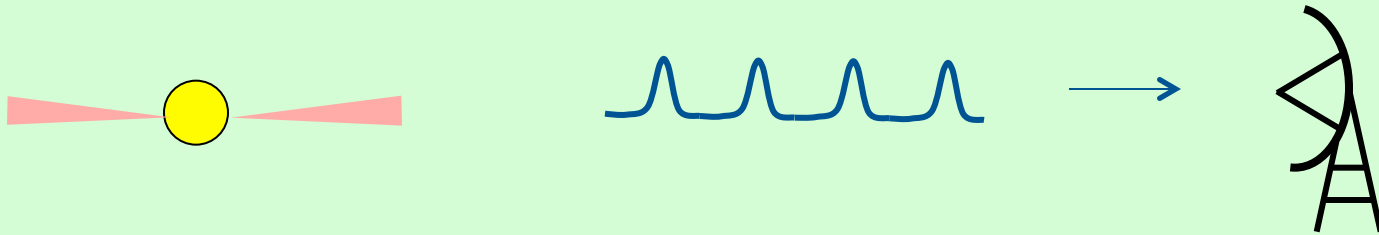
By observing a pulsar at suitable intervals,
it is possible to account for every rotation
of the pulsar.

Example: PSR J1713+0747

First Observed Pulse:	15 Aug. 1992	00:35:00.500421805 \pm 160ns
Last Observed Pulse:	23 Nov. 2013	22:07:41.189191195 \pm 140ns

Pulsar underwent exactly 146,495,457,989 rotations over this time.

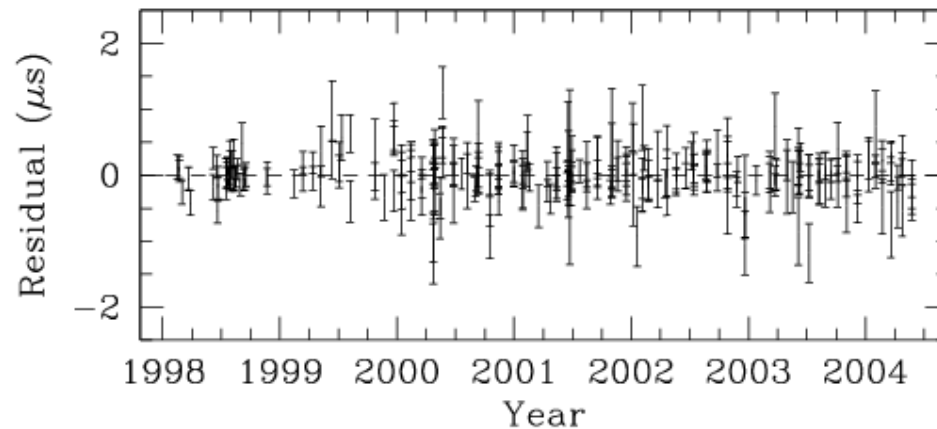
Residuals



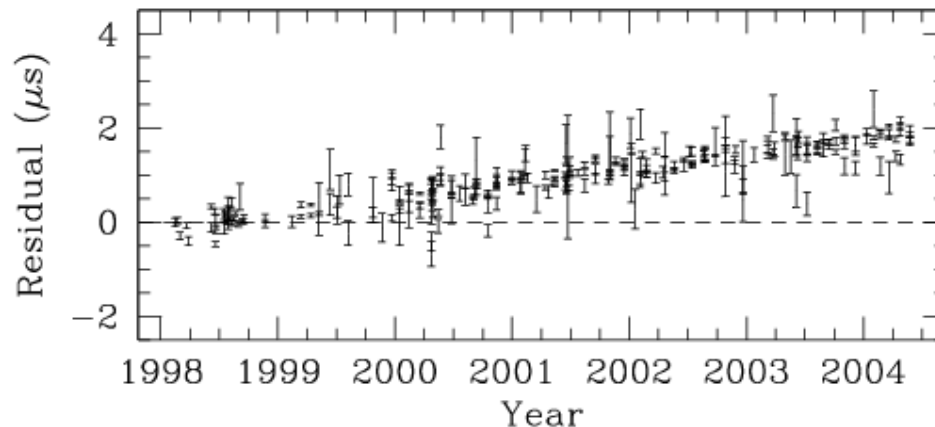
“Residuals” are differences between measured pulses times of arrival and expected times of arrival:

$$\text{residual} = \text{observed TOA} - \text{computed TOA}$$

We hope to detect gravitational wave signals as perturbations of these residuals.



PSR J1713+0747
Splaver et al. 2005
ApJ 620: 405
astro-ph/0410488



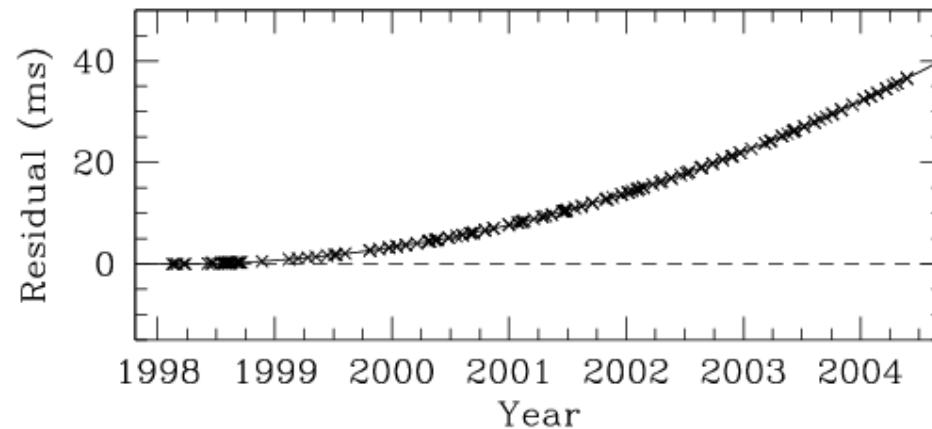
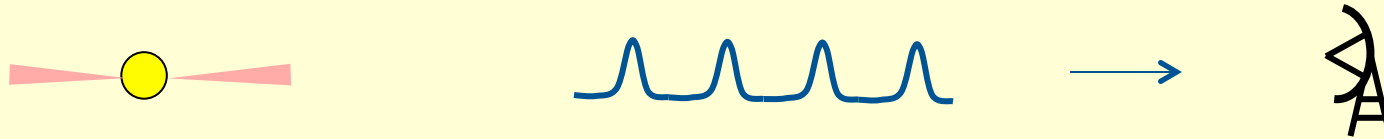
A very-long-period gravitational wave might continuously increase the proper distance traveled by a pulses over a long-term observation program on a pulsar. The plot shows the effect of an increase in travel time of $2 \mu\text{s}$ over 6 years, a distance of only 600 m (compared to pulsar distance of 1.1 kpc, for $\Delta l/l=6 \times 10^{-16}$).

Exactly the same thing would arise if there were no gravitational wave, but the pulse period were slightly longer, 4.57013652508278 ms instead of 4.57013652508274 ms (1 part in 10^{14}).

The period is known only from timing data

⇒ always need to fit out a linear term in timing measurements to find pulse period

⇒ a perturbation due to gravitational waves which is linear in time cannot be detected



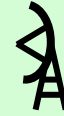
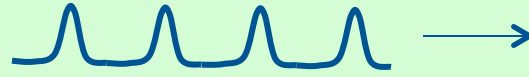
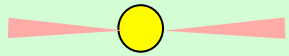
Pulsar rotation *slows down* over time due to magnetic dipole rotation. The ‘spin-down’ rate is not known *a priori*.

The plot shows what the residuals of J1713+0747 look like if we forget to include spin-down. As the pulsar slows down, pulses are delayed by an amount *quadratic* in time.

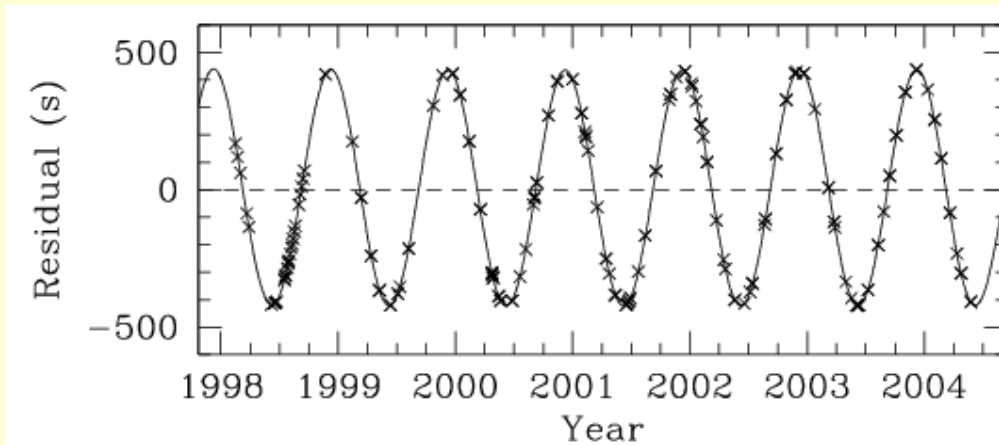
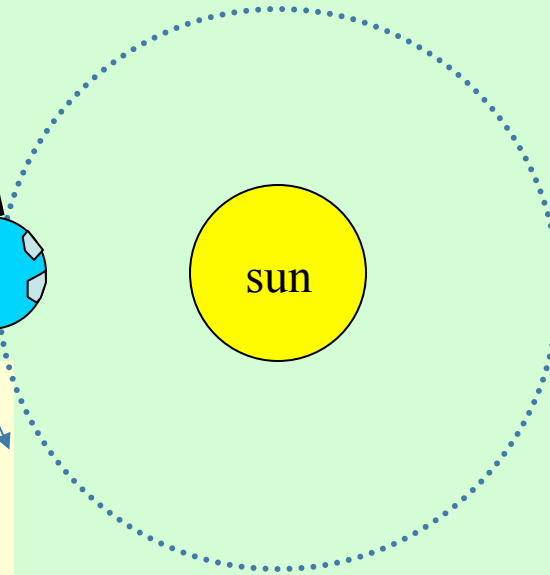
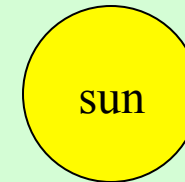
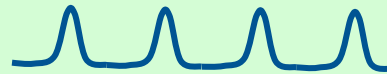
The spin-down rate is known only from timing data

⇒ always need to fit out a quadratic term in timing measurements to find pulse period

⇒ a perturbation due to gravitational waves which is quadratic in time cannot be detected



rotation period
rotation period derivative



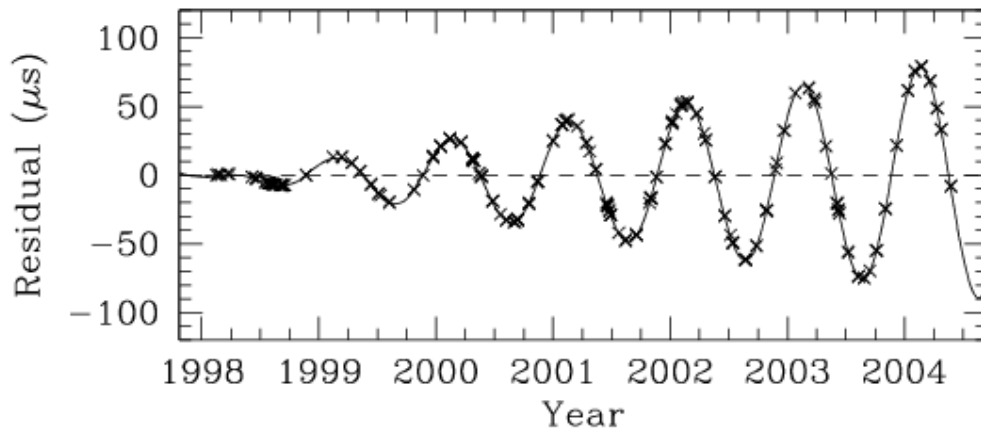
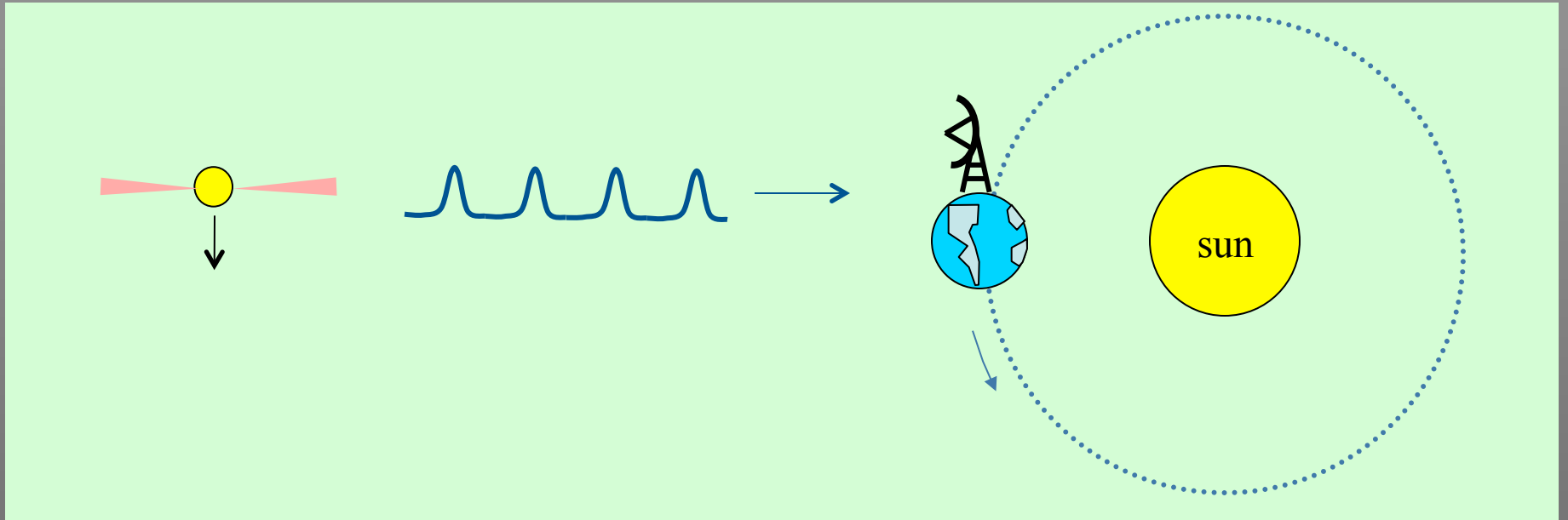
Delays of ~ 500 s due to time-of-flight across the Earth's orbit.

The amplitude and phase of this delay depend on the pulsar position.

Position known only from timing data

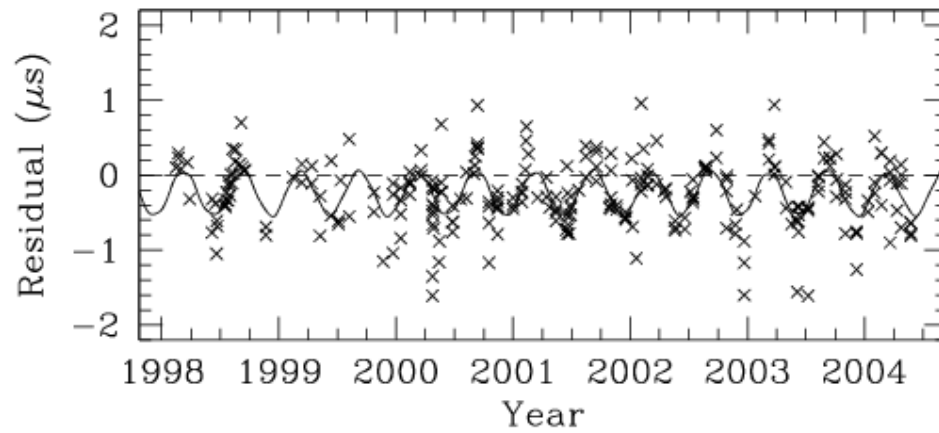
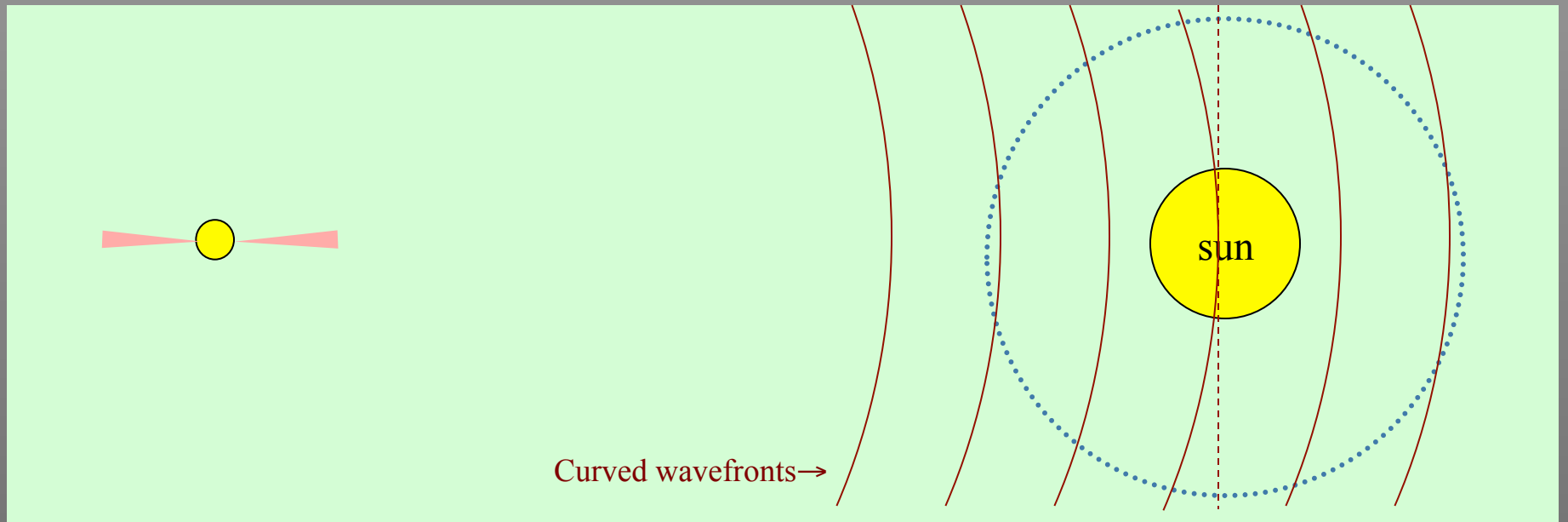
\Rightarrow always need to fit annual terms out of timing solution

\Rightarrow a perturbation due to gravitational waves with ~ 1 yr period cannot be detected



Other astrometric phenomena:

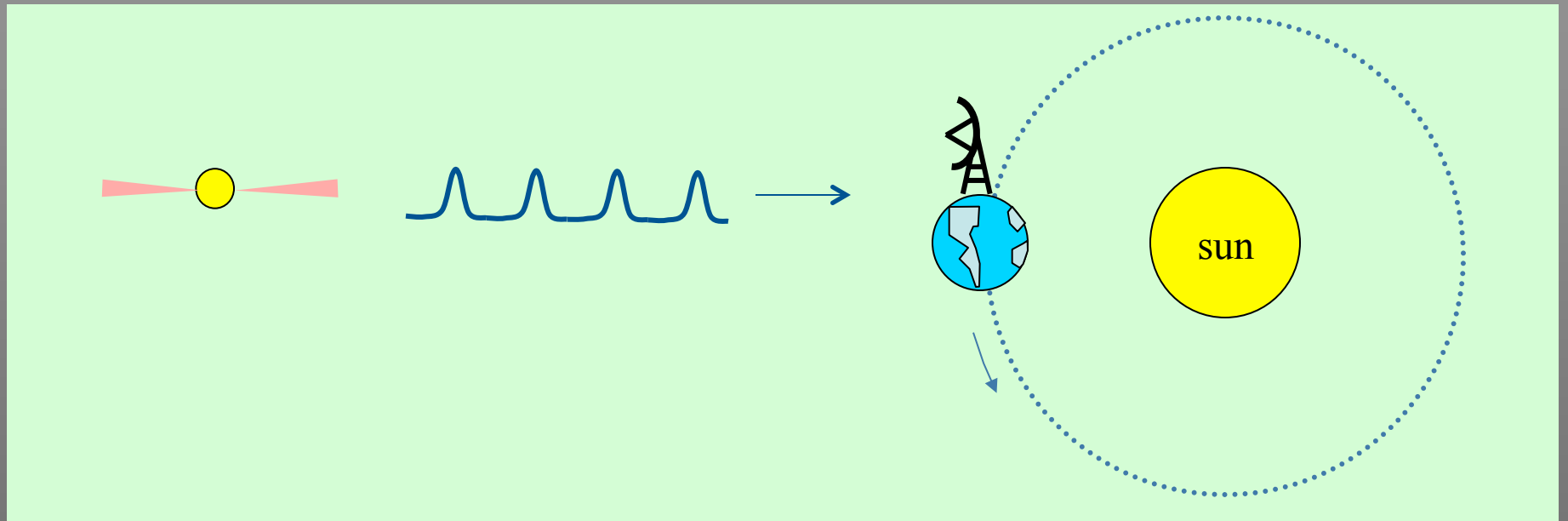
Proper Motion



Other astrometric phenomena:

Proper Motion

Parallax



Measurement of a pulse time of arrival at the observatory is a relativistic event. It must be transformed to an inertial frame: that of the solar system barycenter (center of mass).

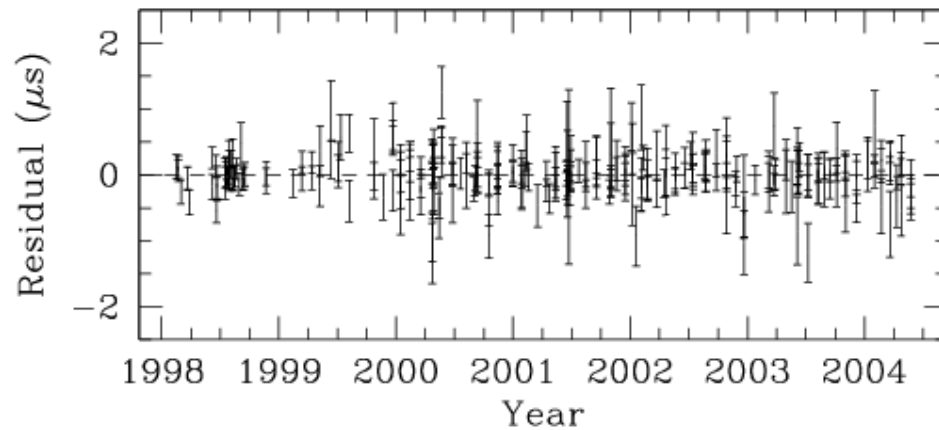
Time transfer:

Observatory clock \rightarrow GPS \rightarrow UT \rightarrow TDB

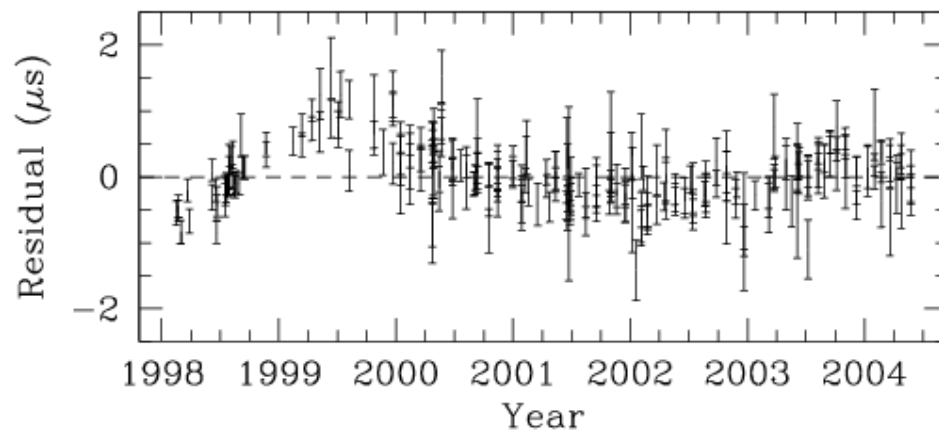
Position transfer:

For Earth and Sun positions, use a solar system ephemeris, e.g., JPL DE405, DE421, etc

For earth orientation (UT1, etc.), use IERS bulletin B

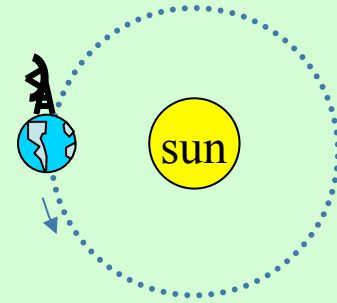
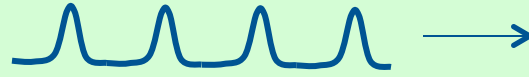
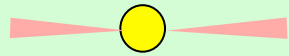


PSR J1713+0747 analyzed using DE 405 solar system ephemeris



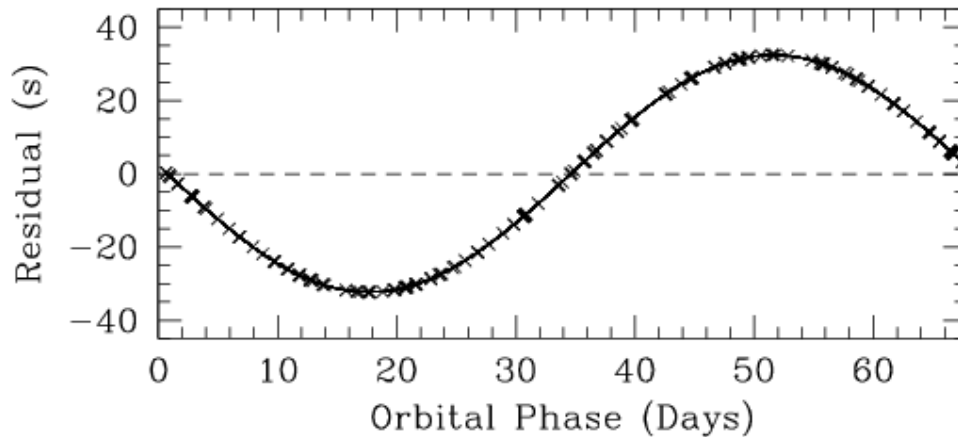
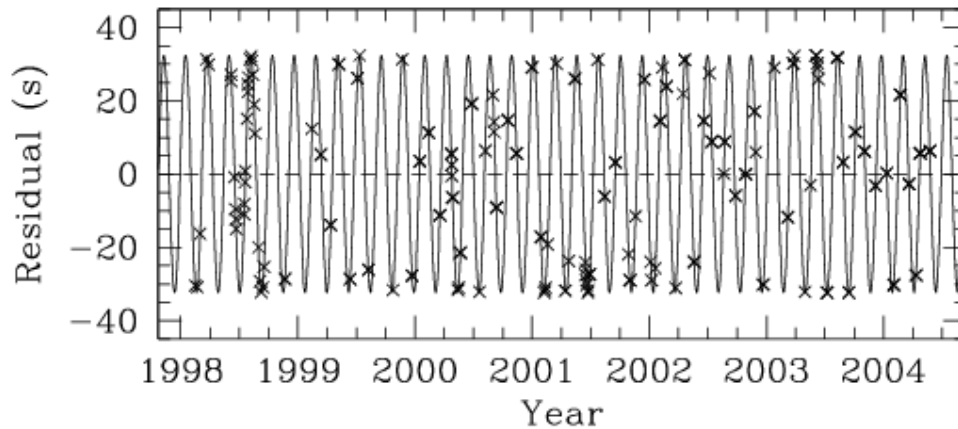
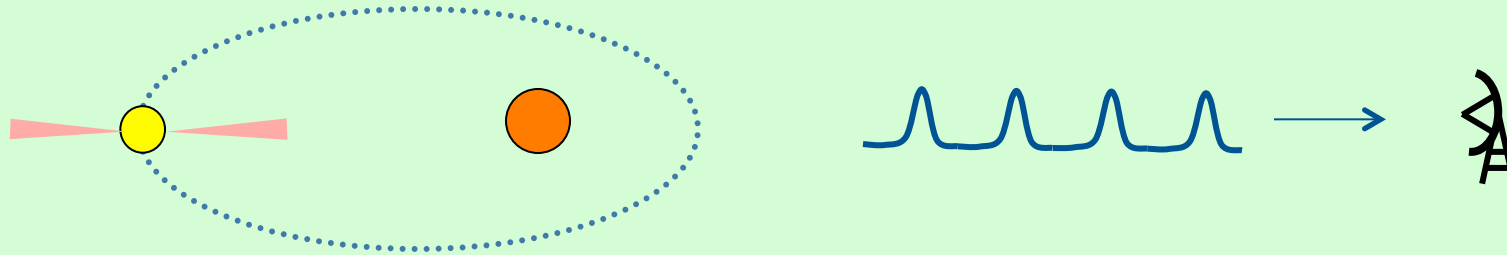
PSR J1713+0747 analyzed using previous-generation DE 200 solar system ephemeris.

$\sim 1 \mu\text{s}$ timing errors
 \Leftrightarrow 300 m errors in Earth position.



rotation period
rotation period derivative

position
proper motion
parallax



Keplerian orbital elements:

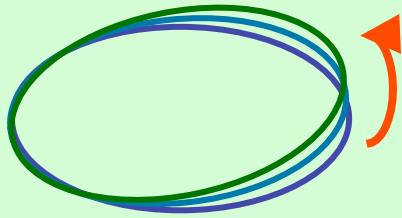
Orbital period

Projected semi-major axis

Eccentricity

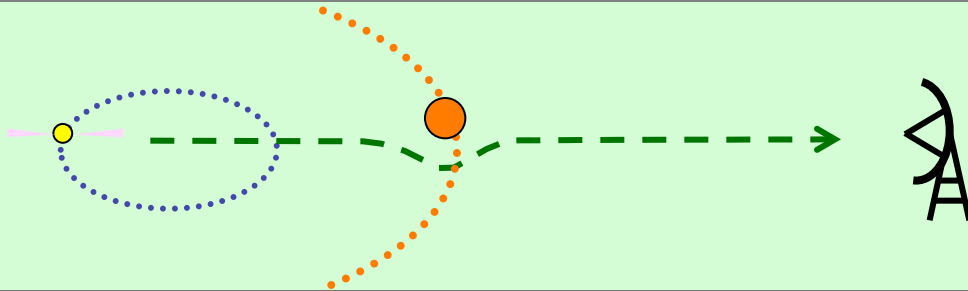
Angle of periastron

Time of periastron passage



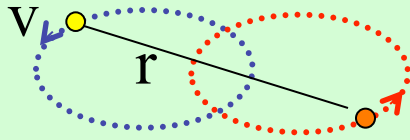
Precession

$$\dot{\omega} = 3 \left(\frac{P_b}{2\pi} \right)^{-5/3} \frac{1}{1-e^2} \left[\frac{G}{c^3} (m_1 + m_2) \right]^{2/3}$$



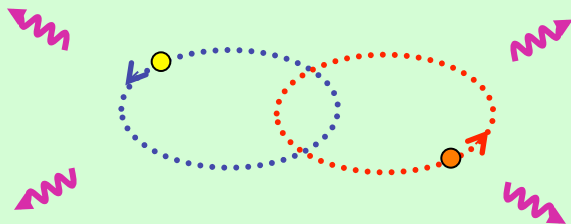
Shapiro Delay

$$\Delta t = 2 \frac{G}{c^3} m_2 \ln [1 - \sin i \sin(\varphi - \varphi_0)]$$



Grav Redshift/Time Dilation

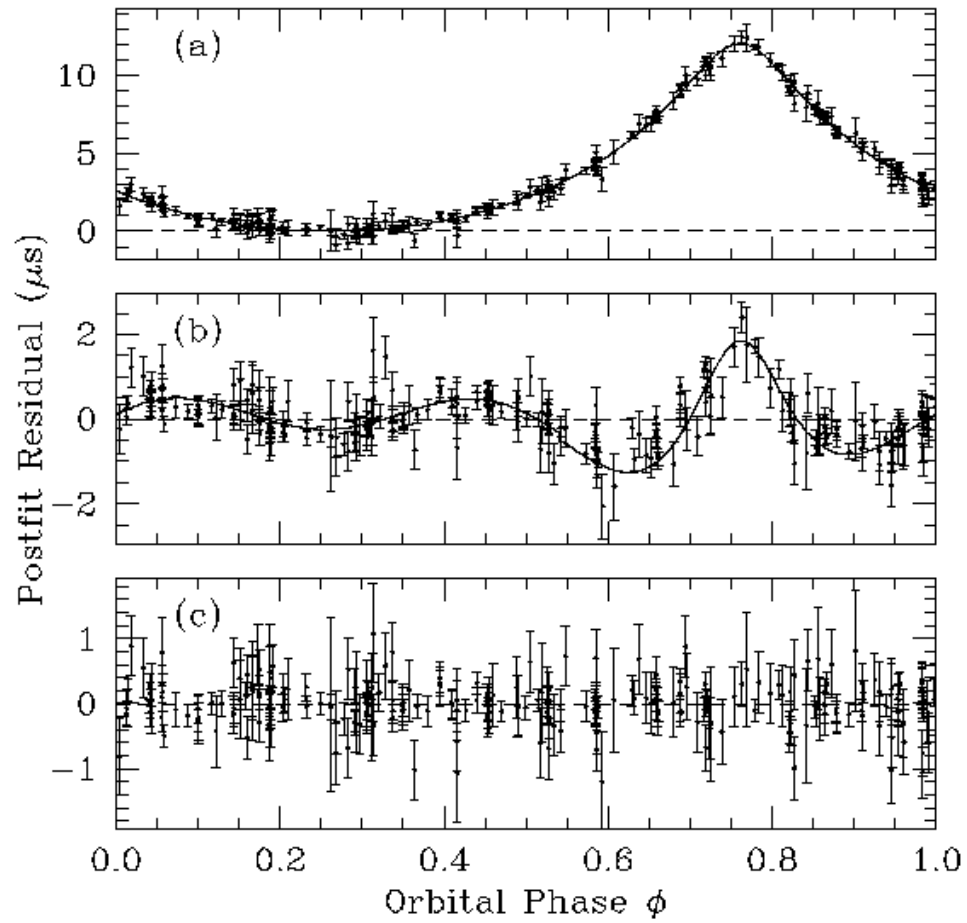
$$\gamma = \frac{G^{2/3}}{c^2} \left(\frac{P_b}{2\pi} \right)^{1/3} e \frac{m_2(m_1 + 2m_2)}{(m_1 + m_2)^{4/3}}$$

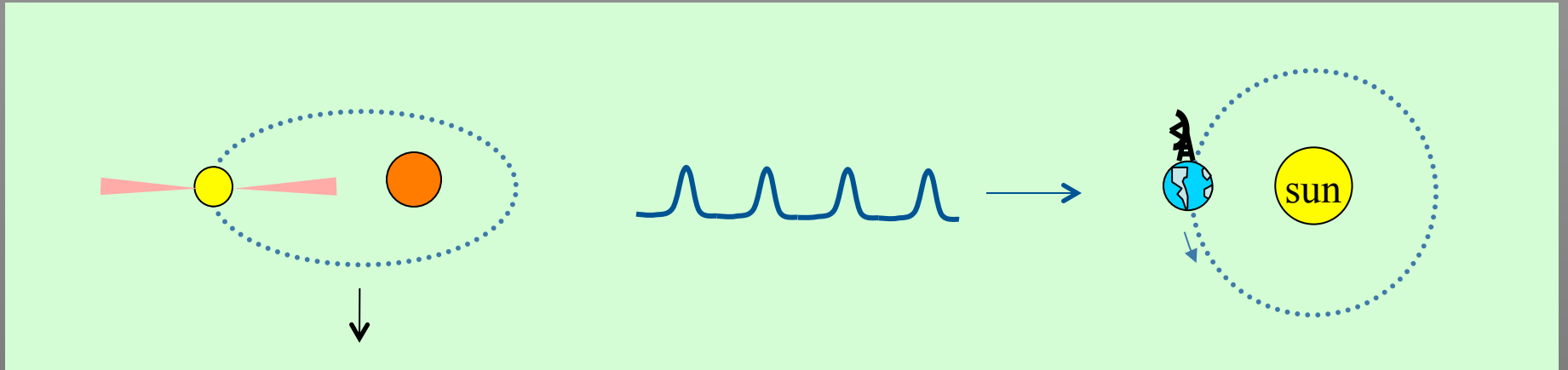


Gravitational Radiation

$$\dot{P}_b = - \left(\frac{192\pi}{5} \right) \frac{G^{5/3}}{c^5} \left(\frac{P_b}{2\pi} \right)^{-5/3} \left(1 + \frac{73}{24} e^2 + \frac{37}{96} e^4 \right) \frac{1}{(1-e^2)^{7/2}} \frac{m_1 m_2}{(m_1 + m_2)^{1/3}}$$

PSR J1713+0747 Shapiro Delay





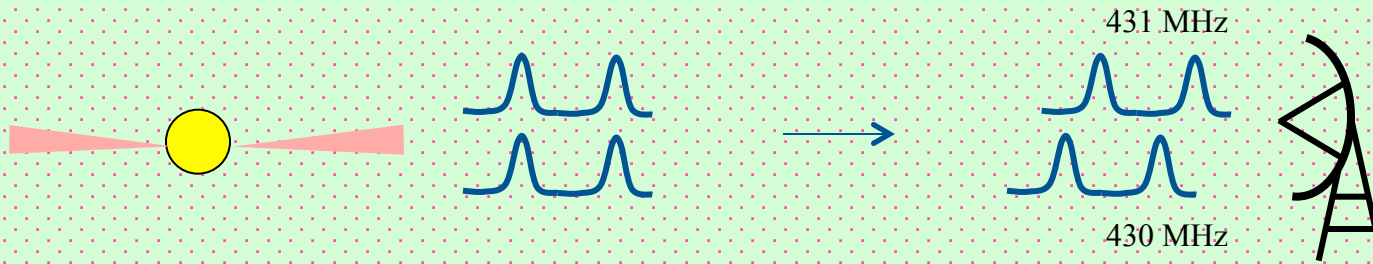
rotation period
 rotation period derivative

Keplerian orbital elements
 relativistic orbital elements

kinematic perturbations of
 orbital elements (secular and
 annual phenomena)

position
 proper motion
 parallax

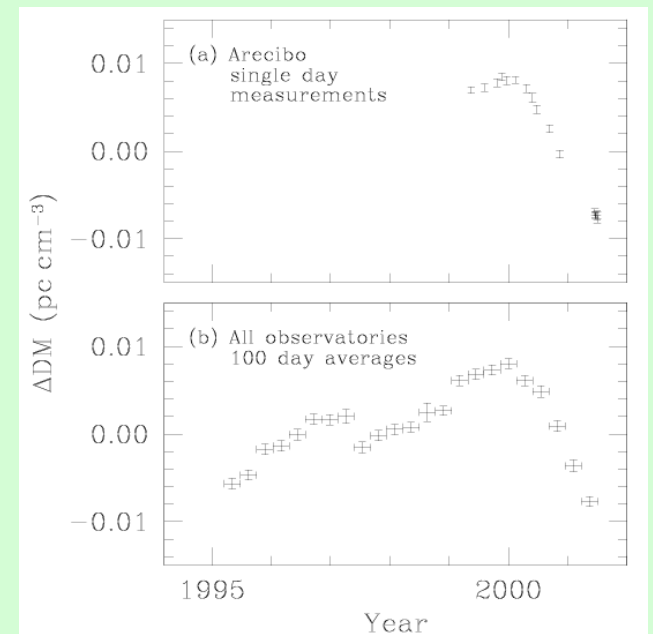
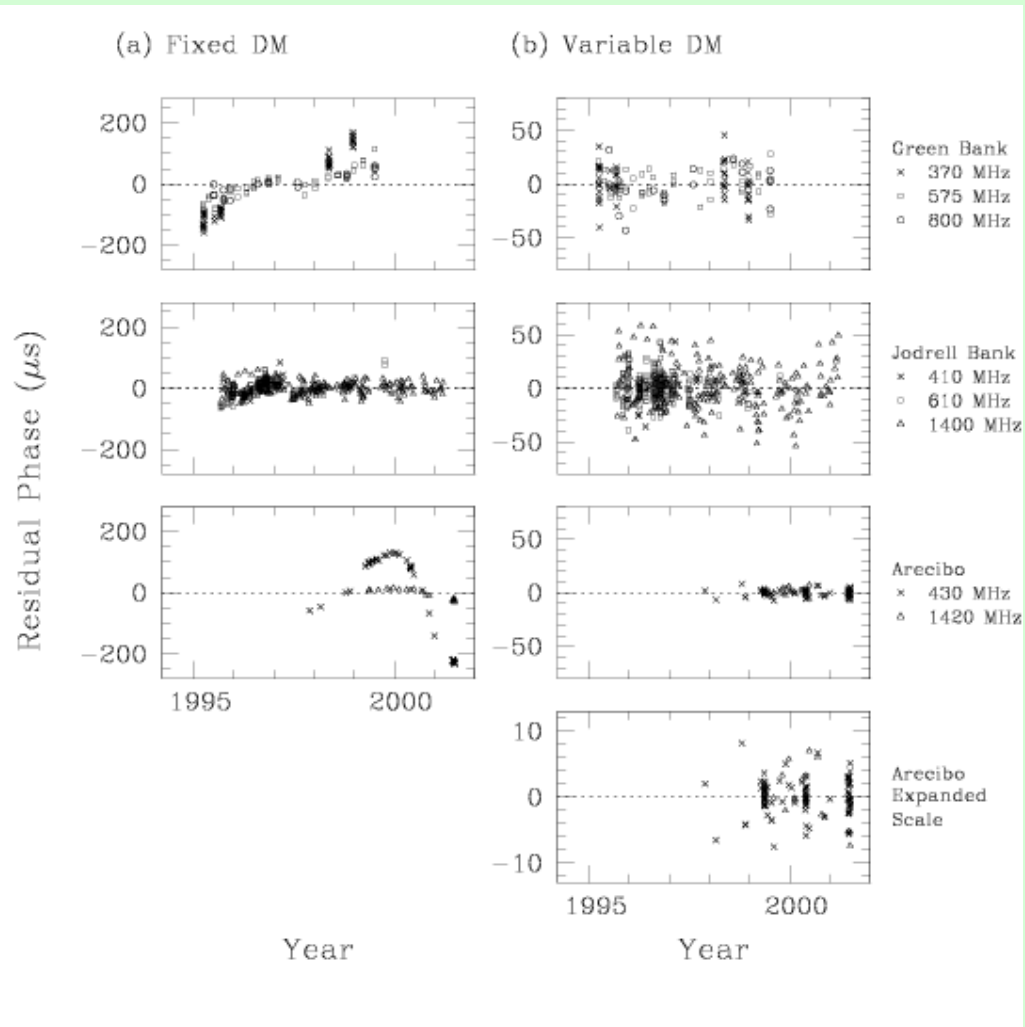
Interstellar Dispersion, Revisited



column density of electrons: $DM = \int n_e(l) dl$

excess propagation time: $t \text{ (sec)} = DM / 2.41 \times 10^{-4} [f(\text{MHz})]^2$

DM Variations in PSR J0621+1002 timing (Splaver et al., ApJ 581: 509, astro-ph/0208281)



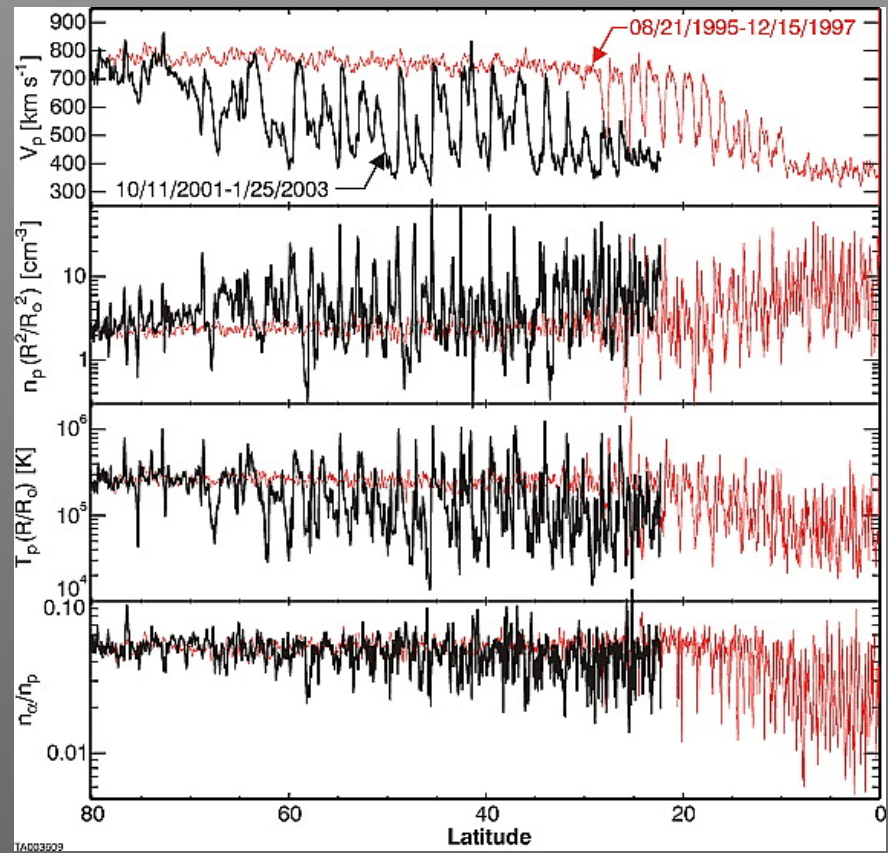
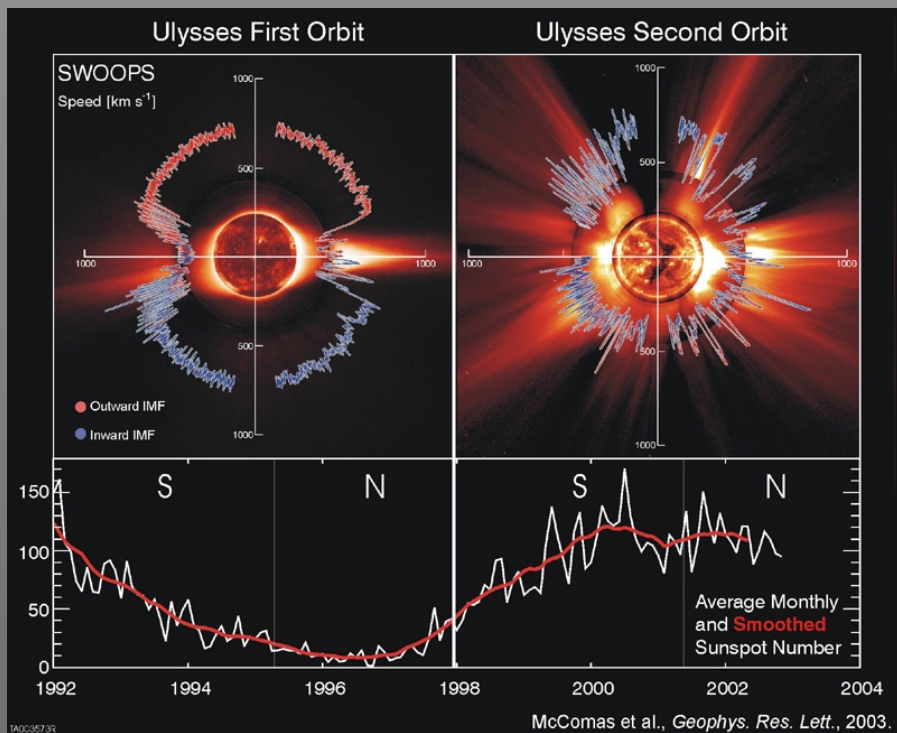
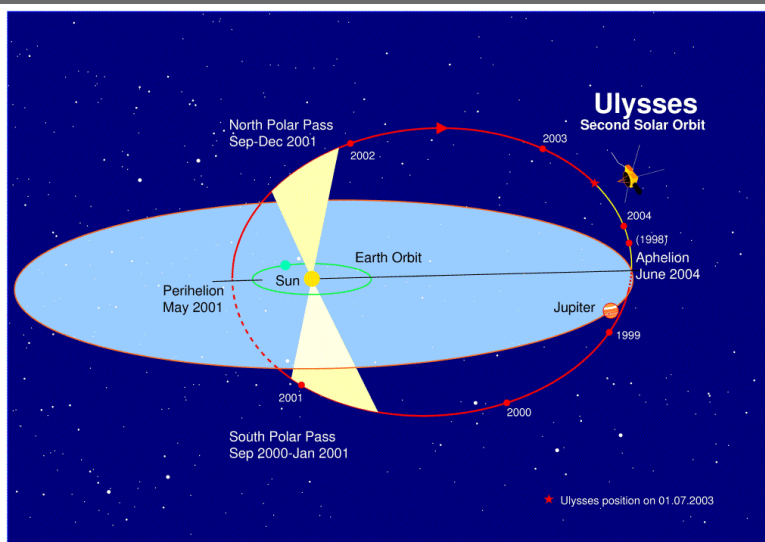
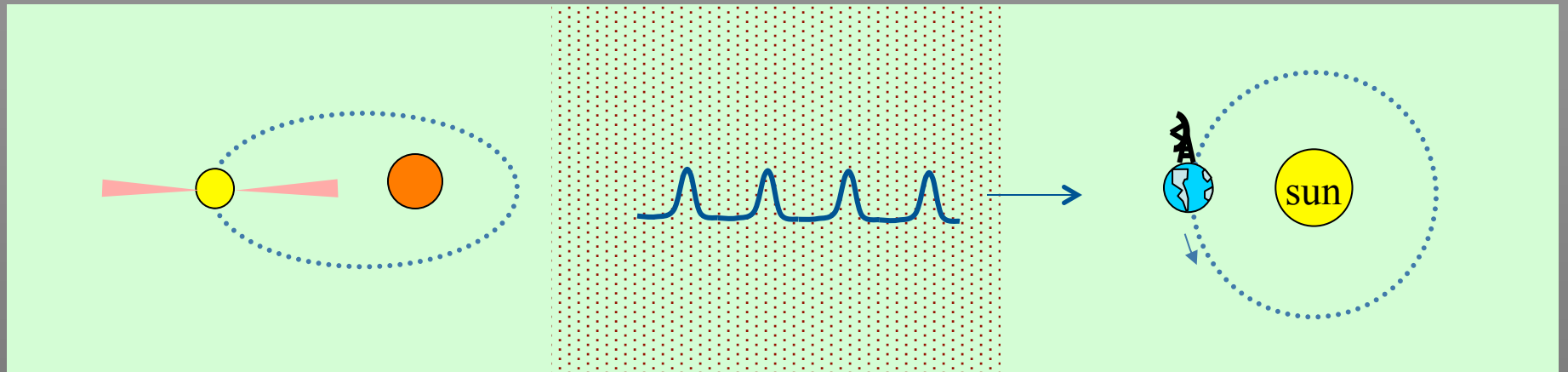


Figure 1. Polar plots of solar wind speed as a function of latitude for Ulysses' first two orbits. Sunspot number (bottom panel) shows that the first orbit occurred through the solar cycle declining phase and minimum while the second orbit spanned solar maximum. Both are plotted over solar images characteristic of solar minimum (8/17/96) and maximum (12/07/00); from the center out, these images are from the Solar and Heliospheric Observatory (SOHO) Extreme ultraviolet Imaging Telescope (Fe XII at 195 Å), the Mauna Loa K-coronameter (700950 nm), and the SOHO C2 Large Angle Spectrometric Coronagraph (white light)

Figure 2. Twelve-hour running averaged solar wind proton speed, scaled density and temperature, and alpha particle to proton ratio as a function of latitude for the most recent part of the Ulysses orbit (black line) and the equivalent portion from Ulysses' first orbit (red line).

D J McComas et al 2003. *Geophys Res Lett* 30, 1517





rotation period
rotation period derivative

Keplerian orbital elements
relativistic orbital elements

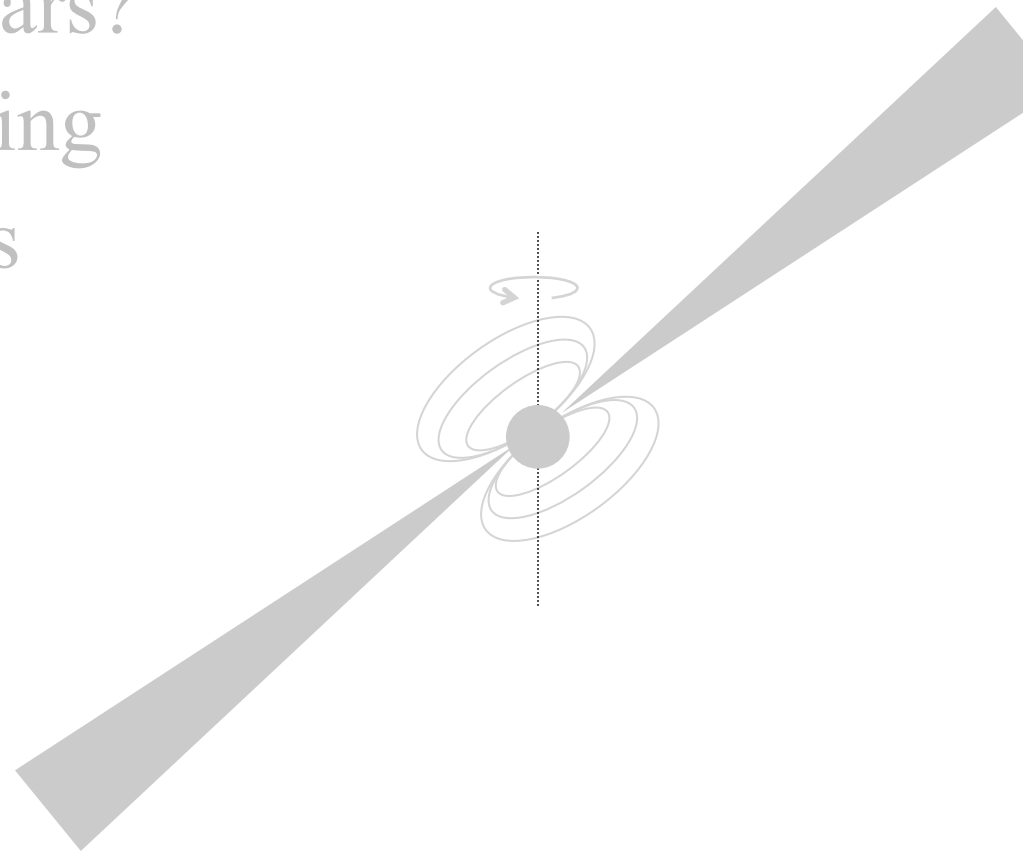
kinematic perturbations of
orbital elements (secular and
annual phenomena)

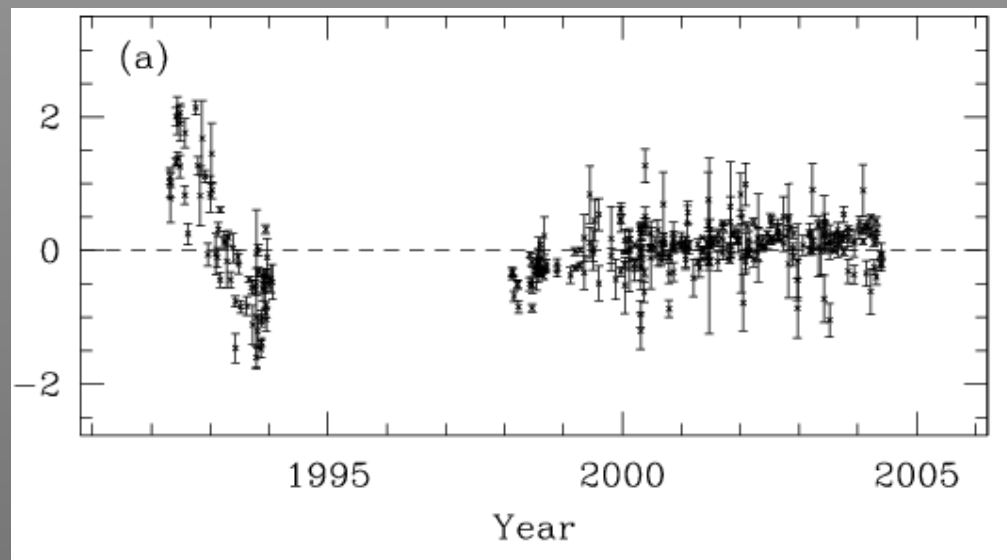
dispersion measure
dispersion meas. variations

position
proper motion
parallax

solar electron density

1. Extremely short overview
2. Measuring TOAs
3. What's in a timing model?
4. Uh oh, timing noise
5. Why millisecond pulsars?
6. Tempo; pulse numbering
7. Show me the residuals

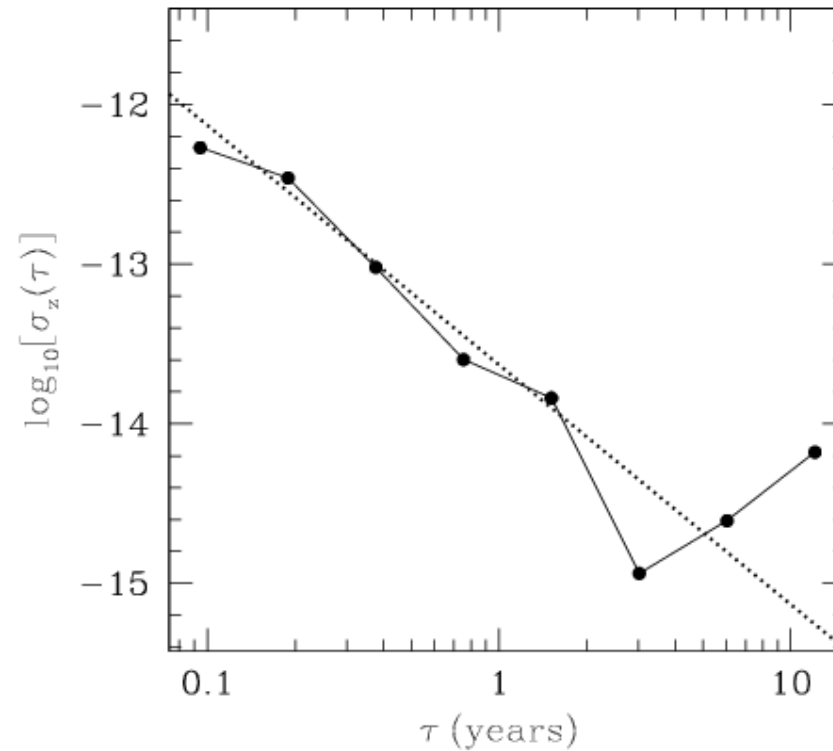




The examples so far are from 6 years of 1713+0747 data. If all the phenomena discussed so far are removed from those pulse arrival times, the remaining residuals look nearly flat.

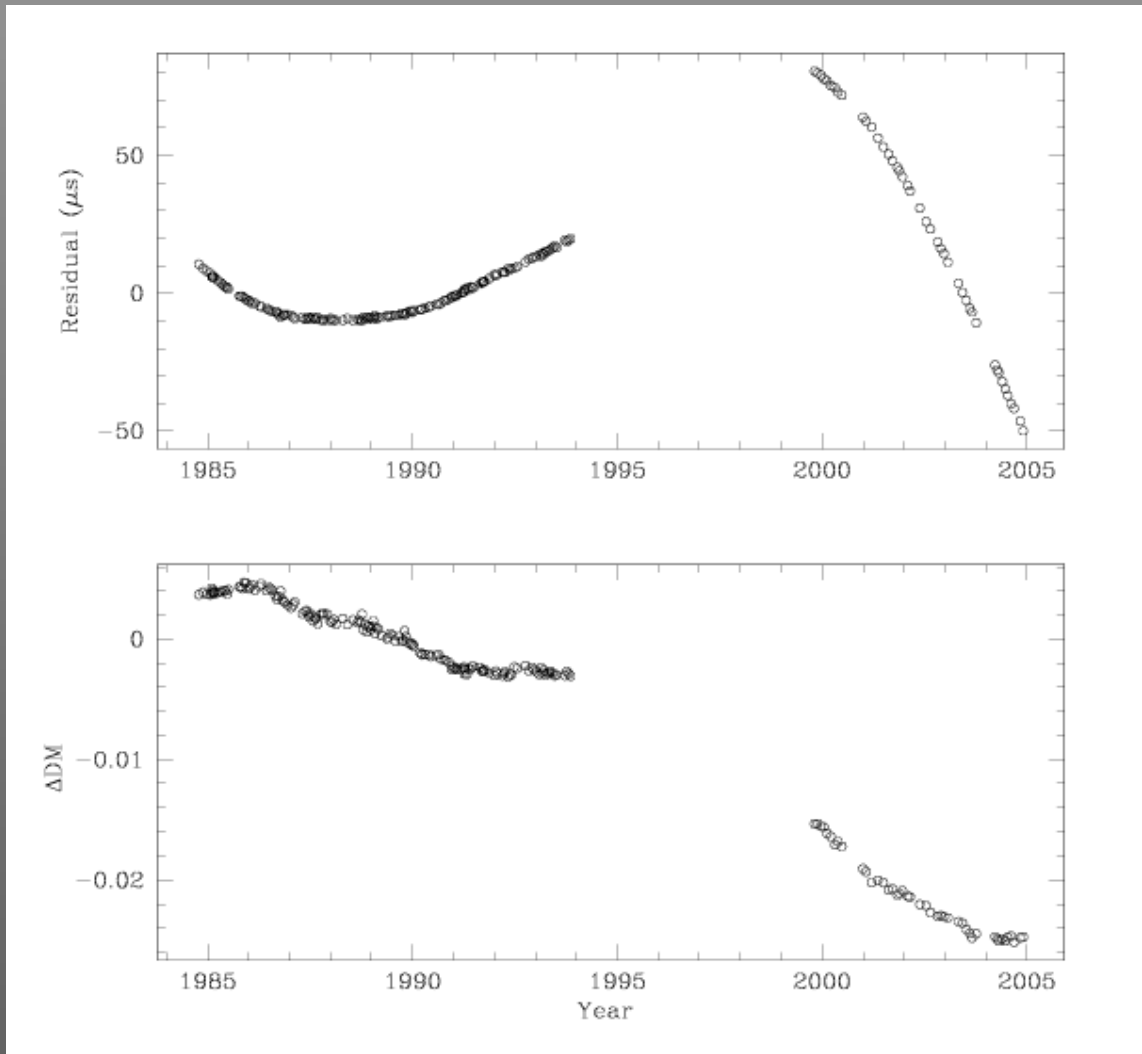
But: incorporating two years of additional data (taken several years earlier), shows that the residuals are *not* flat over longer time scales.

This is a common feature of pulsar timing data, called **timing noise**. Timing noise is probably indicative of irregularities in pulsar rotation; its physical origin remains unclear.

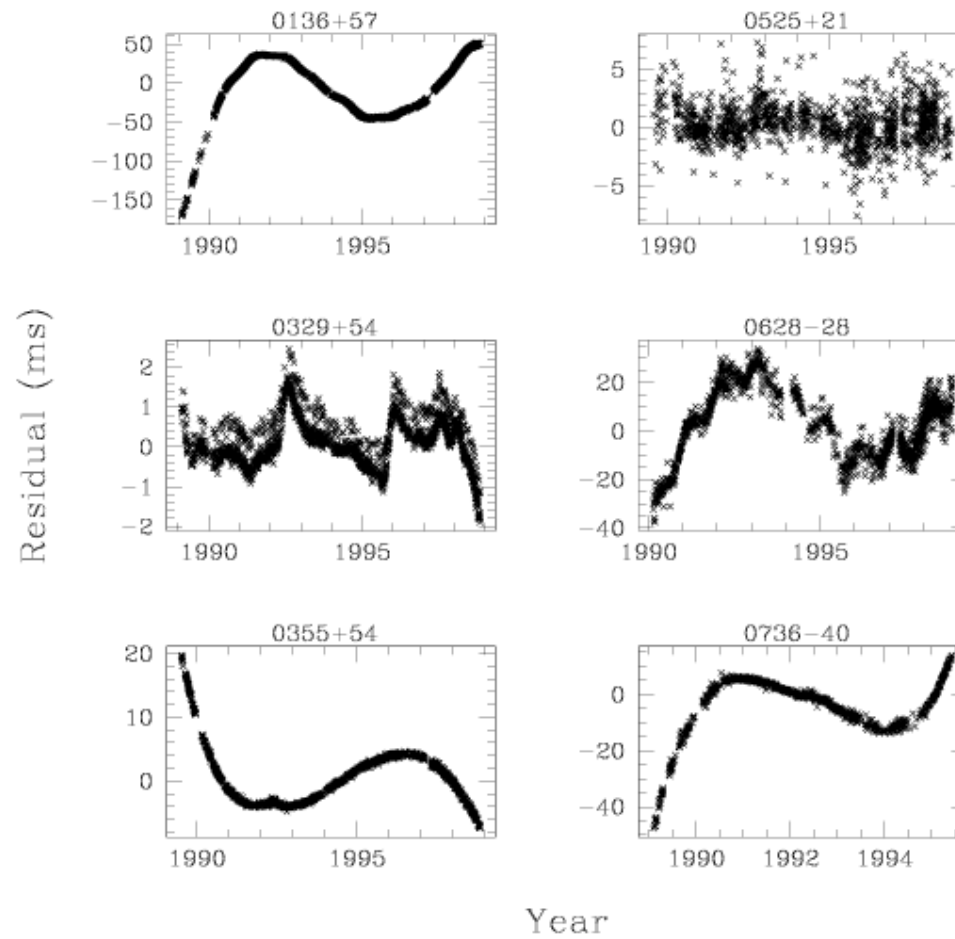


$$\sigma_z(\tau) = \frac{\tau^2 \langle c_3^2 \rangle}{2\sqrt{5}}, \quad r(t) = c_3(t-t_0)^3$$

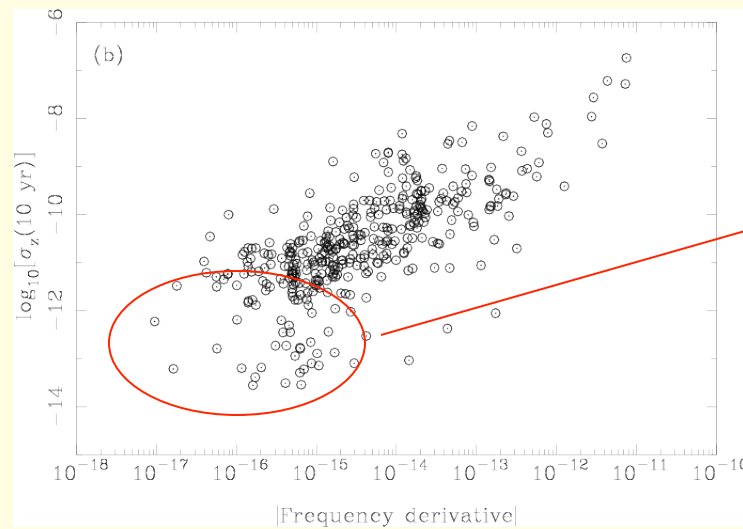
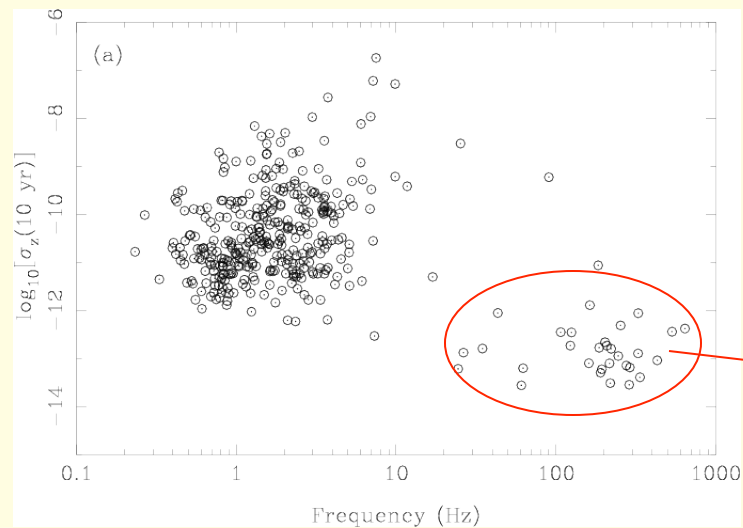
The σ_z timing noise statistic applied to J1713+0747



Timing noise and DM variations of the original millisecond pulsar, B1937+21



Timing noise in several young pulsars.



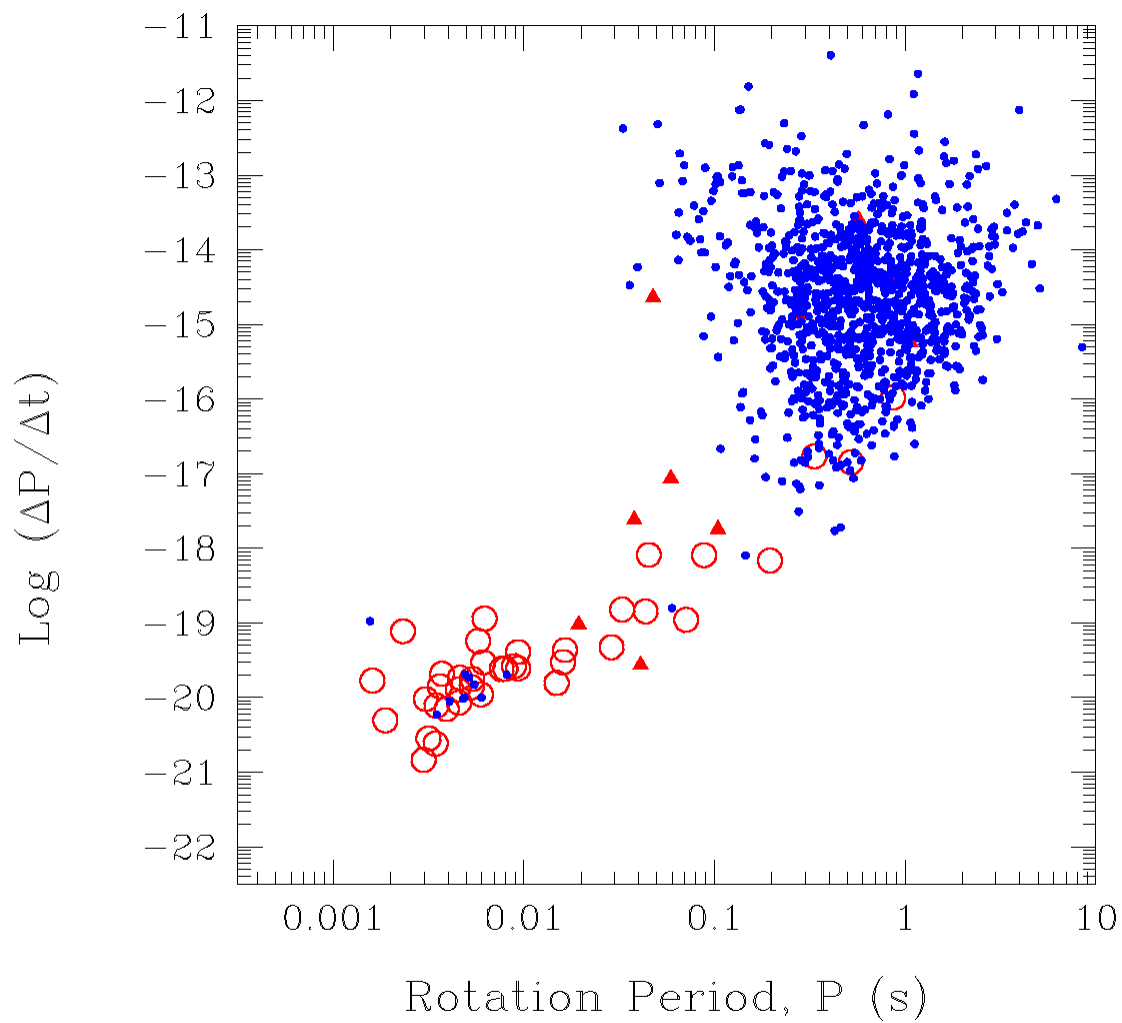
Correlation between timing noise and pulsar spin parameters.

Pulsars with short periods (high spin frequencies) have relatively little noise.

Pulsars with low spin-down rates (small frequency derivatives, small P-dots) have relatively little noise.

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• Single

• Binary

▲ High Eccentricity

○ Low Eccentricity

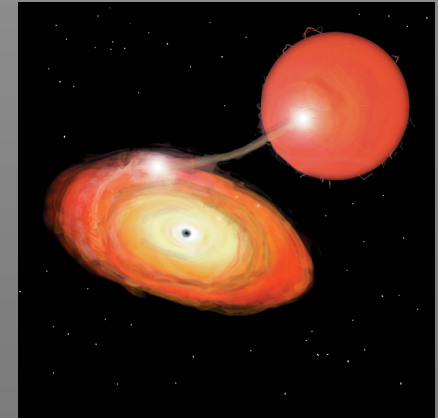
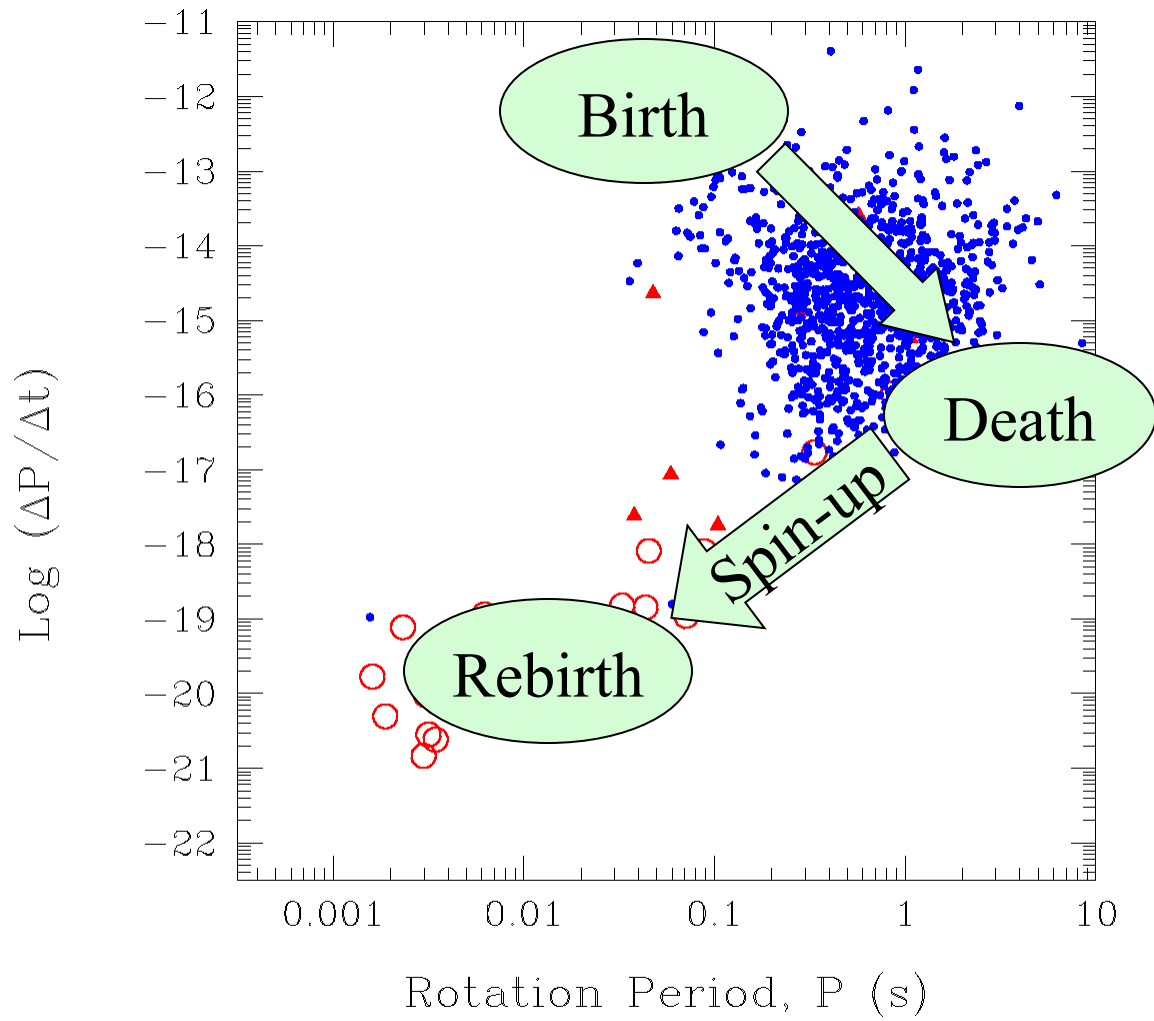
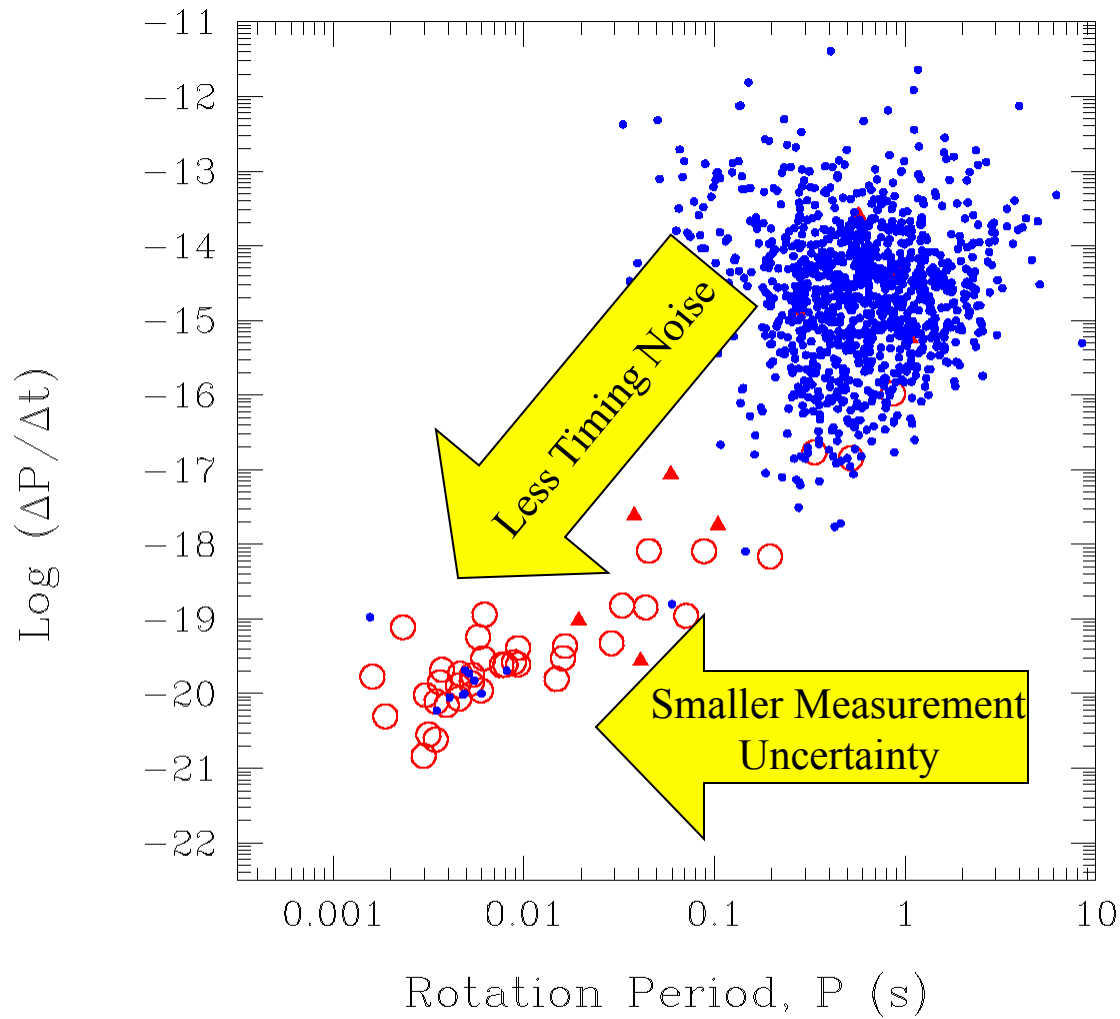


Figure: NASA

- Single
- Binary
- ▲ High Eccentricity
- Low Eccentricity

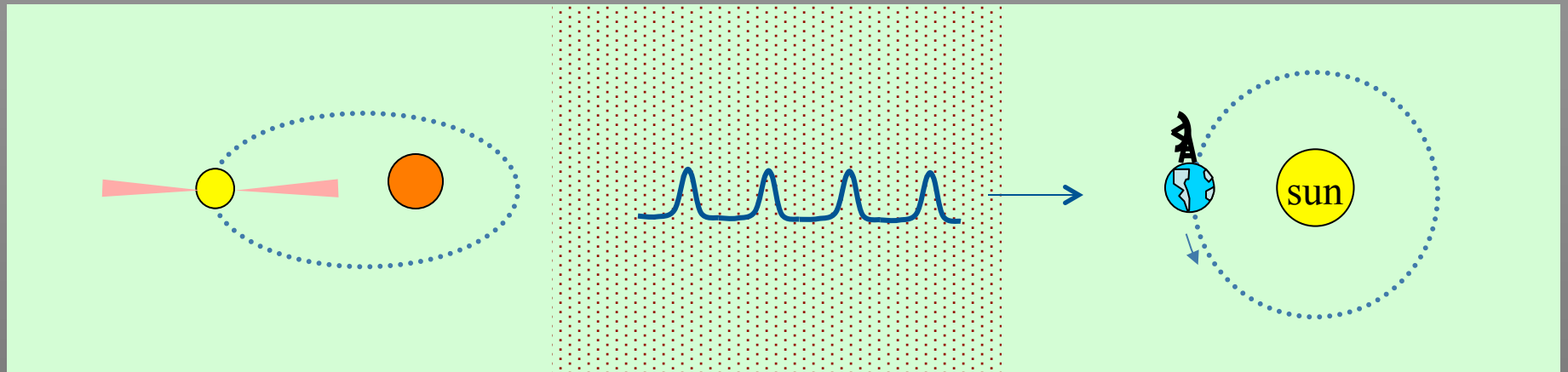


$$\sigma_{TOA} = \left(\frac{T_{sys}}{G} \right) \left(\frac{\eta}{S} \right) \frac{1}{\sqrt{\eta t B n_p}} \eta P$$

- Single
- Binary
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- Low Eccentricity

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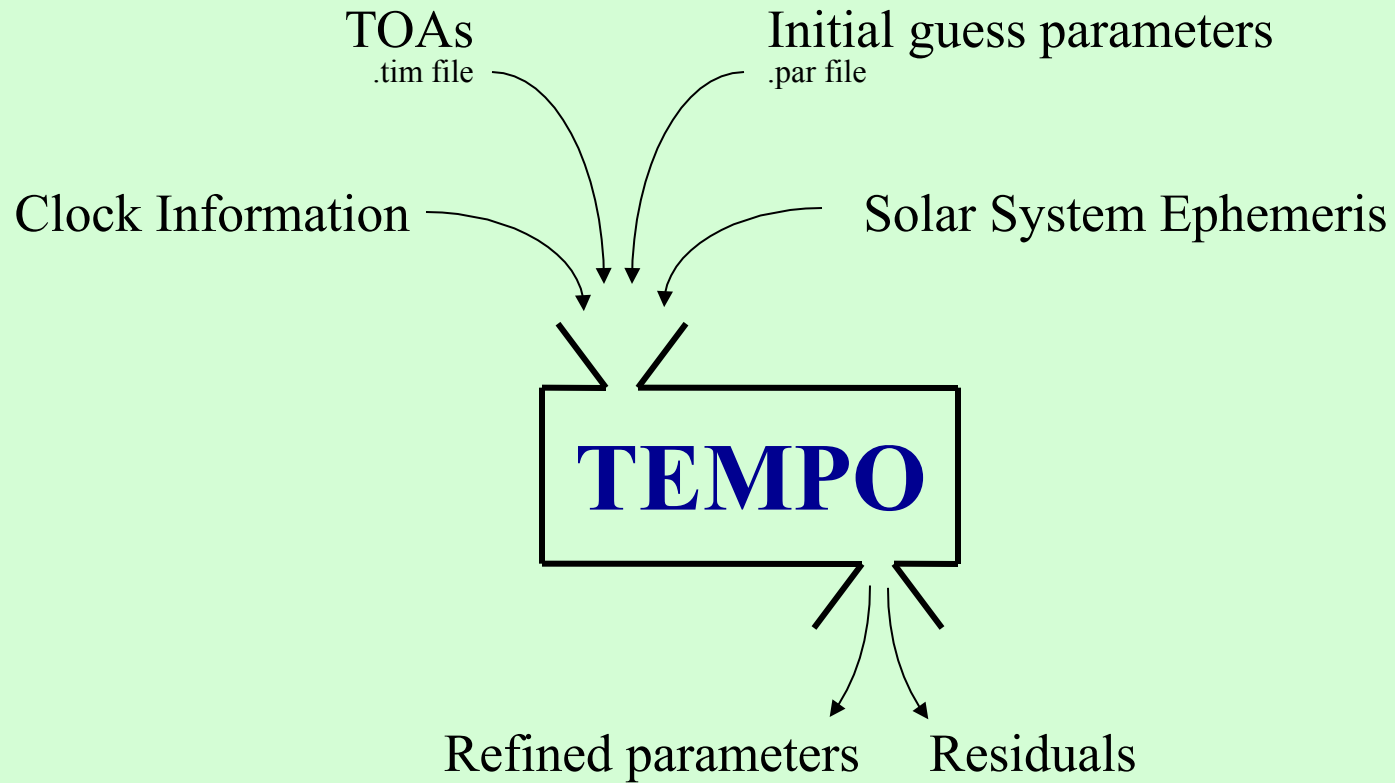
dispersion measure
dispersion meas. variations

position
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parallax

solar electron density

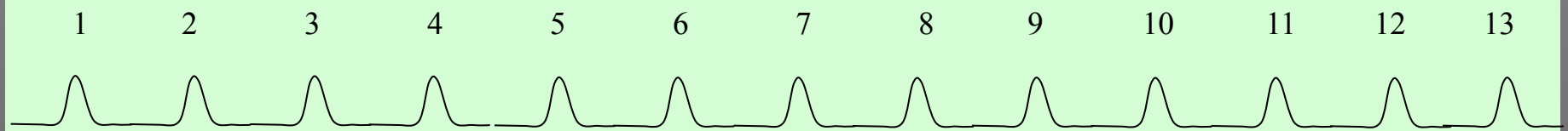
Fit for all of these simultaneously to find the best pulsar timing solution, then examine the residuals for signs of gravitational waves. (Or, better, fit for gravitational wave perturbations at the same time as fitting for all of the above parameters.)

Tempo and Tempo2



Pulse numbering and phase connection

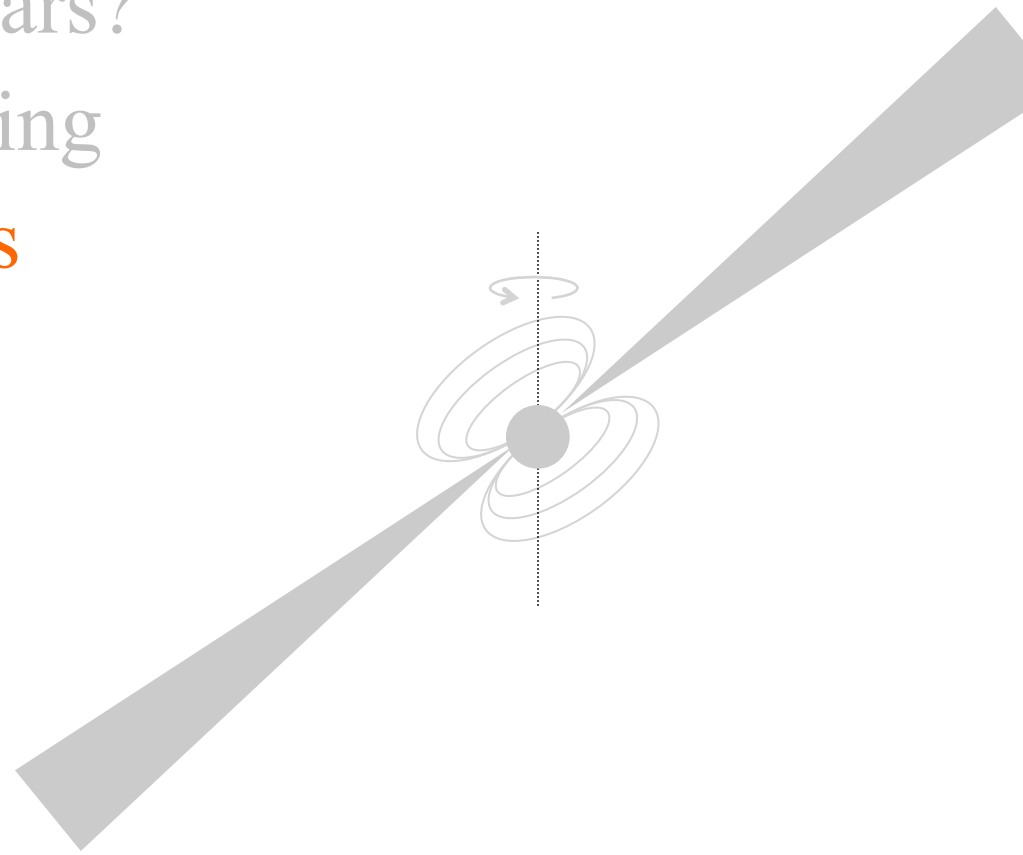
Based on “initial guess” parameters, Tempo assigns a **pulse number** to each measured pulse. It uses these numbers to compute when each pulse “should have” arrived at the telescope, and it refines the parameters to minimize the differences between the observed and computed arrival times. *Accurate pulse numbering is critical to this process.*

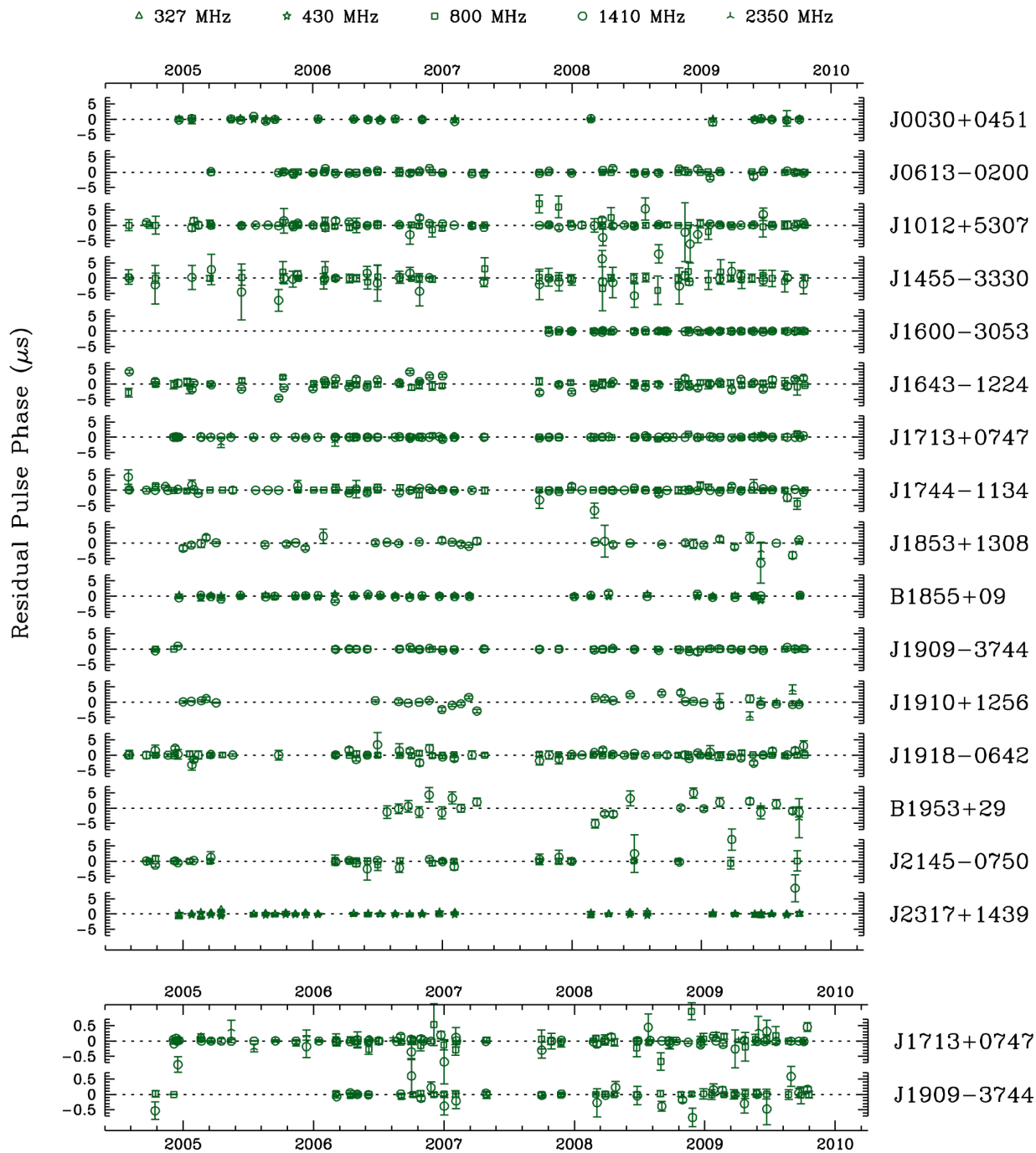


It is not necessary to observe every pulse. If there is a gap in the observed pulse series, it may be possible to accurately extrapolate the pulse numbering across the gap. This is called **phase connecting** the time series. This is challenging to do when a pulsar is newly discovered and the pulsar’s parameters are not well known. It becomes easier as the parameters become established. Gaps of several weeks (~1 billion pulses) are routine, and phase connection can be maintained over gaps of years for a steady millisecond pulsar.



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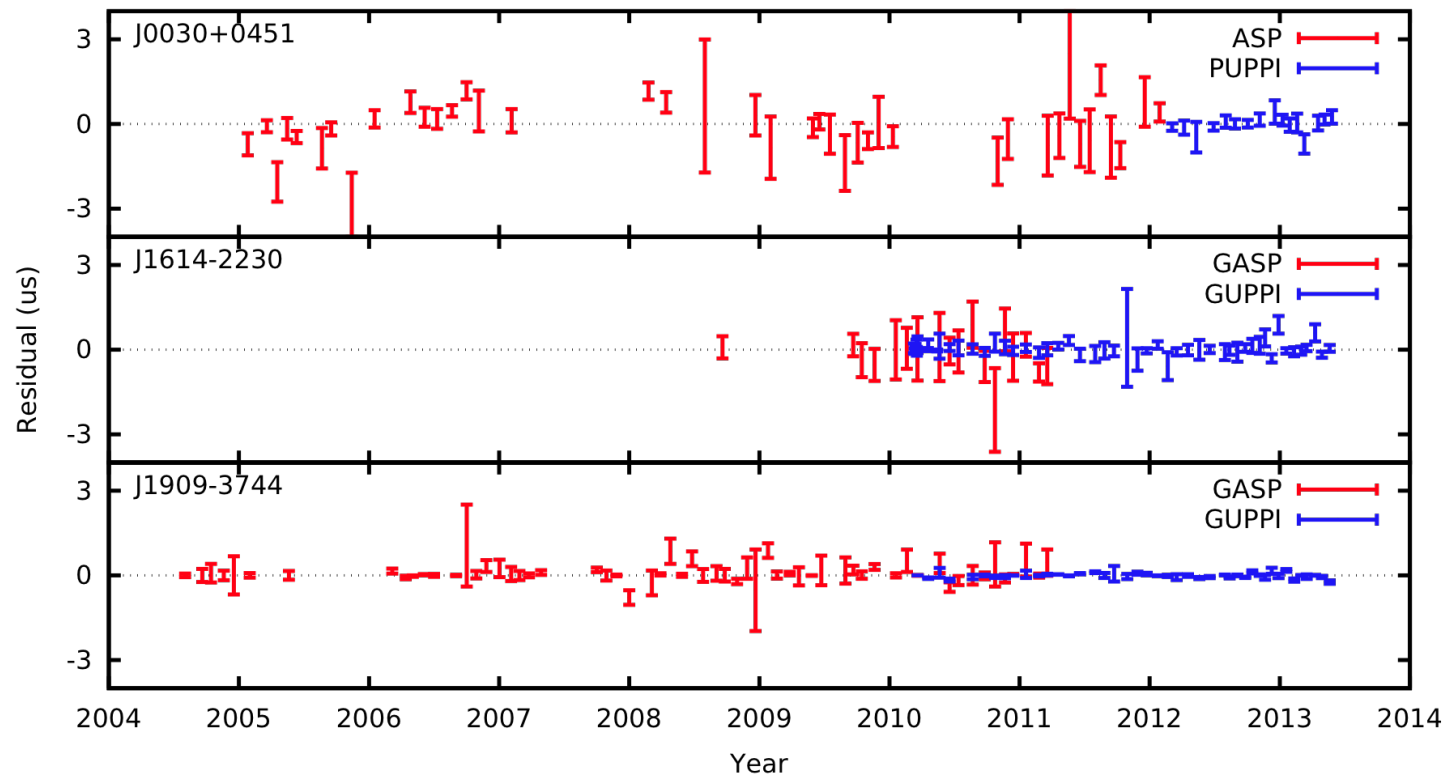




Show Me The Residuals

Five years of timing of sixteen pulsars at Arecibo and Green Bank using the ASP/GASP coherent dedispersion timing systems, showing sub-microsecond residuals for most sources.

Show Me The Residuals



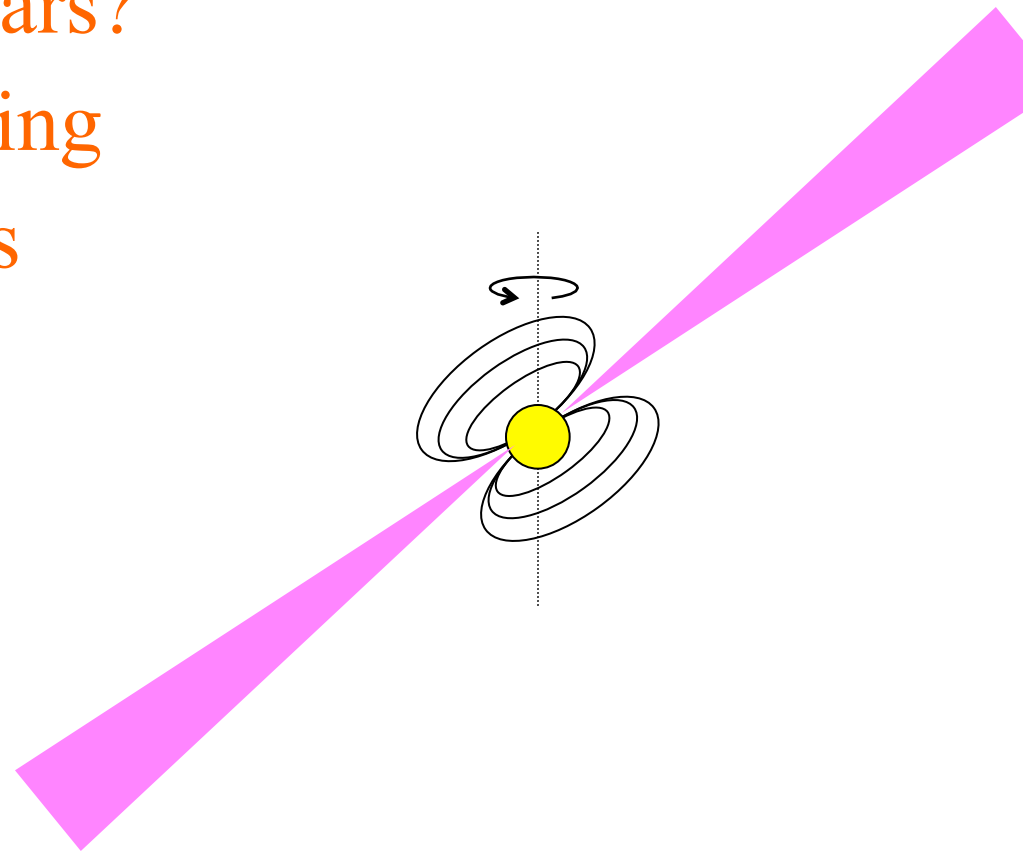
Timing improvement using modern data acquisition machines with very wide bandwidths (i.e., able to simultaneously collect data over a very wide range of the radio spectrum.)

GASP/ASP 64 MHz

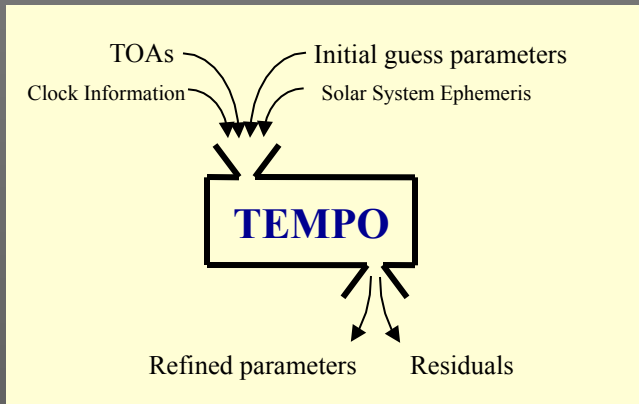
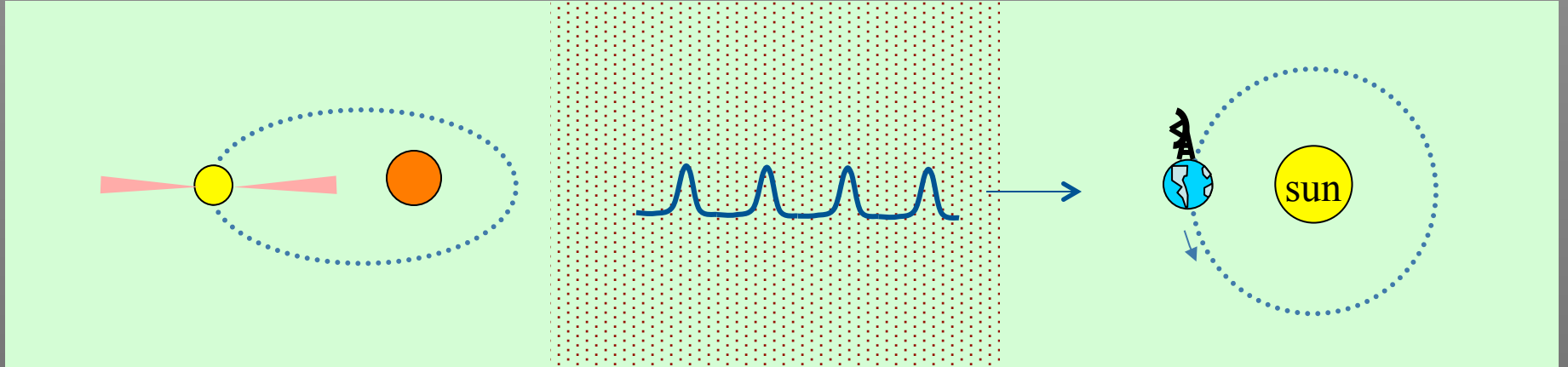
GUPPI/PUPPI 800 MHz

NANOGrav data, work in progress

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Summary



a	3751	1518+49	370.000	50942.02369981804596	69.1	9-May-98
a	3751	1518+49	370.000	50942.02508871578912	74.9	9-May-98
a	3752	1518+49	370.000	50942.02710263928441	107.8	9-May-98

